A MEAN-REVERTING MODEL TO CREATE MACROECONOMIC SCENARIOS FOR CREDIT RISK MODELS

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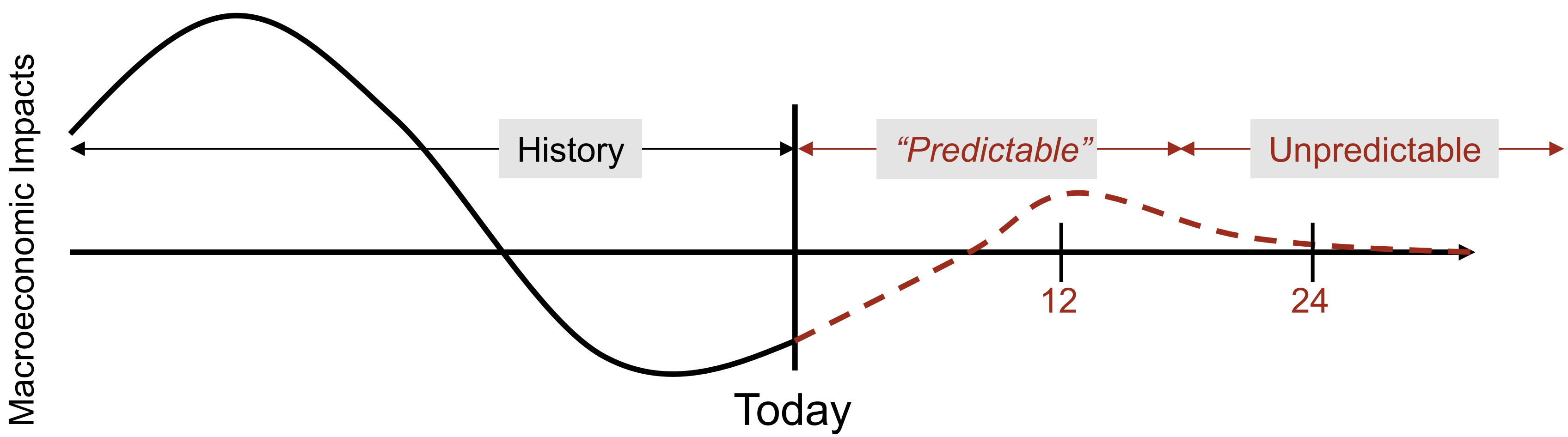
Goals

- Proposed FASB guidelines on loss reserves require lifetime loss forecasts.
- Loan pricing requires a lifetime loss forecast.
- Lifetime loss forecasts require long-range economic scenarios.
- We want a simple way to generate macroeconomic scenarios that start with current conditions and relax onto long-run averages.

Scenarios for Long-term Forecasting

- Use your best macroeconomic assumptions for the first 12 months.
- Relax to the mean over the next 12 months.
- Use the long-term historic average beyond 24 months.



Options for Creating Scenarios

- Damped extrapolation
  - Choose an appealing function
- Mean-reverting process
  - Can provide both mean and variance
  - Creates PIT and TTC economic capital estimates

