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CONFIDENCE-BASED Z-SCORE
(A MEASURE OF BANK SOLVENCY)

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Outline

1. Motivation
2. Existing measures
3. Motivating examples
4. Contribution
5. Results
6. Conclusions

Motivation

- The measurement of financial stability in banking aims at assessing the degree of institution solvency
- In this exercise, it is paramount to explicitly account for the degree of uncertainty in the estimations
- Existing accounting-based indicators do not include estimation error in the evaluation of banks' financial health/soundness

Existing measures

Risk Measures	Source of information: Market based	Source of information: Accounting based
Volatility of stock prices	X	
Distance to default	X	
CDS Spread	X	
Credit ratings	X	
Z-Score		X
Marginal expected shortfall	X	
Equity beta	X	
Equity return volatility	X	
Asset return volatility	X	
Tail risk	X	
Miscellaneous risk measures		X

- We focus on the Z-Score as it is the most used accounting based measure for non-listed institutions to assess the overall risk of bank solvency

Existing measures

- Z-Score uses the following accounting information
  - Return on assets (ROA) – After-tax profits divided by total assets
  - Capital to asset ratio (CAR) – Total equity divided by total assets
- ROA is the random variable
- Capital and total assets are given
- The Z-Score is defined (Boyd and Graham, 1986), under mild assumptions, as the number of the standard deviations of the return on assets necessary to wipe out equity capital
- $PD = 1/Z^2$

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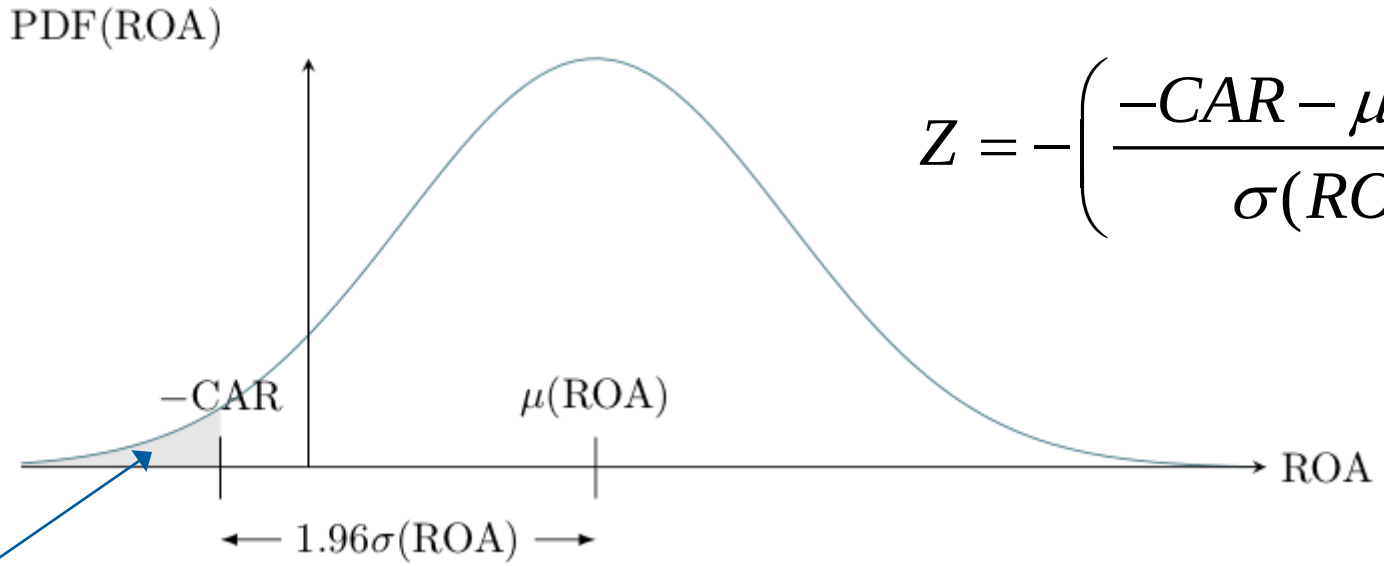


$$Z = -\left(\frac{-CAR - \mu(ROA)}{\sigma(ROA)}\right)$$

Probability that ROA < CAR

Existing methods

k={T-n,….T}	t={1….T}
n=3	T=last period

- A variety of options to compute the Z-Score has been surveyed in Lepetit and Strobel (2013)

