

Analytical Solutions to Multi-period Credit Portfolio Management: A Macroeconomic Approach

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Analytical Solutions to Multi-period Credit Portfolio Management: A Macroeconomic Approach

- **The problem we are solving**

Overcome the challenge of analyzing credit risk dynamically (multi-period) and integrate stress testing (credit risk, PPNR, provisions, mark-to-market) into a unified management framework.

- **Motivation**

Federal Reserve Bank of New York - Staff Report No. 696 – Nov 2014
Supervisory Stress Tests by Beverly Hirtle & Andreas Lehnert.

“The Basel Market Risk Amendment – finalized in 1995 – contained a provision encouraging the use of stress tests to augment the Value at Risk (VaR) measures of computing risk-weighted assets, the denominators in various measures of risk-based capital. VaR models consider the probability distribution of the value of a portfolio of assets. In principle VaR models can be thought of as the result of thousands of individual scenarios, weighted by their probability. In practice however the distributions are not tied to real-world variables other than the observed empirical distributions of the values of various assets.”

Analytical Solutions to Multi-Period Credit Portfolio Management (cont.)

Why is this relevant for banks?

(i) embed stress testing process into the banks' credit management practices (forecasting, provisioning, profitability, risk-based pricing, risk concentration, capital adequacy), (ii) dynamic risk and profitability assessment, (iii) optimal capital management, (iv) risk appetite.

Specifics of what we do

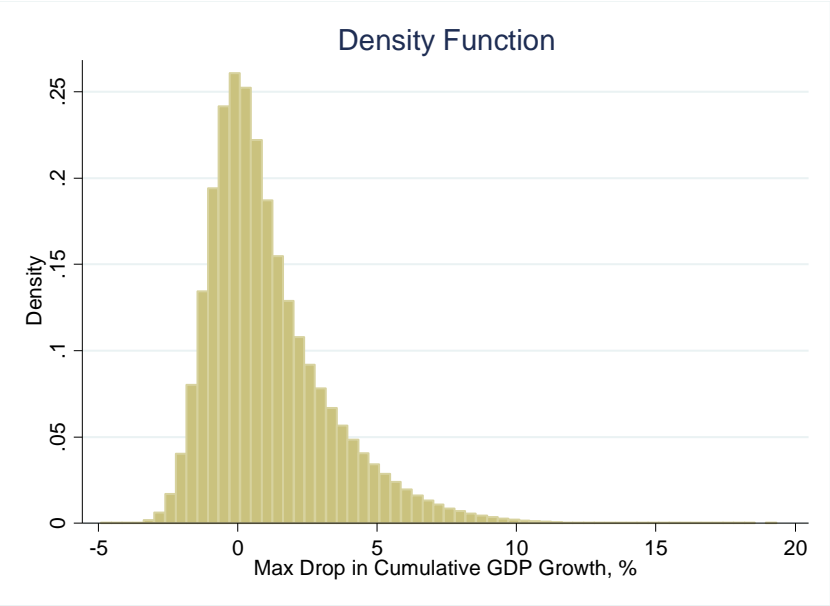
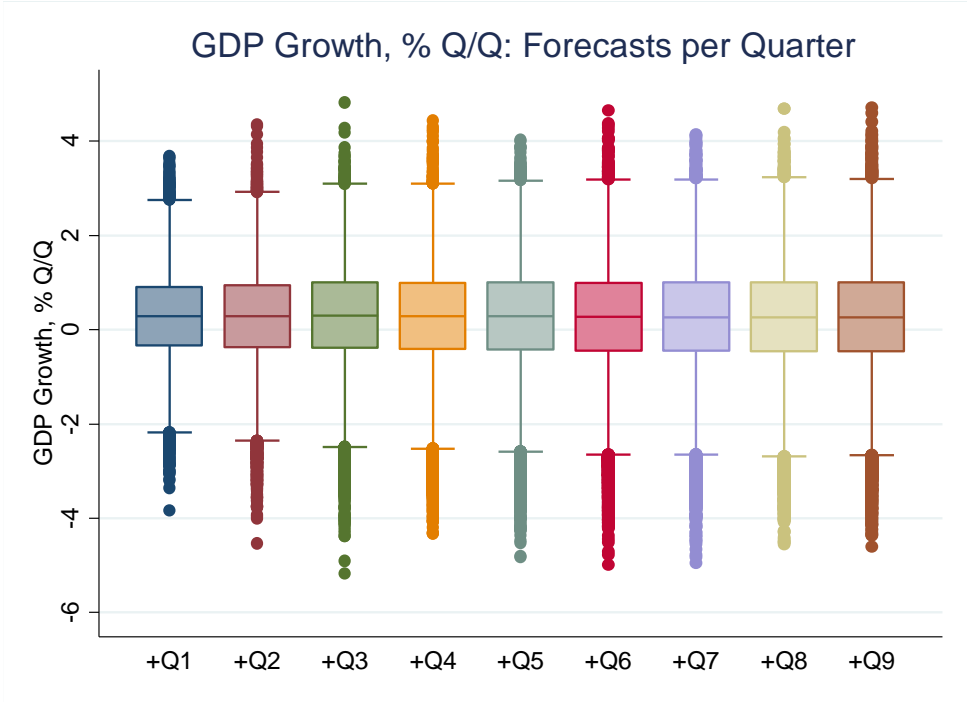
Compute **analytical** solutions to multi-period credit risk management.

- (1) Dynamic simulations of macro scenarios **and probabilities for each path**.
- (2) Multi-period risk parameter simulations (leveraging stress testing models) for conditional credit and mark-to-market parameter realizations.
- (3) Analytical calculation of (intra-period and cumulative) expected and unexpected losses, and the corresponding asset contributions.
- (4) Scenario-specific analysis: embedding ST into the framework.
- (5) Further applications: risk-based pricing, concentration and tail-risk analysis, analytical reverse stress testing, dynamic optimization, integrated mark-to-market and default risk assessment, enhanced Monte Carlo methods.

