Banking System in Crisis: An Economic Capital Viewpoint

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Introduction

how do advanced economic capital models perform in times before and during a financial crisis?

Expected and Unexpected Loss

- Total losses experienced on a loan portfolio by a financial institution depend on the number of defaults from time to time and their severity
- The losses that a bank expects to suffer in a given year are known as Expected Losses (EL)
- Losses that exceed the level of expected losses are known as Unexpected Losses (UL)

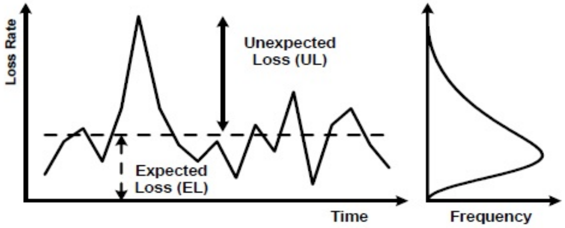

Figure: Decomposition of Loss into Expected and Unexpected[14]

Expected and Unexpected Loss

- how much capital should a bank hold for being protected from such peaks in losses in an economically efficient way?
- Estimate the probability loss distribution for the given portfolio
- Estimate the loss for a given confidence interval

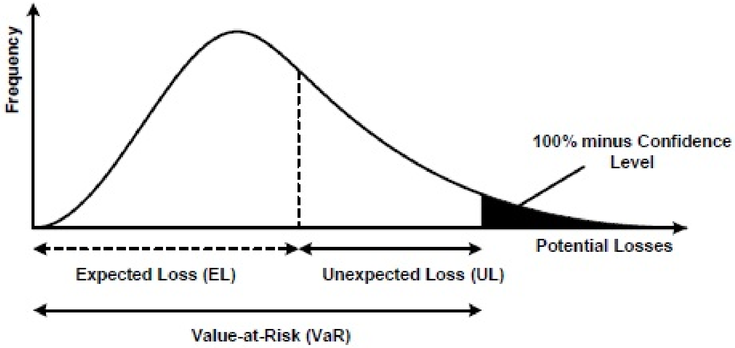

Figure: Value-at-Risk Approach[14]

Economic Capital

Economic Capital

the capital used to provide a cushion against unexpected losses at a specified level over a time horizon is known as economic capital

- The economic capital viewpoint reflects the aim to measure potential changes in the economic value of assets and liabilities
- It internally assesses capital position in relation to total risk (market risk, credit risk, concentration risk etc)
- Is often calculated as Value at Risk
- Is the generally accepted measurement to assess capital during the ICAAP process
- Is used for calculation of risk adjusted profitability estimates such as RAROC, RARORAC, EVA

Existing Methodologies I

- We examine a Corporate Loan portfolio and focus on quantitative approaches that are available for measuring credit and concentration risk
- These approaches try to capture the entire probability distribution of potential losses
- The most associated risk metrics are value-at-risk (**VaR**) and expected shortfall (**ES**)

VaR

a measure of potential losses at a chosen confidence level over a defined time horizon (lacks subadditivity).

ES

the conditional expectation of loss given that the loss is beyond the VaR level (subadditive measure)

Existing Methodologies II

- The credit migration approach to Value-at-Risk
  - J.P. Morgan's CreditMetrics[6, 13] is based on the probability of moving from one credit quality to another, including default, which is used to valuate a firm's assets
  - McKinsey's CreditPortfolioView[16] where the probabilities of default are a function of macro-variables
- The option pricing approach to VaR as in CreditPortfolioManager by KMV, where the actual probability of default for each obligor is derived based on a Merton type models[12]
- We focus on the actuarial approach as proposed by Credit Suisse's CreditRisk+[1, 10] model

CreditRisk+

- Default risk is modeled as opposed to credit migration
- There is no dependence between the default risk and the the capital structure of the firm
- It enables us to analytically compute the portfolio loss distribution without resorting to Monte Carlo simulations
- PD is modelled as a linear combination of sector default random variables that are gamma distributed:

$$p_i^s = p_i(w_{0i} + \sum_{k=1}^{K} w_{ki} S_k)$$

where w_{ki} are the weights of each sector that affects the probability of default, and S_k are the gamma distributed sector random variables with a mean of 1 and variance $V[S_k] = \sigma_k^2$.