Method of Analysis

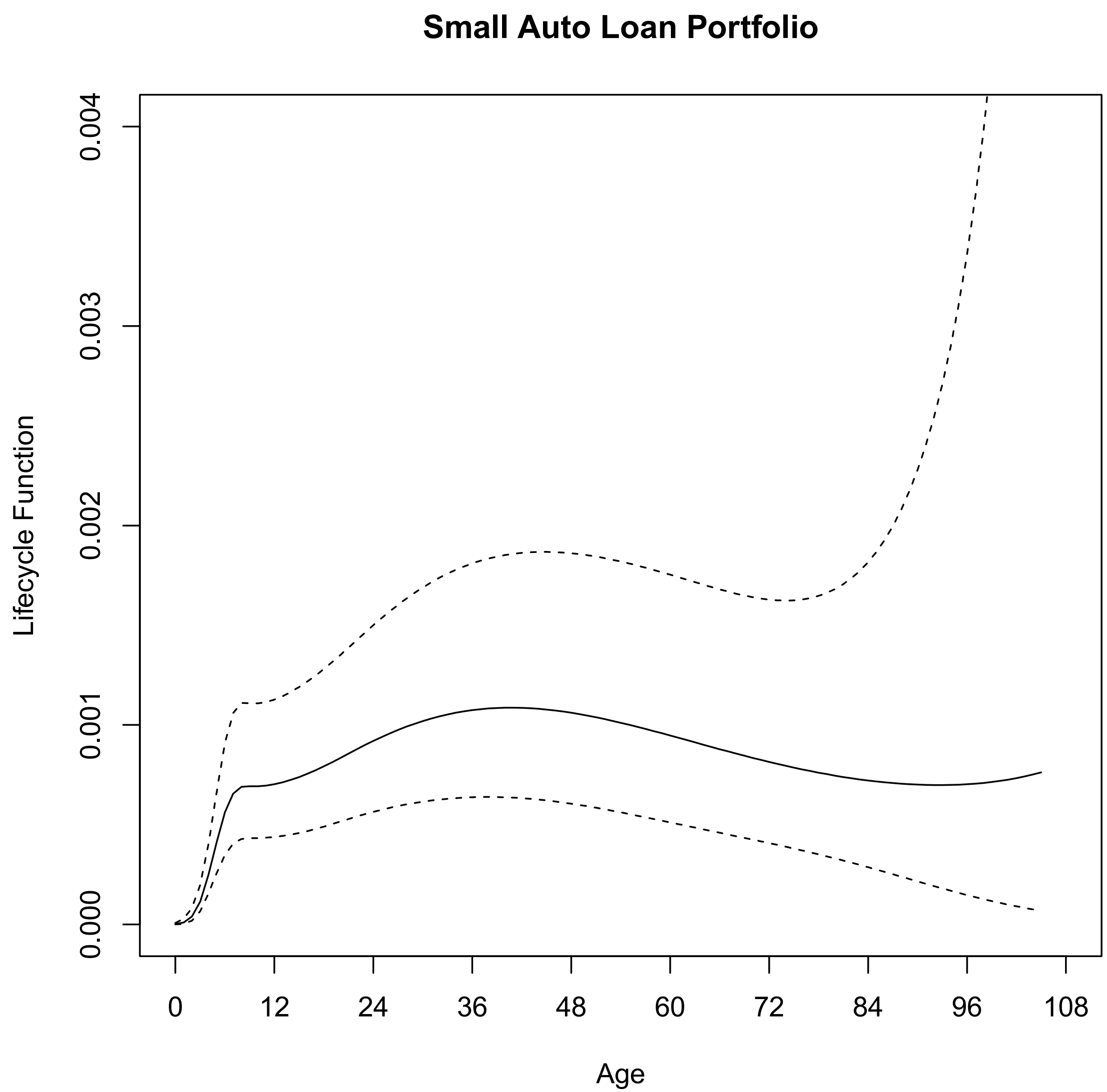
1. Create a forecast model that accepts macroeconomic scenarios.
2. Collect the macroeconomic sensitivity of the credit risk model into a single index.
3. Calibrate a mean-reverting model to the macroeconomic sensitivity index.
4. Obtain a macroeconomic scenario from economist(s).
5. Overlay the mean-reverting model onto the macroeconomic scenario.