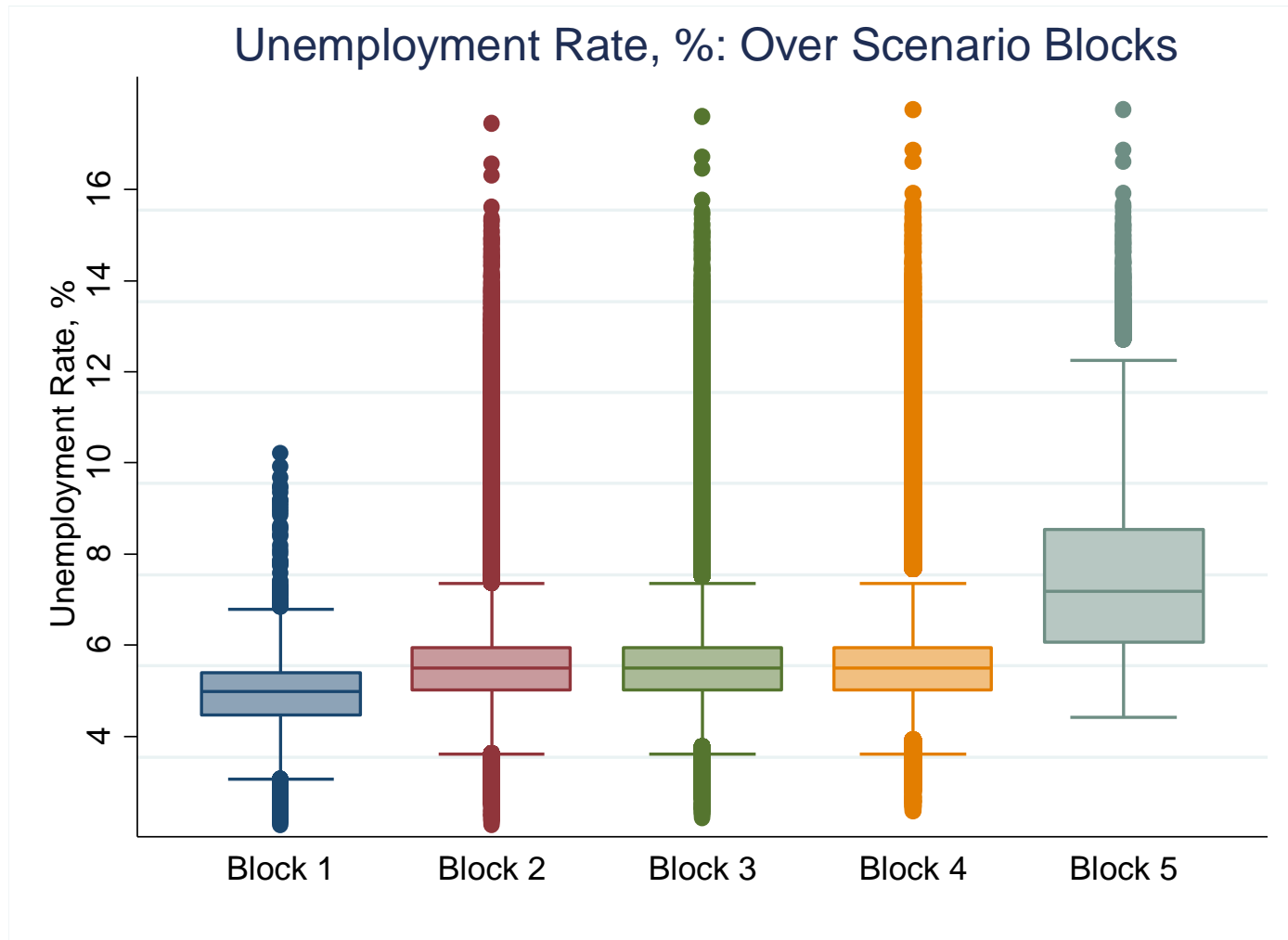
1

Dynamic simulations of macro scenarios and probabilities for each path

Severity of Scenarios



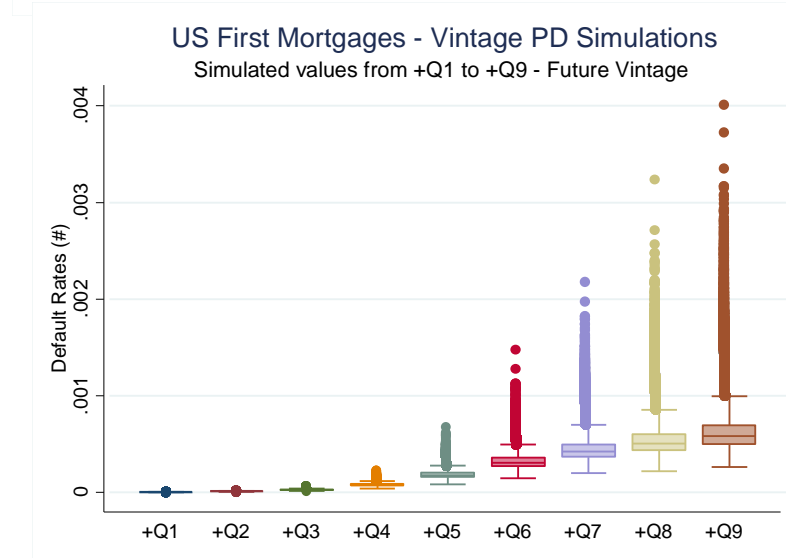
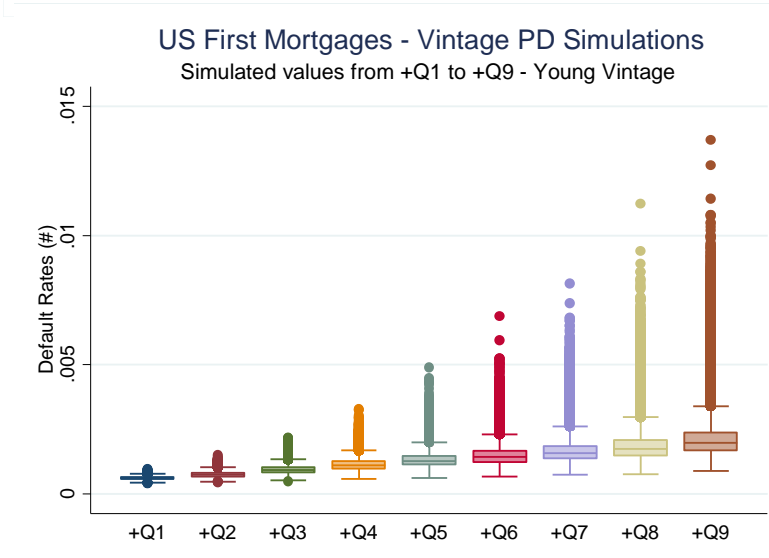
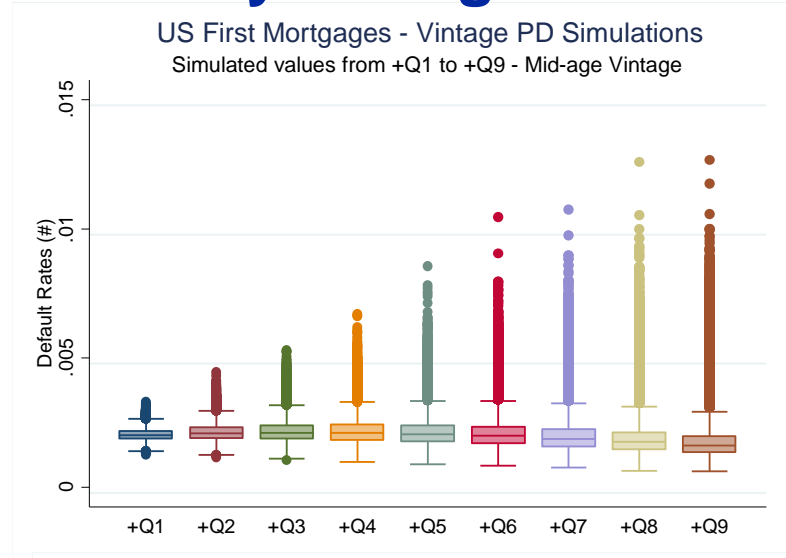
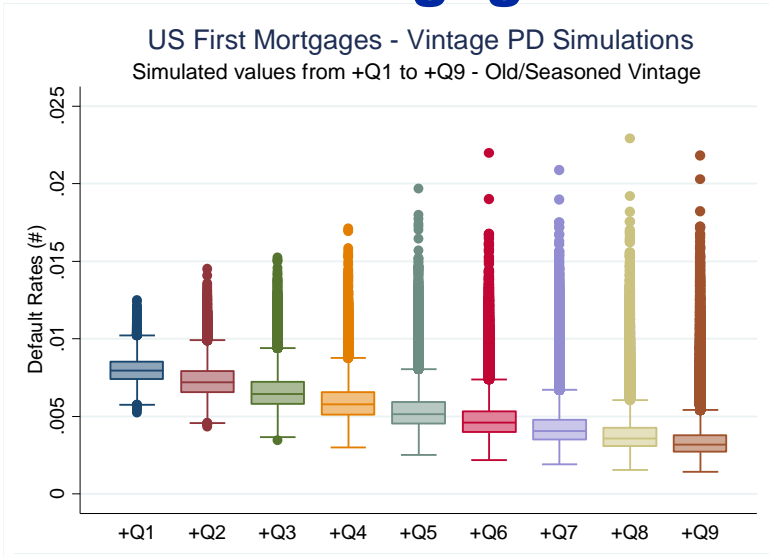
Severity of Scenarios (cont.)