Boyd et al. (2006) ⟹ $$Z1 = \frac{\mu_{ROA,k} + \mu_{CAR,k}}{\sigma_{ROA,k}}$$

Yeyati and Micco (2007) ⟹ $$Z2 = \frac{\mu_{ROA,k} + \mu_{CAR,T}}{\sigma_{ROA,k}}$$

Hesse and Cihák (2007) ⟹ $$Z3 = \frac{ROA_T + CAR_T}{\sigma_{ROA,t}}$$

Boyd et al. (2006) ⟹ $$Z4 = \frac{ROA_T + CAR_T}{|\, ROA_T - \mu_{ROA,t} \,|}$$

Lepetit and Strobel (2013) ⟹ $$Z5 = \frac{\mu_{ROA,t} + CAR_T}{\sigma_{ROA,t}}$$

Existing methods - Limitations

- Z3 ⟹ considers the current ROA and the standard deviation over the whole horizon therefore it is not clear which the random variable is

- Z4 ⟹ sigma instantaneous is a non-standard measure of variability and therefore you cannot use it in a probabilistic analysis, i.e. not clear the link with the probability of default

Existing methods - Limitations

- Z5 does not reflect the fact that the magnitude of returns can be associated with higher variance (heteroscedasticity)

- No existing methods reflects the degree of estimation error associated with the available data

Motivating example

t={1….T}
T=last period

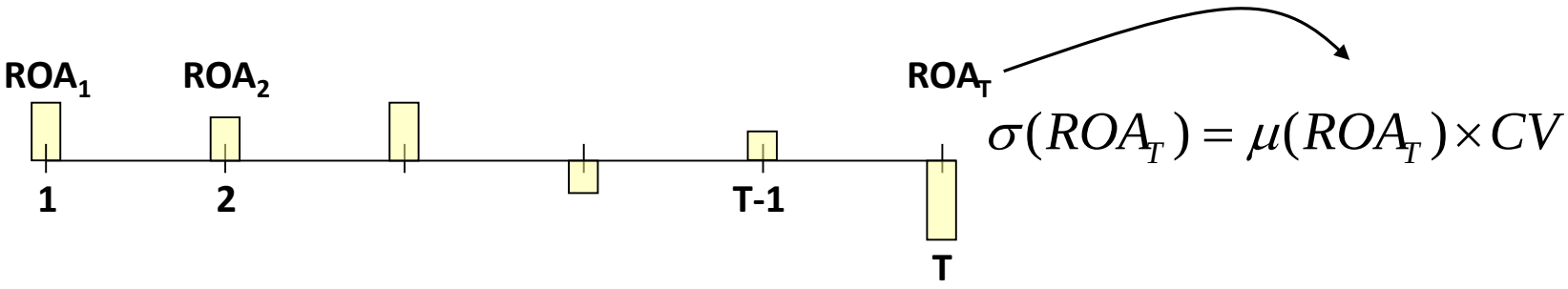
➢ Example 1

Data:

σ_{ROA}=standard deviation of ROA over the sample period

μ_{ROA}=mean of ROA over the sample period

CV=Coefficient of variation $= \sigma_{ROA,t} / \mu_{ROA,t}$



Motivating example

- Example 2

ROA Bank1

2005 S1	2005 S2	2006 S1	2006 S2	2007 S1	2007 S2	2008 S1	2008 S2
0.0098	0.0088	0.0108	0.0127	0.0098	0.0069	0.0117	0.0155

ROA Bank 2

2005	2006	2007	2008
0.0800	0.1200	0.0640	0.0800
0.0100	0.0150	0.0080	0.0100

	St. Dev.	α	LO Bound	Average	UP Bound
Bank 1	0.0026	0.05	0.0838	0.0860	0.0882
Bank 2	0.0239	0.05	0.0480	0.0860	0.1240

Contribution

t={1….T}
T=last period

- We introduce a new method (Z6) based on the coefficient of variation of the ROA

$$Z6 = \frac{\mu_{ROA,T} + CAR_T}{CV \times \mu_{ROA,T}}$$

- We complement existing methods (Z1, Z2, Z5) and Z6 by building confidence intervals around them in order to reflect the degree of estimation error