CreditRisk+ Extensions I

- The original method used Panjer recursions[15] which are numerically unstable for large loan portfolios - Giese[5] improved upon that by using a recursive computation of exponential and logarithmic polynomials (see also [7])
- The original model assumes that the sector default rates are independent, but in reality these are affected by macroeconomic indicators and are in some part correlated. To address that:
  - In Bürgisser et al[2] a single sector model is calculated, with an adjusted portfolio default rate standard deviation according to sector correlations: if we denote by EL_i the expected loss for sector $i \in \{1, \ldots, K\}$, then the relative default frequency σ^2 is computed by:

$$\sigma^2 = \frac{\sum_{k=1}^{K} \sigma_{S_k}^2 EL_k^2 + \sum_{k \neq l} corr(S_k, S_l)\sigma_{S_k,S_l} EL_k EL_l}{EL^2} \quad (1)$$

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CreditRisk+ Extensions II

- The hidden gamma model of Giese[5] is consistent with the common sense narrative that a macroeconomic factor induces correlation to sector default rates
- ...but at the same time, the covariance structure the model can describe is restricted: the common risk factor is $\hat{Y} \sim Gamma(\theta, 1)$, and in [5] it is shown that for sector variances σ_k^2, bounds on θ are induced:

$$0 \leq \theta \leq \min_k \left\{ \frac{1}{\sigma_k^2} \right\}$$

which lead to considerable restrictions to the covariance structure the model can describe[8].

CreditRisk+ Extensions III

- In [8] the model was generalized by giving up the condition that the coefficient of Y_k must be the same as that of $\hat{Y}$, ie

$$S_k = \delta_k Y_k + \gamma_k \hat{Y}$$

  which allows a wider range of sector covariance structures
- Fischer & Dietz in [4] extended the model further by considering several additive T_L background factors

The Common Background Vector Model I

Following [4], the default indicator of counterparty A is approximated by the following Poisson variable:

$$\lambda_A^S = p_A(w_{A0} + w_{A1}\hat{S}_1 + \cdots + w_{AK}\hat{S}_K + w_{A,K+1}T_1 + \cdots + w_{A,K+L}T_L)$$

where the sector impact is determined by weights w_{Aj} and $w_{A0} = 1 - w_{Ak} - \cdots - w_{A1}$, and where we have K dependent sector variables $\hat{S}_K$ and L T_L common factors that connect the underlying independent K sector variables $S_1 \ldots S_K$:

$$\hat{S}_K = \delta_k S_k + \sum_{l=1}^{L} \gamma_{lk} T_l$$

with $S_k \sim \Gamma(\theta_k, 1)$ and $T_l \sim \Gamma(\hat{\theta}_l, 1)$ for $k = 1, \ldots, K$ and L weights $w_{A,K+l} = \sum_{k=1}^{K} w_{Ak}\gamma_{lk}$ for $l = 1, \ldots, L$

The Common Background Vector Model II

The theoretical sector VCV derives as:

$$E[\hat{S}_k] = \delta_k \theta_k + \sum_{l=1}^{L} \gamma_{lk} \hat{\theta}_l$$

$$Var[\hat{S}_k] = \delta_k^2 \theta_k + \sum_{l=1}^{L} \gamma_{lk}^2 \hat{\theta}_l$$

$$cov(\hat{S}_i, \hat{S}_j) = \sum_{l=1}^{L} \gamma_{li} \gamma_{lj} \hat{\theta}_l$$

The original CreditRisk+ model then can be applied where the parameters of the model have been appropriately substitubed by θ_j, θ_l and δ_j, where $j = 1, \ldots, K + L$ and $l = 1, \ldots, L$ as described in [4].