2

Dynamic simulations of credit risk parameters (retail credit as leading example)

US First Mortgage Portfolio – Quarterly Vintages



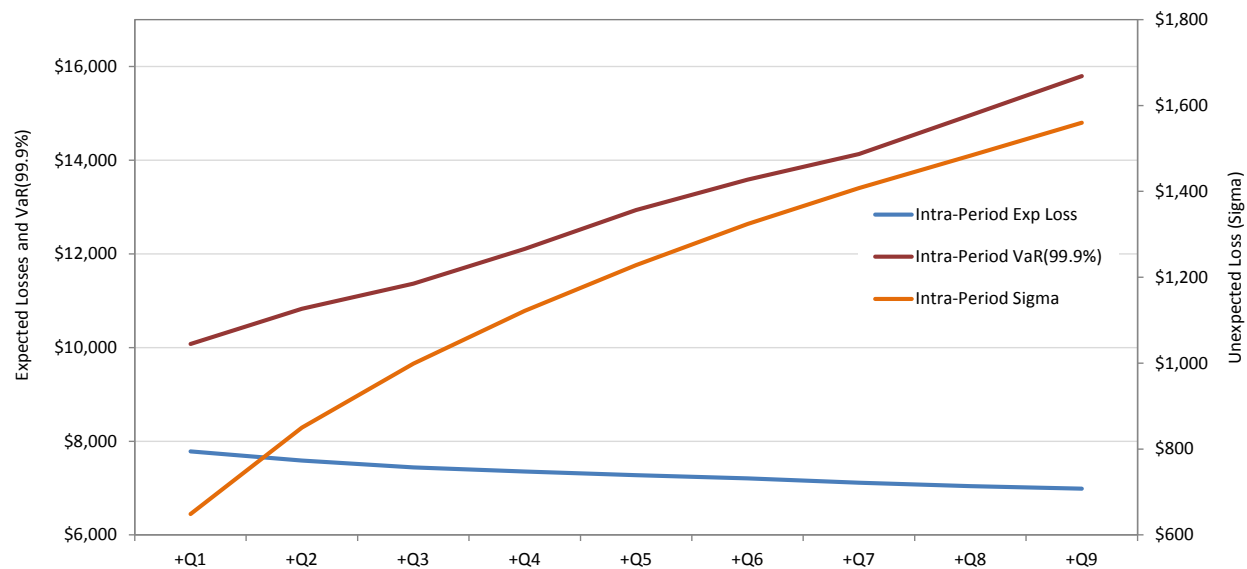
3

Analytical calculation of (intra-period and cumulative) expected and unexpected losses, asset contributions.

US First Mortgage Portfolio – Loss Metrics

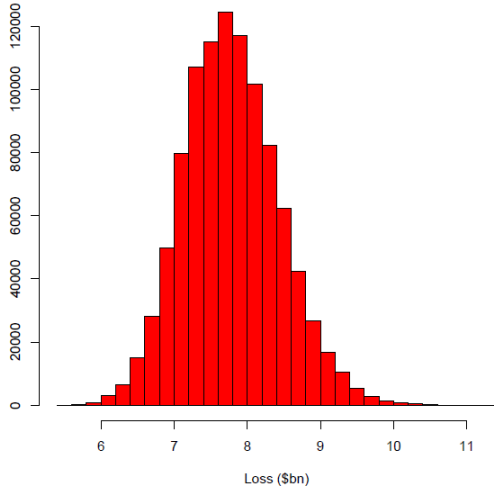
	Intra-Period Exp Loss	Intra-Period Sigma	Intra-Period VaR(99.9%)	Cumulative Exp Loss	Cumulative Sigma	Cumulative VaR(99.9%)
+Q1	\$ 7,784.9558	\$ 648.5309	\$ 10,078.3132	\$ 7,784.9558	\$ 648.5309	\$ 10,078.3132
+Q2	\$ 7,587.3311	\$ 850.0719	\$ 10,828.4443	\$ 15,372.2869	\$ 1,364.6327	\$ 20,248.9755
+Q3	\$ 7,442.0702	\$ 999.0752	\$ 11,365.3219	\$ 22,814.3572	\$ 2,193.1532	\$ 30,983.0449
+Q4	\$ 7,354.4725	\$ 1,121.8841	\$ 12,104.1981	\$ 30,168.8297	\$ 3,118.9971	\$ 42,307.3145
+Q5	\$ 7,274.9933	\$ 1,228.1673	\$ 12,933.1039	\$ 37,443.8230	\$ 4,124.4656	\$ 54,375.8396
+Q6	\$ 7,206.7848	\$ 1,323.8555	\$ 13,583.7713	\$ 44,650.6077	\$ 5,207.1490	\$ 66,748.3775
+Q7	\$ 7,113.0817	\$ 1,407.5700	\$ 14,128.6149	\$ 51,763.6895	\$ 6,352.9917	\$ 79,727.7836
+Q8	\$ 7,041.5606	\$ 1,483.2394	\$ 14,961.4969	\$ 58,805.2500	\$ 7,553.4249	\$ 93,484.9177
+Q9	\$ 6,989.0686	\$ 1,559.7431	\$ 15,795.7796	\$ 65,794.3186	\$ 8,805.0694	\$ 107,915.7781