Contribution

t={1….T}
T=last period

- We complement existing methods (Z1, Z2, Z5) and Z6 by building confidence intervals around them to reflect the degree of estimation error

$$\mu \in \left[\bar{\mu} + t_{n-1,\alpha/2} \frac{1}{\sqrt{n}} s, \bar{\mu} + t_{n-1,1-\alpha/2} \frac{1}{\sqrt{n}} s \right] = \left[\mu_L, \mu_U \right]$$

$$\sigma^2 \in \left[\frac{(n-1)s^2}{\chi^2_{n-1,1-\alpha/2}}, \frac{(n-1)s^2}{\chi^2_{n-1,\alpha/2}} \right] = \left[\sigma_L^2, \sigma_U^2 \right]$$

$$Z5^{CB} = \frac{\mu_{ROA,t,L} + CAR_T}{\sigma_{ROA,t,U}}$$

Sample

Country	2005	2006	2007	2008	2009	2010	2011	2012	2013	Total
ARGENTINA	37	37	41	40	42	41	40	39	35	352
AUSTRIA	192	201	215	197	198	208	212	197	138	1,758
BRAZIL	58	58	63	65	64	63	66	67	65	569
CANADA	13	13	13	13	13	10	43	61	57	236
CHINA	29	40	56	70	84	94	113	123	128	737
FRANCE	97	142	151	157	165	165	184	178	166	1,405
GERMANY	1,078	1,218	1,208	1,218	1,227	1,291	1,330	1,353	1,128	11,051
INDIA	48	49	54	55	57	57	58	58	54	490
INDONESIA	34	33	36	39	40	43	45	45	46	361
ITALY	4	464	474	487	493	493	509	507	420	3,851
JAPAN	550	555	547	551	545	547	559	549	538	4,941
MEXICO	19	19	27	33	33	35	35	41	41	283
REPUBLIC OF KOREA	2	7	3	3	3	3	16	17	17	71
RUSSIA	339	497	659	716	708	783	798	789	761	6,050
SAUDI ARABIA	9	9	9	9	9	9	9	9	9	81
SOUTH AFRICA	7	10	11	13	12	13	13	13	14	106
SPAIN	17	78	82	84	86	90	89	93	54	673
TURKEY	11	13	21	23	23	23	23	25	24	186
UNITED KINGDOM	59	78	80	83	83	87	95	106	91	762
USA	6,635	6,776	6,921	7,033	6,978	6,866	6,795	6,736	6,711	61,451
Total	9,238	10,297	10,671	10,889	10,863	10,921	11,032	11,006	10,497	95,414

Preliminary results (I)

- We classify the observations into the deciles of the distributions of the confidence-based Z5 and Z6 and the non-confidence-based indicators
- In principle, being equal the measurement error for all banks, we should obtain the same classification

e.g. by using confidence-based Z5, 499 banks fall 7 positions in the ranking (compared to the ordinary Z5)

Difference in deciles base vs confidence based

Diff.	-9	-8	-7	-6	-5	-4	-3	-2	-1	0
Z5	0	0	499	496	369	685	1,134	2,050	6,574	61,476
Z6	0	0	1	0	3	10	46	558	15,549	72,228

Difference in deciles base vs confidence based

Diff.	9	8	7	6	5	4	3	2	1
Z5	0	0	0	0	0	0	0	0	25,130
Z6	174	92	111	105	145	239	520	1,283	7,349

Preliminary results (II)

- For each Z-score, when confidence intervals overlap, we can't distinguish banks in terms of their risk
- So, the smaller the number of overlaps the better (i.e. risk rankings are more conclusive)

Year	Average number of overlaps (per institution)				
	# of institutions	Z1	Z2	Z5	Z6
2005	375	344	332	255	160
2006	458	383	356	281	199
2007	486	410	379	279	221
2008	490	429	393	270	209
2009	489	417	383	248	189
2010	501	394	366	239	201
2011	519	395	378	250	203
2012	532	424	406	250	194
2013	537	352	345	269	172
Global average		394	371	260	194