1. Create a Forecast Model

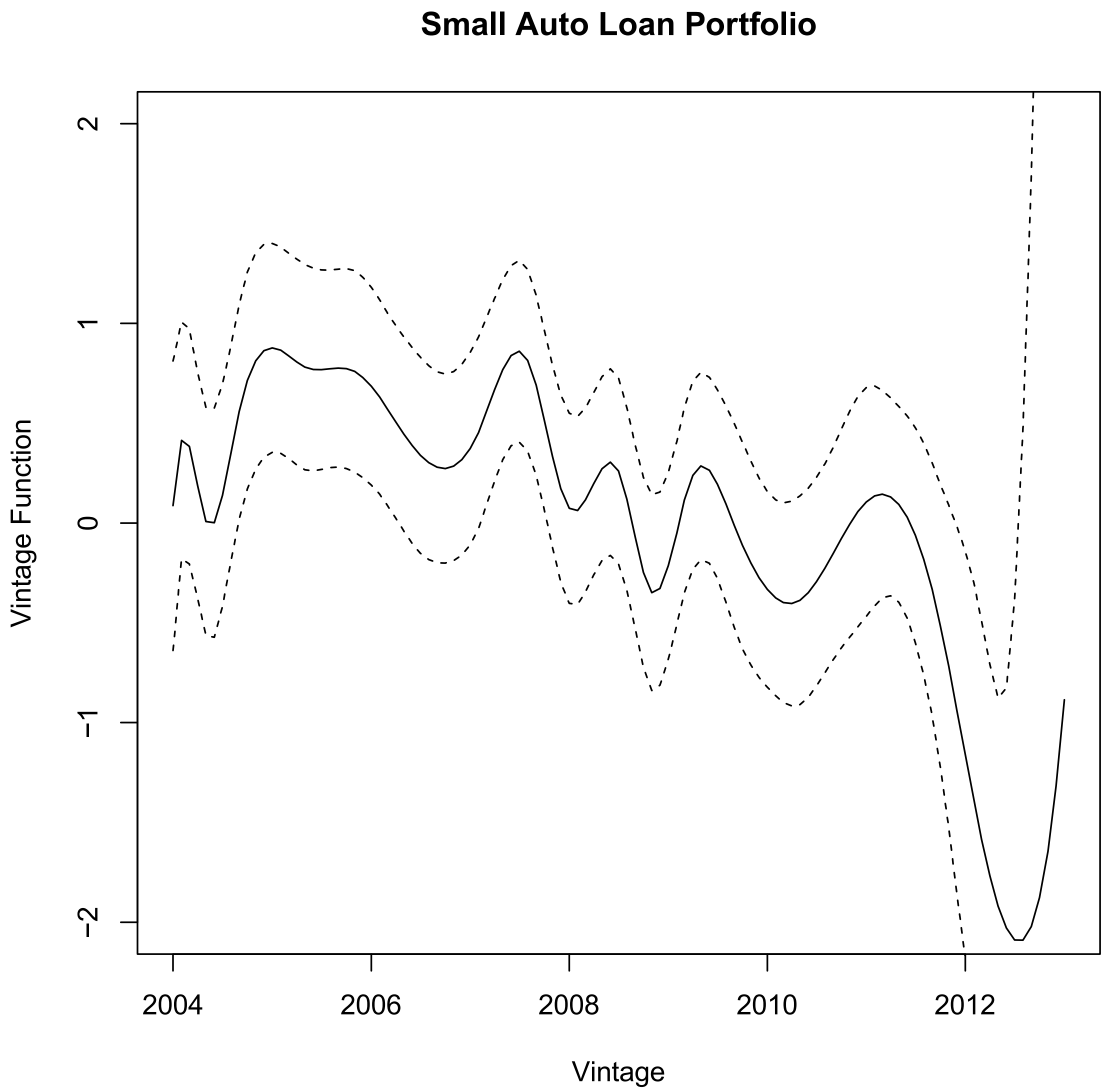
- For lifetime forecasting, choose a model that captures the lifecycle(a), environment(t), and credit risk(v).
- In this example, we use a GLM-AVT model. A loan-level version of an Age-Period-Cohort model.

$$\log\left(\frac{p_i(a,v,t)}{1-p_i(a,v,t)}\right) = f(a)+g(v)+h(v), \quad a=t-v$$