Intra-Period Expected Losses, σ and VaR(99.9%) (Millions of USD)

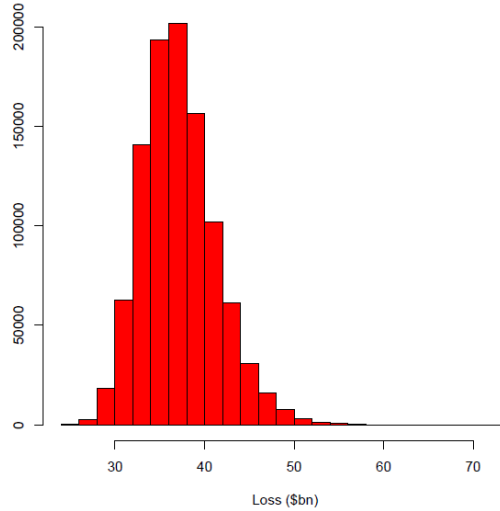


US First Mortgage Portfolio – Loss Metrics

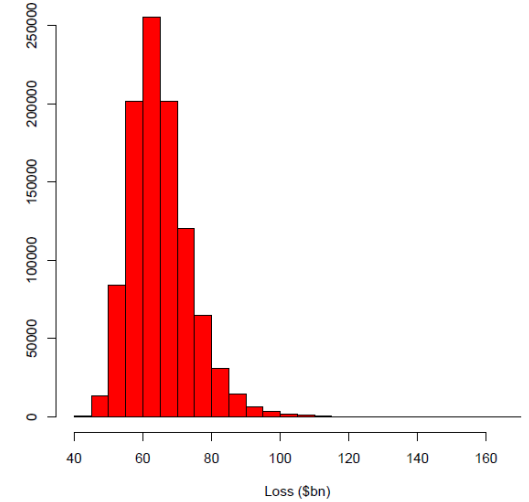
Q1 cumulative loss



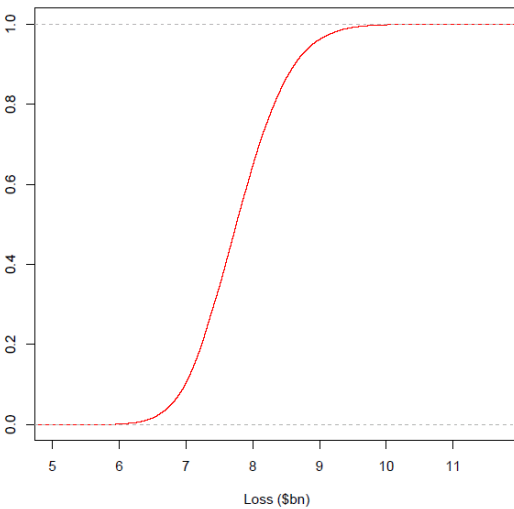
Q5 cumulative loss



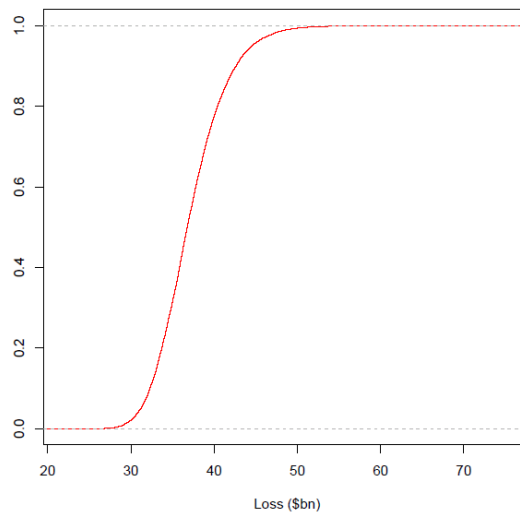
Q9 cumulative loss



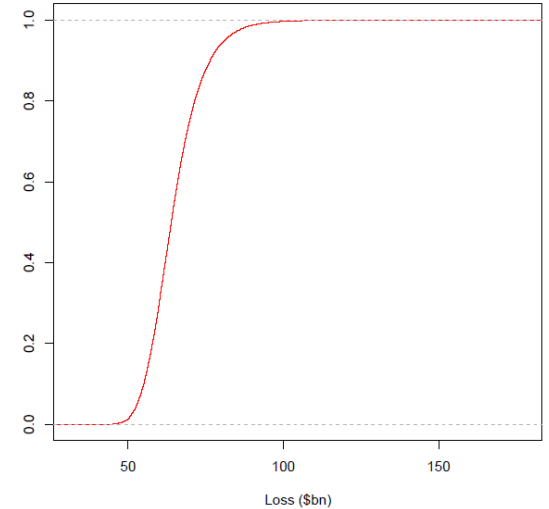
Q1 cumulative loss



Q5 cumulative loss