The Common Background Vector Model III

The unknown parameters are chosen such that we minimize the distance between the observed and modeled VCV matrix, under some constraints:

$$\min_{\hat{\theta}_k,\theta_l,\delta_k,\gamma_{lk}} \sum_{k=1}^{K}(\sigma_k^2 - \delta_k^2\hat{\theta}_k - \sum_{l=1}^{L}\gamma_{lk}^2\theta_l)^2 + \sum_{k=1}^{K}\sum_{j=1}^{k-1}(\sigma_{kj} - \sum_{l=1}^{L}\gamma_{lk}\gamma_{lj}\theta_l)^2$$

$$\text{s.t.} \tag{2}$$

$$\delta_k\theta_k + \sum_{l=1}^{L}\gamma_{lk}\hat{\theta}_l = 1 \qquad \forall k \in \{1 \dots K\} \tag{3}$$

$$w_{A,K+l} \geq 0 \qquad \forall A \in \mathcal{A} \text{ and } \forall l \in \{1 \dots L\} \tag{4}$$

$$\theta_k, \hat{\theta}_l, \delta_k \geq 0 \qquad \forall k \in \{1 \dots K\} \text{ and } \forall l \in \{1 \dots L\} \tag{5}$$

where by $\mathcal{A}$ we denote the set of all counterparties.

Implementation

- Results were derived from two CreditRisk+ implementations (i) the original CreditRisk+ as adjusted in [2], i.e. using the equation (1) to determine sector correlation , (ii) the CBV model as described in the previous section
- Both models are fit for quickly computing loss distributions via deriving their PGF with minimal pre-computation, using FFT as described in [11] for implementation (i), and the recursion scheme introduced by Giese for implementation (ii) following [4]
- Implementation (i) runs instantly, and a multithreaded Java implementation (parallelizing computations for each sector) for (ii) has runtime of a few seconds to a few minutes, for corporate portfolios ranging from a few thousand to hundreds of thousands of loans (having solved the minimization problem in a pre-computation step)

Data and Pre-Computation

- Segments of corporate portfolios (portfolios resulting from mergers were excluded) for the years before and during the economic crisis in Greece
- Exposures ranged from a few Euros to millions of Euros
- To specify the observed VCV matrix, the general rule was to use time series in corporate defaults T=18 months before the year in examination, where available
- We restricted ourselves to 23 industry sectors (or factors, in terms of the model) along with an idiosyncratic factor and 3 common background factors (i.e. $L = 3$)
- The empirical VCV matrix may not be positive semidefinite (PSD) which leads to non-convexity problems during the optimization step described in (2). There exist a number of methodologies for such a transformation to the nearest PSD matrix (see e.g. [9, 3])

Empirical Results I

- We provide VaR and ES figures for the above mentioned datasets, using implementation (i) (CreditRisk+ as adjusted in [2]), (ii) (the CBV model) plus results relating the economic capital with the regulatory capital associated with each sample/year.