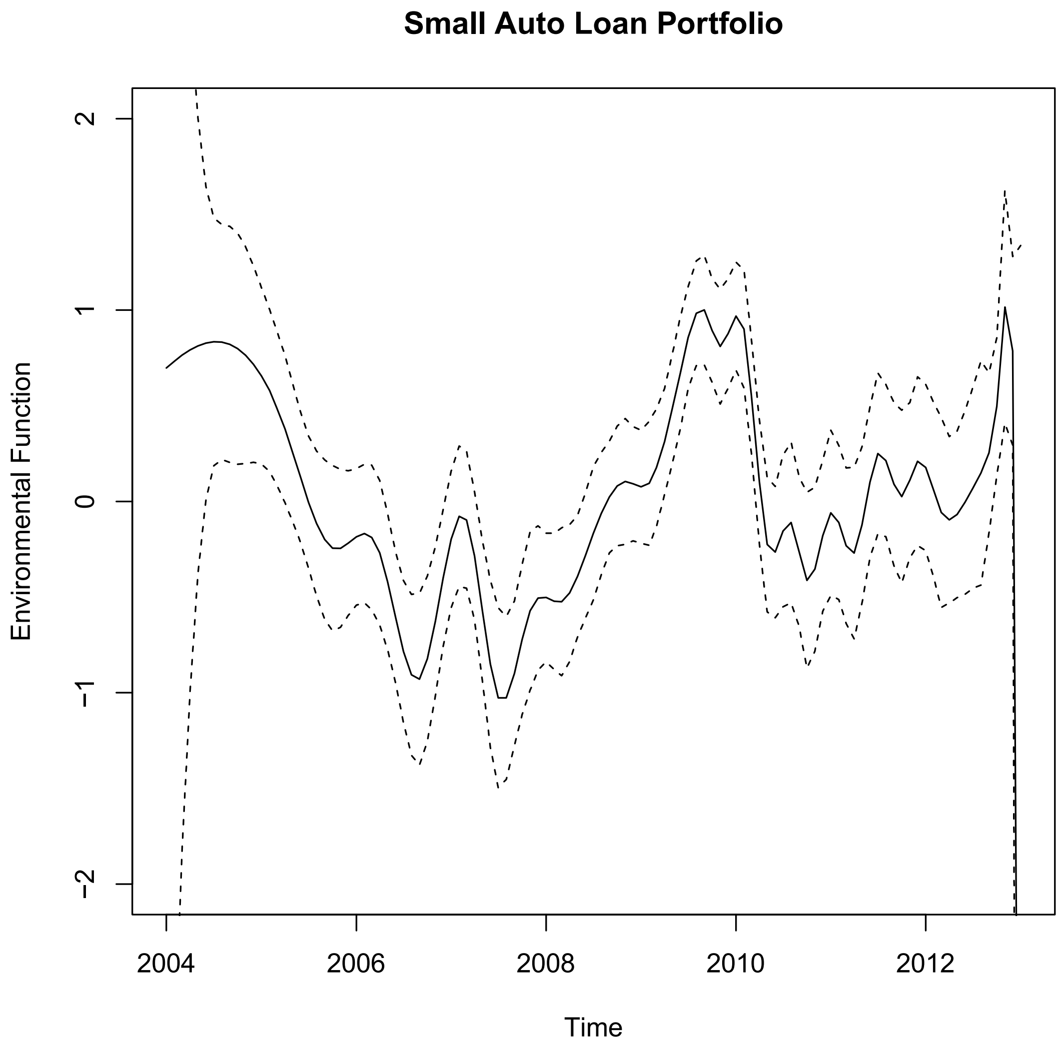
Lifecycle Function Example



Credit Risk Function Example



Environment Function Example



2. Create a Macroeconomic Sensitivity Index

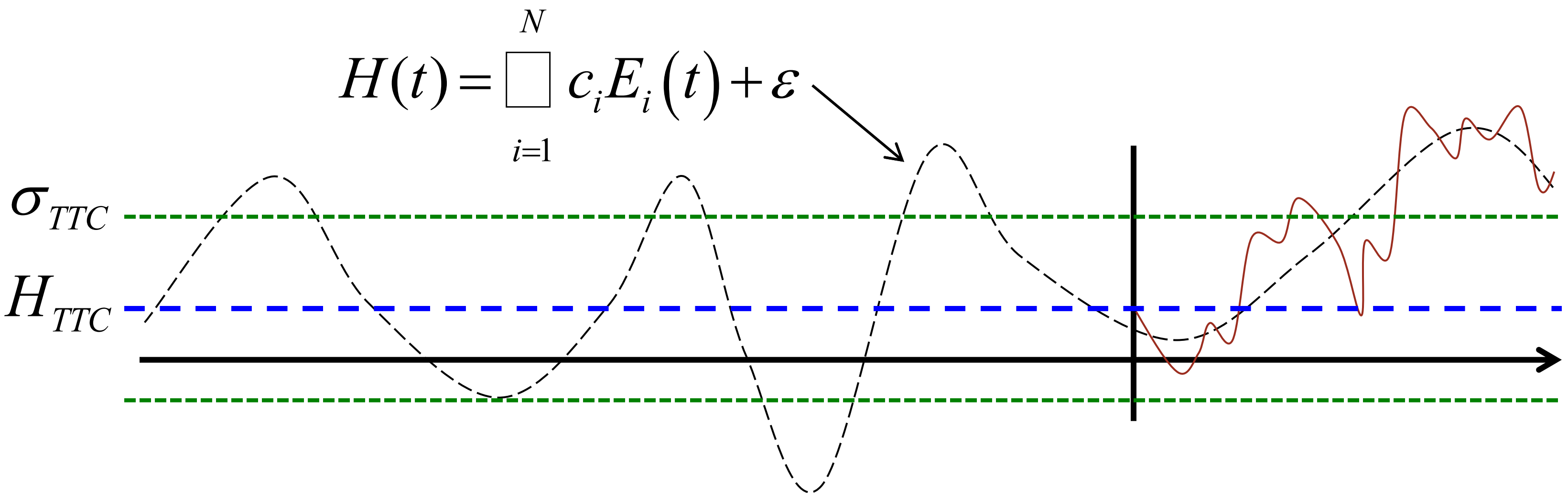
- The method applies to any model that accepts macroeconomic inputs such that they can be collected into single function:
  - Logistic regression
  - Cox PH
  - GLM
  - Age-Period-Cohort
  - GLM-AVT
  - LR-AVT

- Define $$H'(t) = \sum_{i=1}^{N} c_i E_i\left(t\right) + \varepsilon_t$$

as the environmental function – the macroeconomic sensitivity index. Assumes $H'(t)$ is normally distributed.

Through-the-Cycle Estimates

- Extrapolate the macroeconomic sensitivity backward through previous economic environments.
- Compute the TTC values from the fit to macroeconomic data *H'(t)*, not the short amount of observed history *H(t)*.



3. Calibrate a Mean-Reverting Model

- Most common is the Ornstein-Uhlenbeck process:

$$dx_t = \theta(\mu - x_t)dt + \sigma dW_t$$

- Where x_t is the time series being simulated, μ is the long run mean, σ is the deviation about this mean, θ is the relaxation rate to the mean, and W_t is a Wiener process.
- The Vasicek model is an Ornstein-Uhlenbeck process.
- The Ornstein-Uhlenbeck model is the only stationary, Gaussian, and Markovian model possible.

Mean-Reverting Models in Discrete Time

- Our data is always discrete time, and almost always uniformly spaced in time.
- A discrete-time simplification is

$$\Delta x_t = \theta(\mu - x_t)\Delta t + \varepsilon_t$$

$$\mu = d - \frac{\sigma^2}{2\theta}, \ \ \varepsilon_t \approx N(0,\sigma)$$

- Where d is the drift term. From this, we get

$$E(x_t) = (1 - e^{-\theta(t-t_0)})\mu + e^{-\theta(t-t_0)} x_{t_0} \xrightarrow[t\to\infty]{} \mu$$

$$Var(x_t) = \frac{\sigma^2}{2\theta}(1 - e^{-2\theta(t-t_0)}) \xrightarrow[t\to\infty]{} \frac{\sigma^2}{2\theta}$$