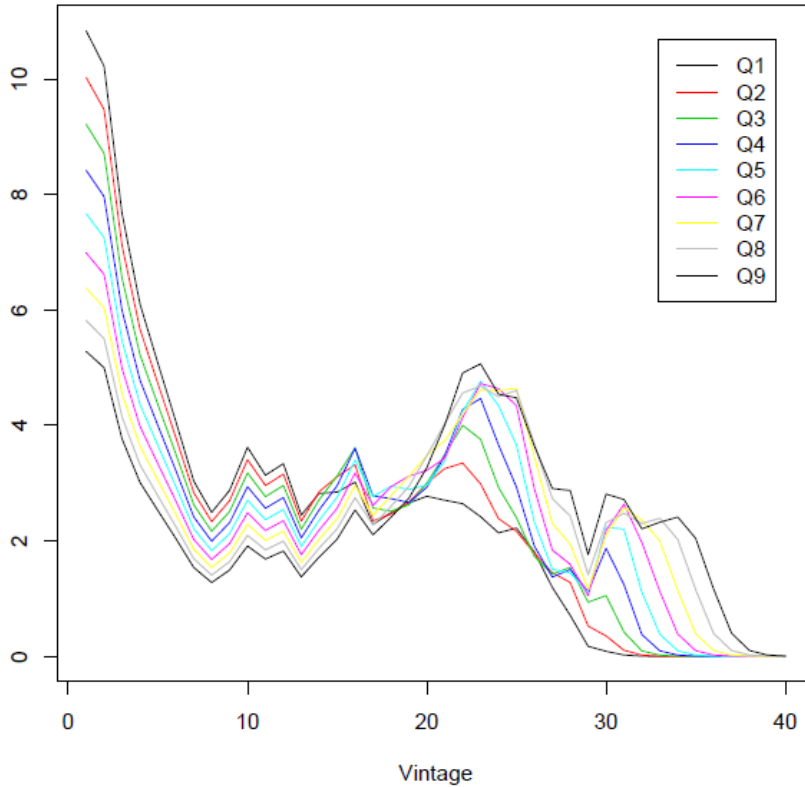


Q9 cumulative loss

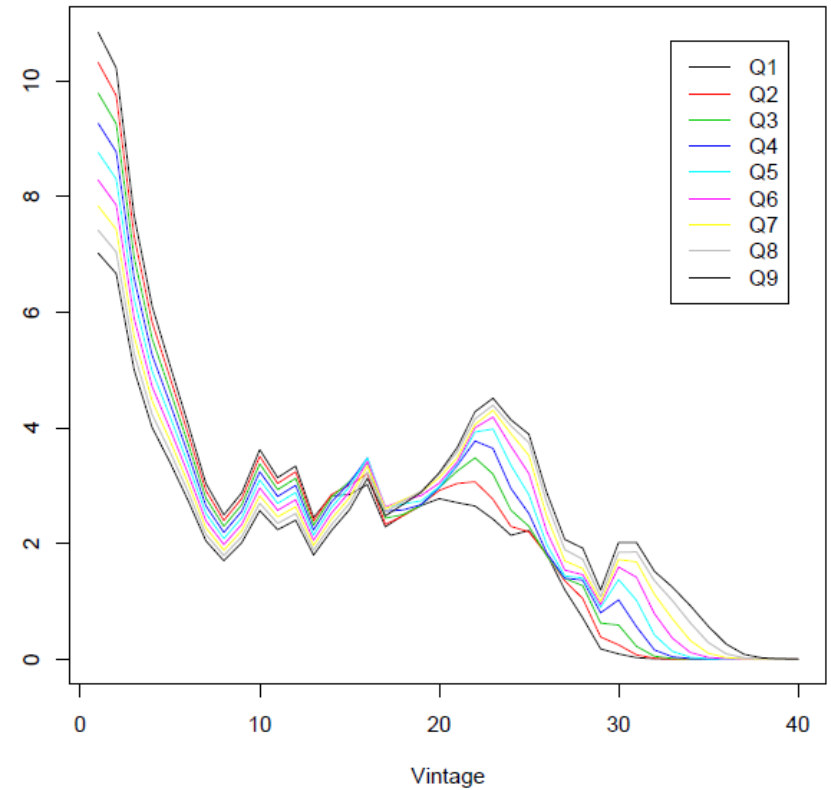


US First Mortgage Portfolio – Loss Metrics

RCs to cumulative EL



RCs to Cumulative Loss Volatility



4

Scenario-specific analysis: Embedding ST into dynamic portfolio analysis

US First Mortgages – Cumulative Scenario Losses

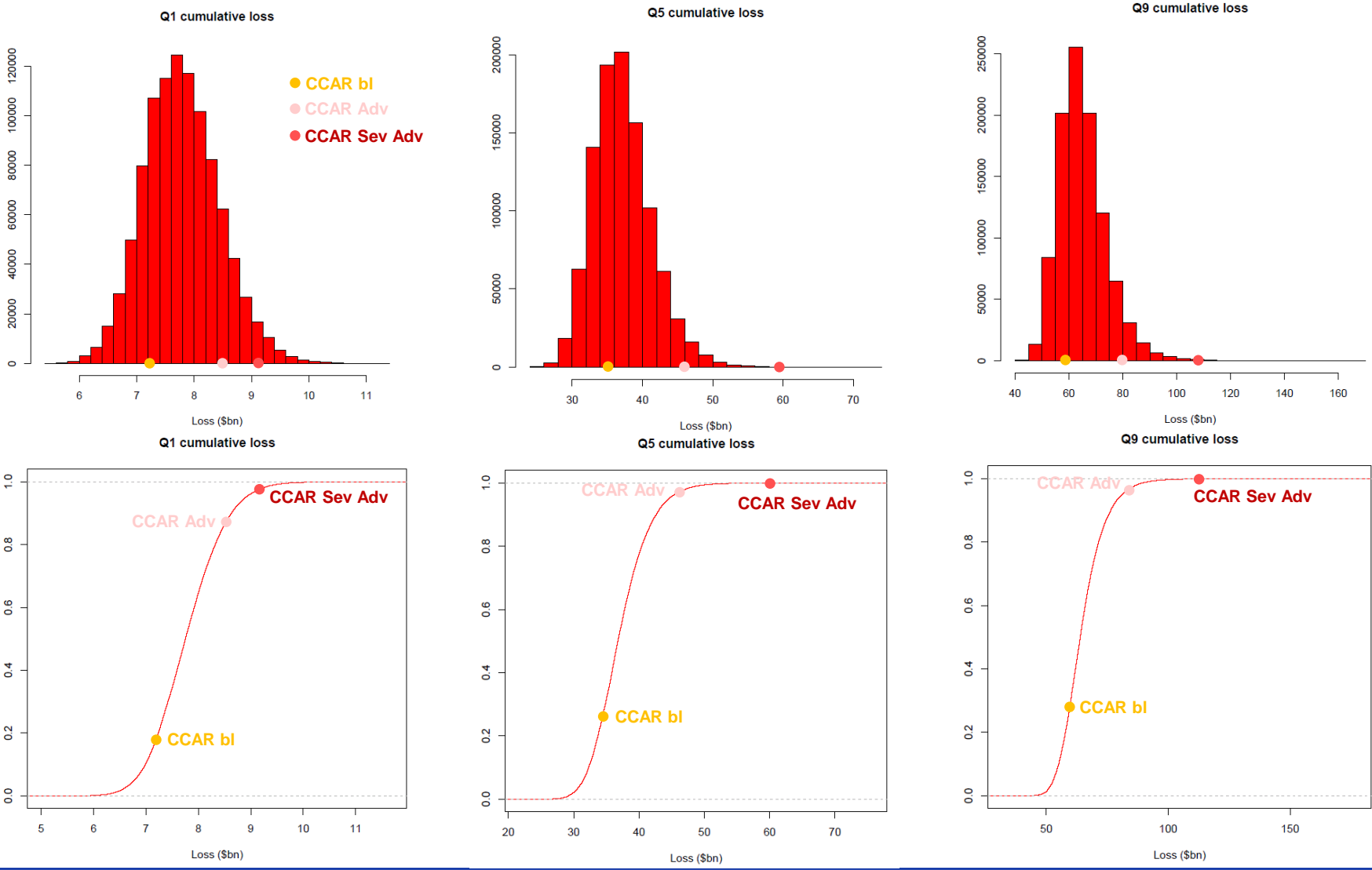
Scenario-Specific Cumulative Losses over time (MCC output)