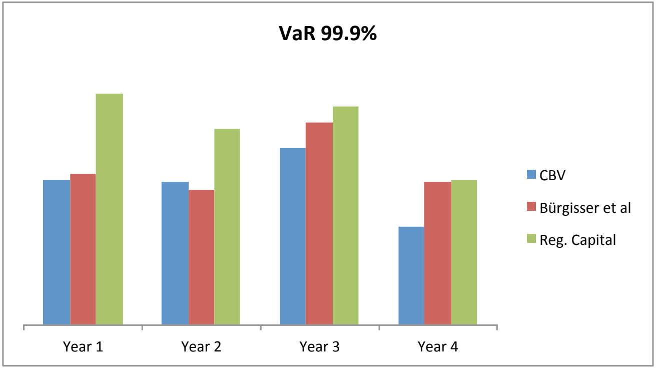
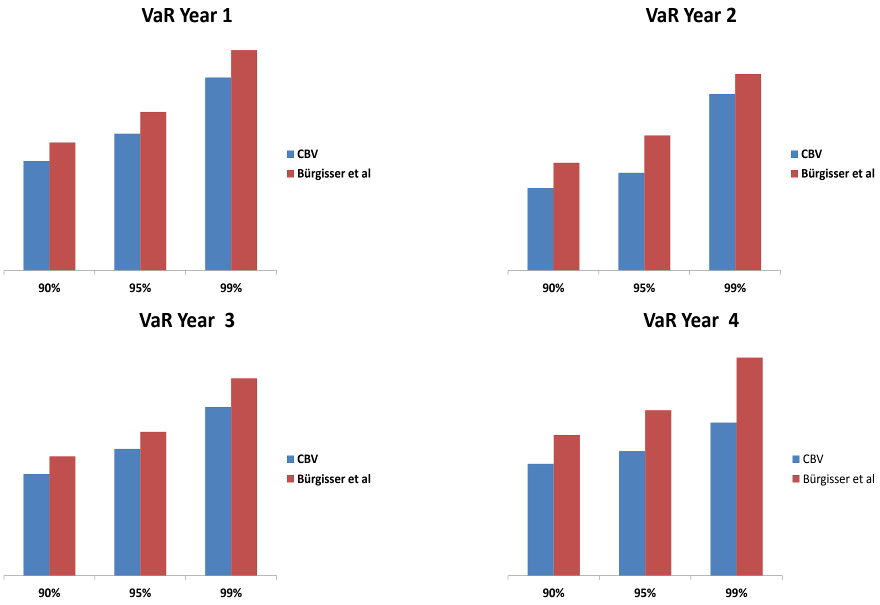
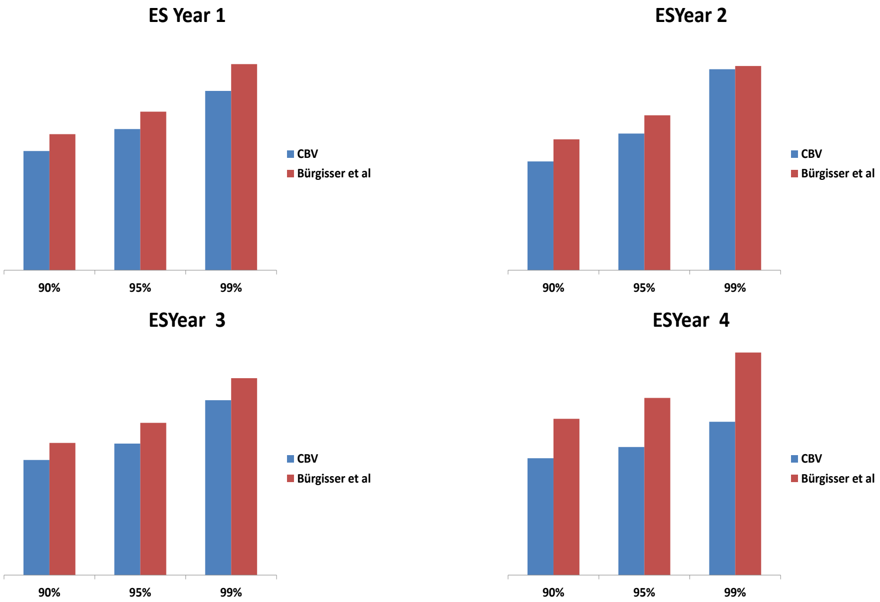


Figure: VaR at 99.9% with different methodologies.

Empirical Results II



Empirical Results III



Conclusions

- CreditRisk+ assumes sectors are independent: we implemented and adjusted two methodologies for estimating VaR where this issue is being addressed
- The first (Burgisser et.al.)[2] is a well-known methodology that attempts to model sector correlation, while the second one (CBV [4]) is a very recent method which attempts to incorporate the actual correlation between each pair of factors in our model
- The first model overestimates the risk due to the common correlation factor which is driven by particular sectors which are extremely correlated in downturn conditions
- Numerically stable recursion schemes as introduced in [5] and well-posed optimisation setups (by calibrating our model using PSD vcv matrices[9, 3]) are critical in successful implementations of the CBV model

Future Research

- Among the next steps in such an analysis is to omit the gamma-distributed sectors and introduce new appropriate distributions, since there is no empirical evidence for such a selection
- Extreme Value Theory can be used to estimate tail risks, especially for the Expected Shortfall metric

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