Mean-Reverting Models with A-V-T

- Model the environmental impacts

$$\Delta H(t) = \theta(\mu - H(t))\Delta t + \varepsilon_t, \;\; \varepsilon_t \approx N(0,\sigma)$$

$$E(H(t)) = (1 - e^{-\theta(t-t_0)})\mu + e^{-\theta(t-t_0)} H(t_0) \xrightarrow[t\to\infty]{} \mu = H_{TTC}$$

$$Var(H(t)) = \frac{\sigma^2}{2\theta}(1 - e^{-2\theta(t-t_0)}) \xrightarrow[t\to\infty]{} \frac{\sigma^2}{2\theta} = \sigma^2_{TTC}$$

- The previous Through-the-Cycle analysis provides H_{TTC} and σ^2_{TTC} .

Connecting PiT and TTC

- The half-time (time to relax half-way to the mean) is

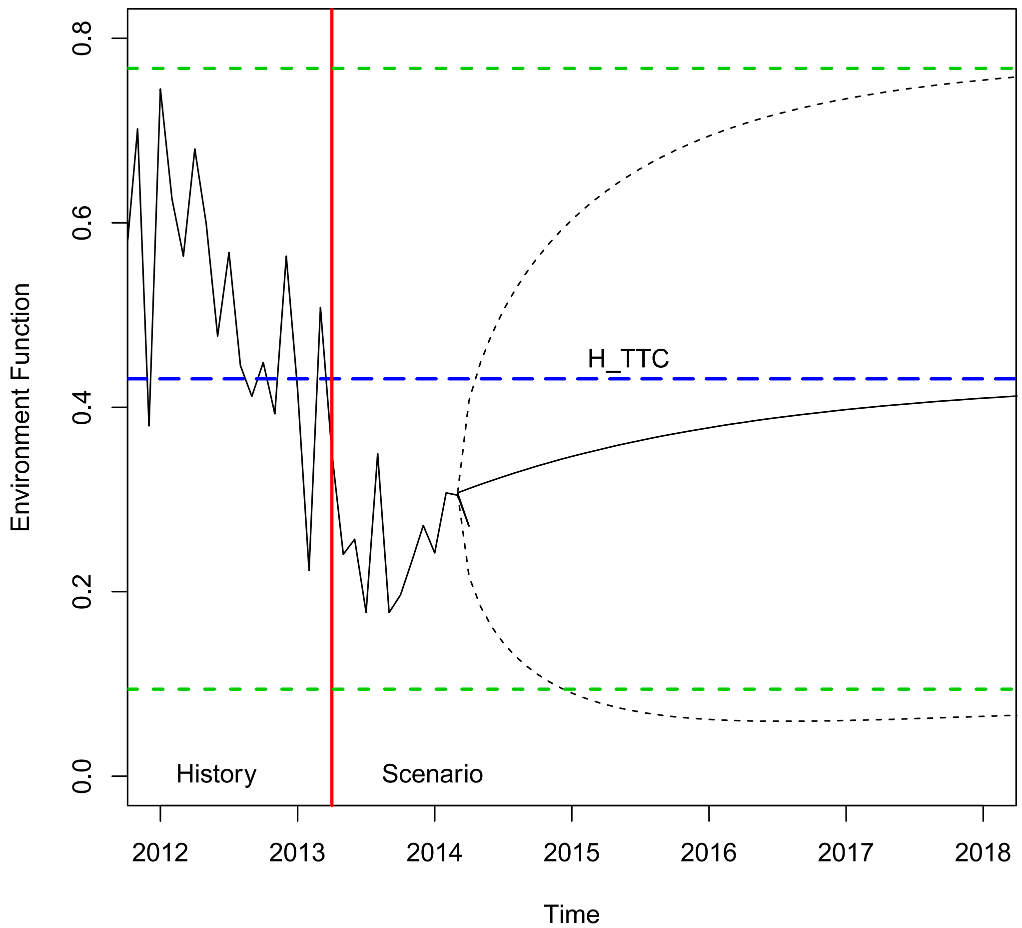
$$t_{1/2} = \frac{\ln(2)}{\theta}$$

- If we assume that $t_{1/2}$ is approximately 1.5 years (based upon Monte Carlo experiments), then