	CCAR Baseline	CCAR Adverse	CCAR Severely Adverse	ECCA's Baseline	ECCA's s1	ECCA's s2	ECCA's s3	ECCA's s4	ECCA's s5	ECCA's s6
+Q1	\$ 7,232.9020	\$ 8,498.6685	\$ 9,173.4777	\$ 7,355.2682	\$ 7,355.6346	\$ 7,355.6346	\$ 7,355.6346	\$ 7,355.6346	\$ 7,355.6346	\$ 7,355.5408
+Q2	\$ 14,257.5338	\$ 17,940.8468	\$ 20,399.0835	\$ 14,300.3248	\$ 14,199.2785	\$ 14,793.8070	\$ 16,178.1381	\$ 16,609.8792	\$ 14,721.8985	\$ 14,749.7852
+Q3	\$ 21,091.9610	\$ 27,563.2185	\$ 32,994.9114	\$ 20,960.1226	\$ 20,464.3961	\$ 22,081.8591	\$ 26,916.6141	\$ 28,335.3669	\$ 21,865.6050	\$ 22,007.2487
+Q4	\$ 27,816.9351	\$ 37,047.0213	\$ 46,296.8187	\$ 27,437.8960	\$ 26,544.2066	\$ 29,248.4545	\$ 38,484.9117	\$ 41,403.1589	\$ 28,975.0811	\$ 30,088.6691
+Q5	\$ 34,406.5024	\$ 46,157.3768	\$ 59,486.5298	\$ 33,796.6555	\$ 32,525.2174	\$ 36,293.6484	\$ 49,833.8217	\$ 54,890.3760	\$ 35,995.7029	\$ 39,871.8138
+Q6	\$ 40,858.0799	\$ 54,983.3008	\$ 71,969.6092	\$ 40,029.5324	\$ 38,479.7494	\$ 43,129.8369	\$ 60,706.4973	\$ 68,013.2971	\$ 42,893.2674	\$ 50,664.9719
+Q7	\$ 47,216.9322	\$ 63,658.3171	\$ 84,099.8995	\$ 46,270.8379	\$ 44,358.1512	\$ 49,707.3865	\$ 70,766.6219	\$ 80,577.3767	\$ 49,723.6893	\$ 61,452.2176
+Q8	\$ 53,421.8923	\$ 72,154.3765	\$ 95,528.2296	\$ 52,484.7535	\$ 50,315.5577	\$ 56,019.5191	\$ 80,329.6951	\$ 92,670.4991	\$ 56,512.7693	\$ 72,040.5363
+Q9	\$ 59,602.6835	\$ 80,626.3743	\$ 106,408.0386	\$ 58,606.4104	\$ 56,232.3471	\$ 62,215.5086	\$ 89,370.3160	\$ 104,633.0555	\$ 63,265.6943	\$ 82,196.9864

Prob(Cumulative Losses <= Scenario-Specific Cumulative Losses)

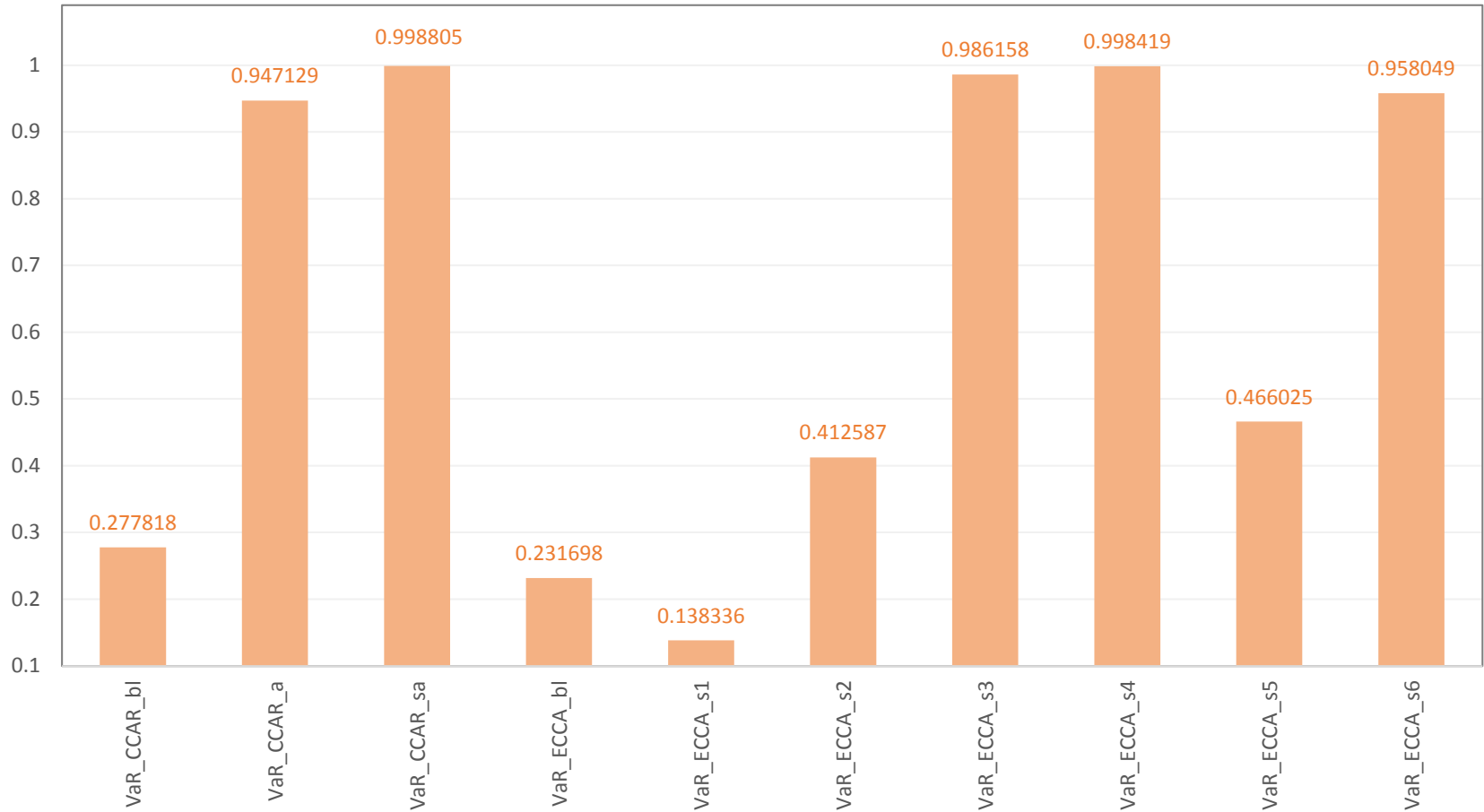
	CCAR Baseline	CCAR Adverse	CCAR Severely Adverse	ECCA's Baseline	ECCA's s1	ECCA's s2	ECCA's s3	ECCA's s4	ECCA's s5	ECCA's s6
+Q1	0.19926	0.86432	0.97673	0.26567	0.26586	0.26586	0.26586	0.26586	0.26586	0.26582
+Q2	0.21753	0.96094	0.99924	0.22824	0.20338	0.36408	0.74231	0.83049	0.34415	0.35199
+Q3	0.23275	0.97534	0.99978	0.21042	0.14411	0.41393	0.96075	0.98751	0.37204	0.39988
+Q4	0.24821	0.97694	0.99993	0.20638	0.11976	0.43719	0.98946	0.99834	0.39955	0.54948
+Q5	0.26141	0.97201	0.99989	0.20775	0.11394	0.45008	0.99376	0.99921	0.41742	0.76821
+Q6	0.26556	0.96541	0.99980	0.20765	0.11485	0.44924	0.99370	0.99927	0.42910	0.88858
+Q7	0.27193	0.95873	0.99963	0.21521	0.11952	0.44080	0.99188	0.99920	0.44197	0.93348
+Q8	0.27540	0.95260	0.99932	0.22232	0.12918	0.42279	0.98954	0.99886	0.45349	0.95168
+Q9	0.27782	0.94713	0.99881	0.23170	0.13834	0.41259	0.98616	0.99842	0.46603	0.95805

US First Mortgages – Cumulative Scenario Losses



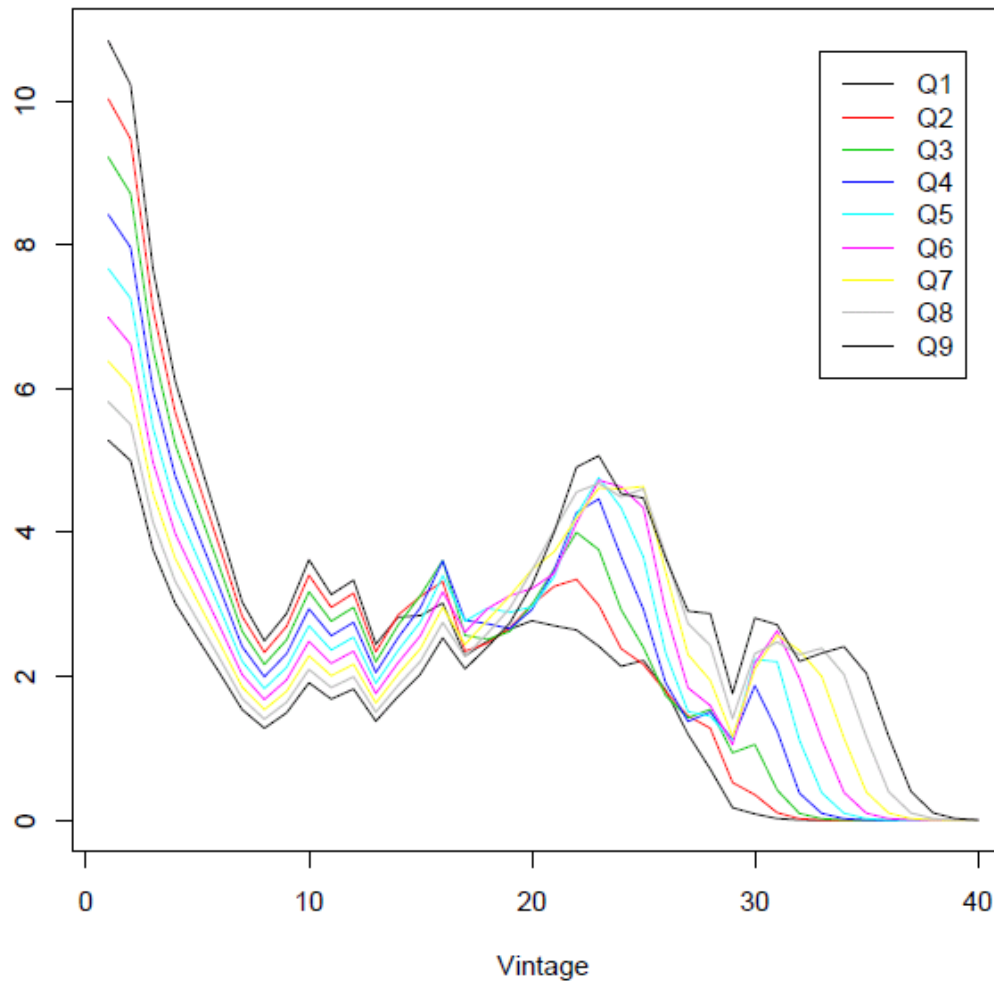
US First Mortgages – Cumulative Scenario Losses

Prob(Cumulative L ≤ Scenario Specific Cumulative Losses)
+Q9 (End of Stress Testing Period)



US First Mortgages – Cumulative Scenario Losses

RCs to cumulative CCAR Severely Adverse



5

Practical Applications: RAROC, Optimization, Optimal Importance Sampling, and Reverse Stress Testing

Risk Adjusted Pricing (RAROC)

Analytical calculation of the portfolio volatility up to the new deal's maturity allows us to instantaneously price.

The Sharpe ratio of the new portfolio with the new deal (or deals) should be larger than without:

$$\frac{\mu^* - r}{\sigma(L^*)} > \frac{\mu - r}{\sigma(L)} \quad \rightarrow \quad \text{BUY}$$

Portfolio Optimisation

What is the portfolio composition n_i that minimises the portfolio loss volatility given a level of expected loss (and hence return) $EL = L$?

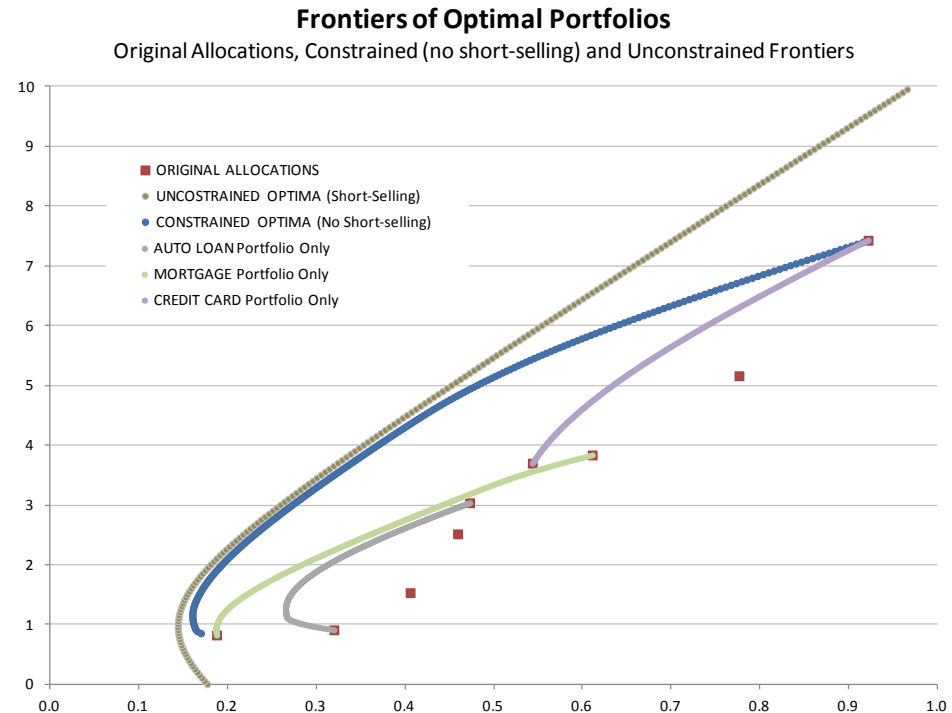
Using the Lagrange multipliers:

$$\Lambda(n_i, \lambda) = \sigma(L; n_i) + \lambda(EL(n_i) - L)$$

Thus:

$$n_i C_i + \lambda EL_i^* = 0$$

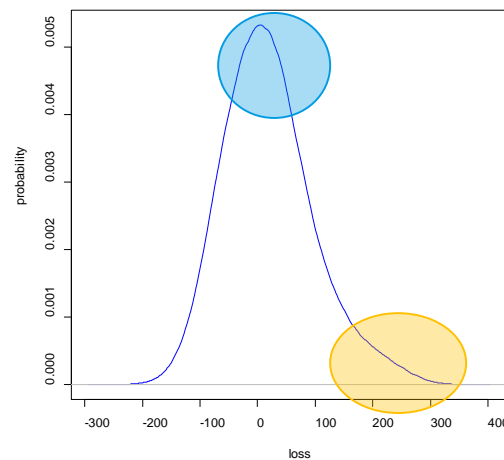
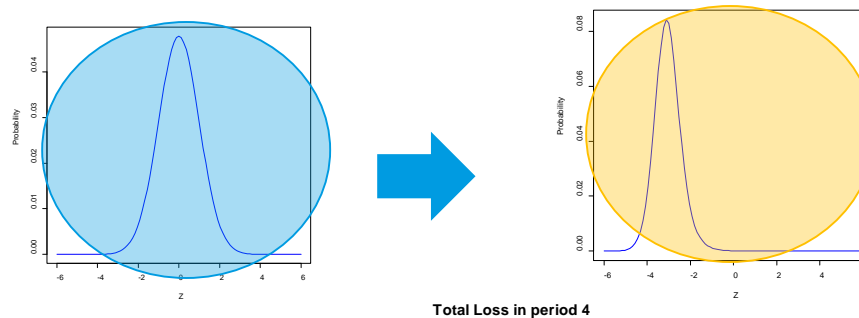
$$\sum_{i=1}^N n_i EL_i^* - L = 0$$



- Extend the current framework to study DYNAMIC OPTIMISATION (infinite horizon) **Recursive Dynamic Programming** (Bellman Equations) and the study of the optimal solutions to the underlying stochastic difference equations

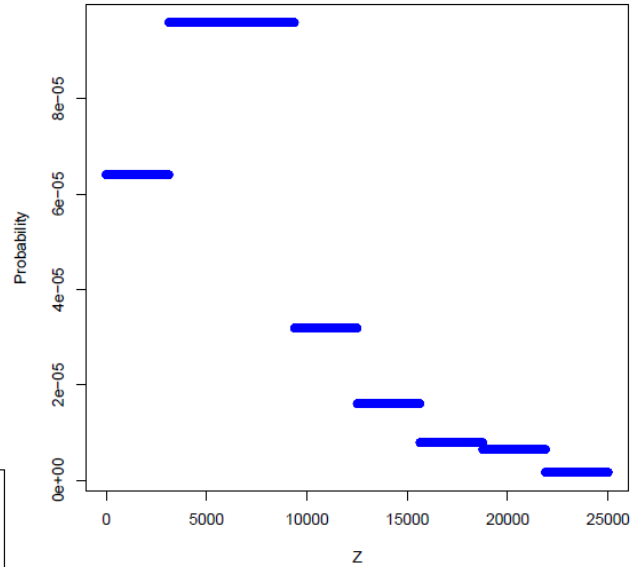
Optimal Importance Sampling

Instead of drawing random numbers with unconditional probability from conditional distribution w_z draw from conditional $w_z(\alpha)$. After the simulation reweigh the loss distribution using $\text{weight} = \frac{w_z}{w_z(\alpha)}$

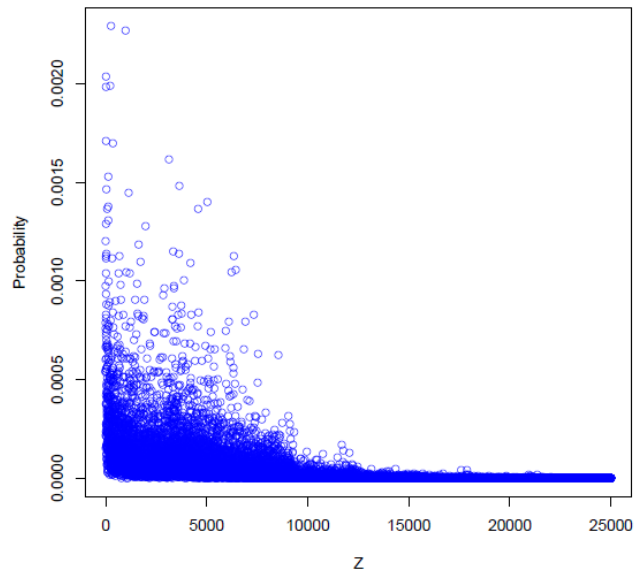


Analytical Reverse Stress Testing

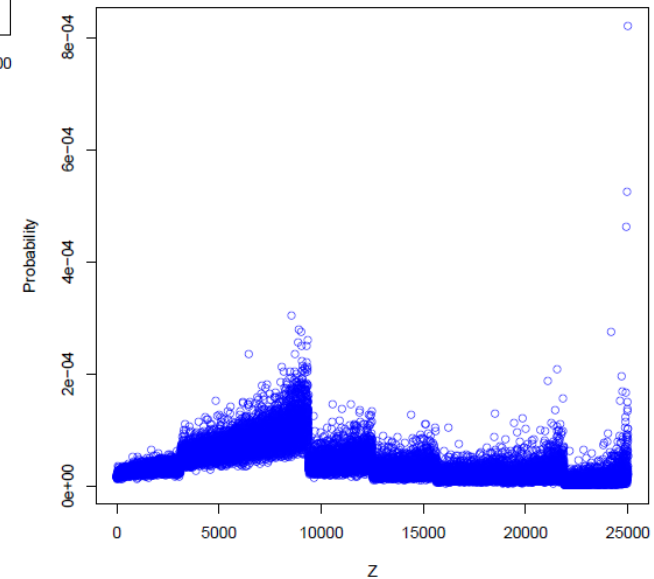
P(Z) conditional on L = EL



P(Z) conditional on L < EL



P(Z) conditional on L > EL



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