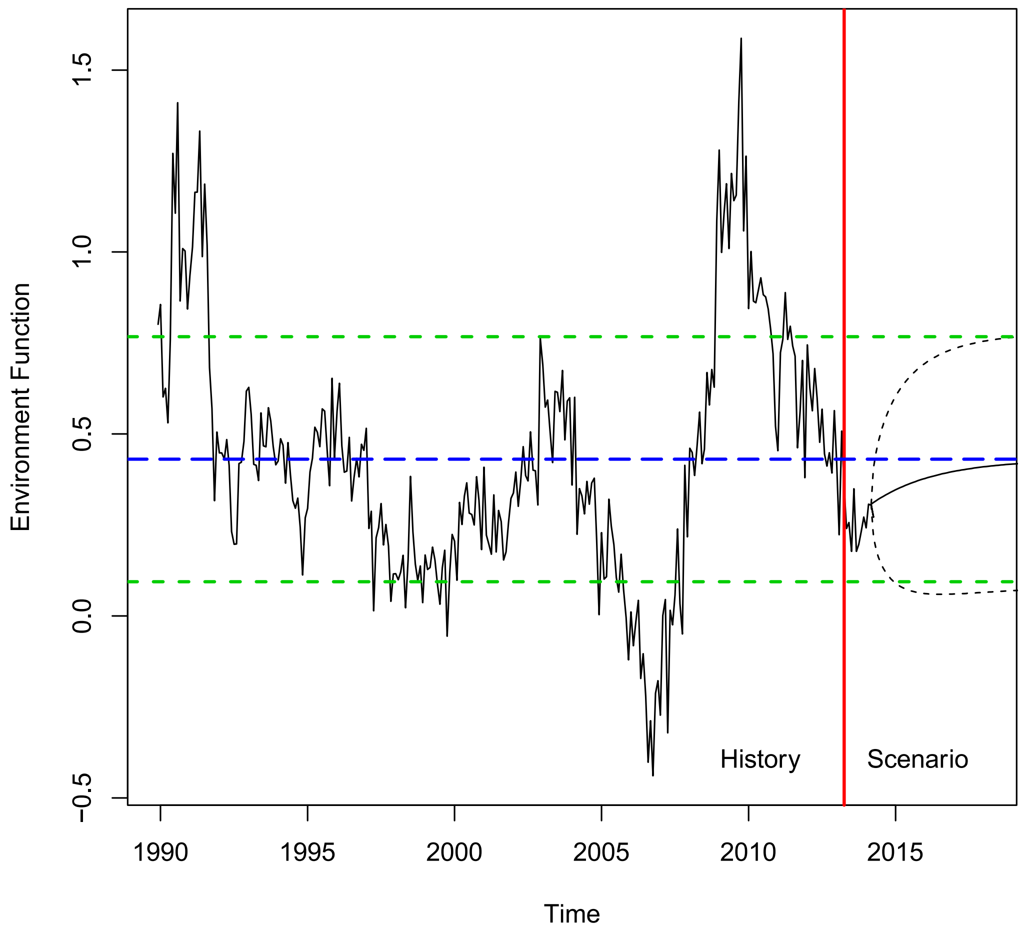
$$\sigma^2 = \frac{4}{3}\ln(2)\sigma_{TTC}{}^2, \quad \mu = H_{TTC}$$

- And we have a fully specified mean-reverting model.

5. Application of Mean-Reverting Model



5. Application of Mean-Reverting Model



Conclusions

- This does a reasonable job at creating a distribution of possible futures.
  - Relaxes only the TTC mean.
  - Converges to the TTC variance.
- Can be used to
  - Generate a single mean scenario for forecasting or pricing.
  - Generate extreme scenarios for PIT and TTC economic capital.

Model Extensions

- Markovian processes depend only on the current state.
- Macroeconomic time series show autocorrelation out to 6 to 12 months.
- A more accurate model might include lagged dependencies.
- Such models are significantly more complex.

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