In any year, **Z6** has the smallest number of overlaps

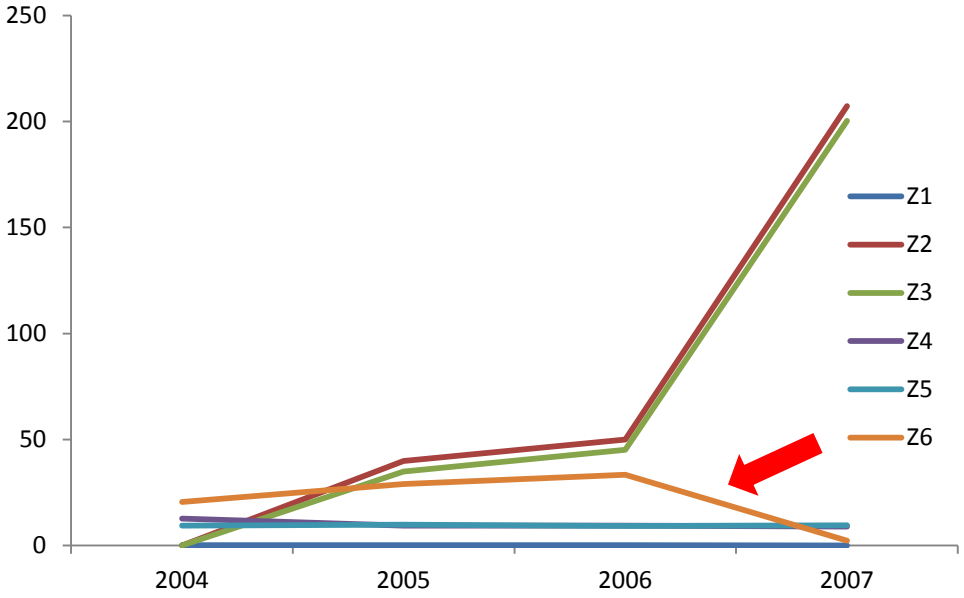
Case studies

- Information on distress events (manually collected from news)
- UK banks
- According to accounting data available, we calculated the value of different Z-scores measures before the events
- Note: Low Z-scores associated with high probability of default (distress). $PD = 1/Z^2$
- So, the best Z-score measures should decrease (just) before the events

Case 1 - RBS

- Events: Emergency funds from BoE and bailout
- Dates: 17/10/2008, 13/11/2008, 19/1/2009 and 3/11/2009
- Sources: FT, The Guardian, London Lite

Year	Z1	Z2	Z3	Z4	Z5	Z6
2004	0.0626	0.0000	0.0000	12.5955	9.3175	20.4839
2005	0.0470	39.8889	34.8073	9.4030	9.7761	28.9744
2006	0.0465	49.8897	45.1385	9.3723	9.1560	33.2900
2007	0.0442	207.2515	200.3222	8.8701	9.4631	2.1639

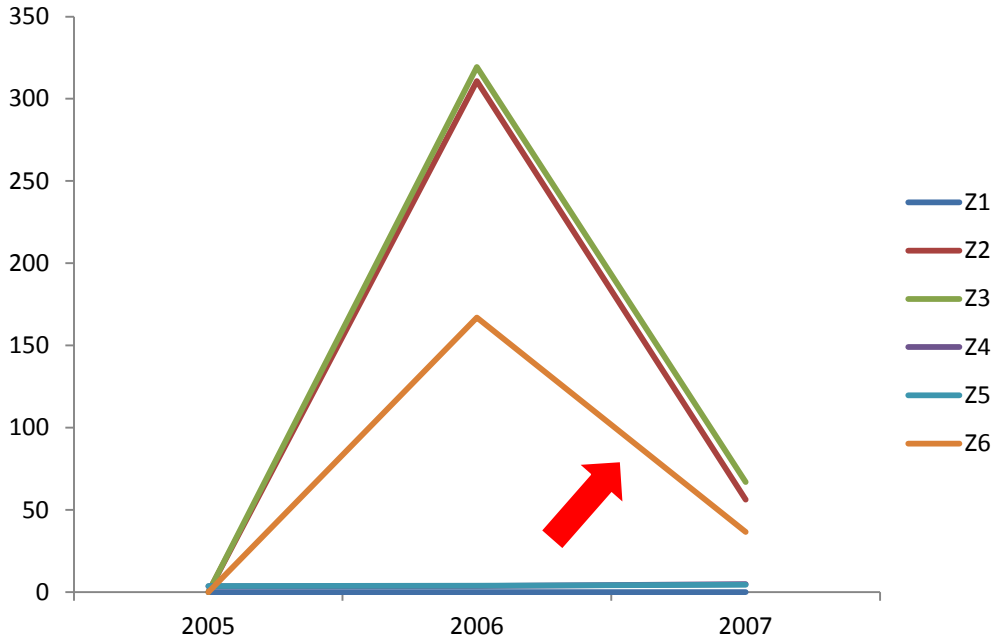


The lower the riskier

Case 2 - HBOS

- Event: Emergency funds from BoE
- Date: 13/11/2008
- Source: FT

Year	Z1	Z2	Z3	Z4	Z5	Z6
2005	0.0224	0.0000	0.0000	3.6174	3.7294	0.0000
2006	0.0241	310.7622	319.3503	3.8057	3.9930	167.0025
2007	0.0330	56.3704	66.9032	4.9984	4.5945	36.6136

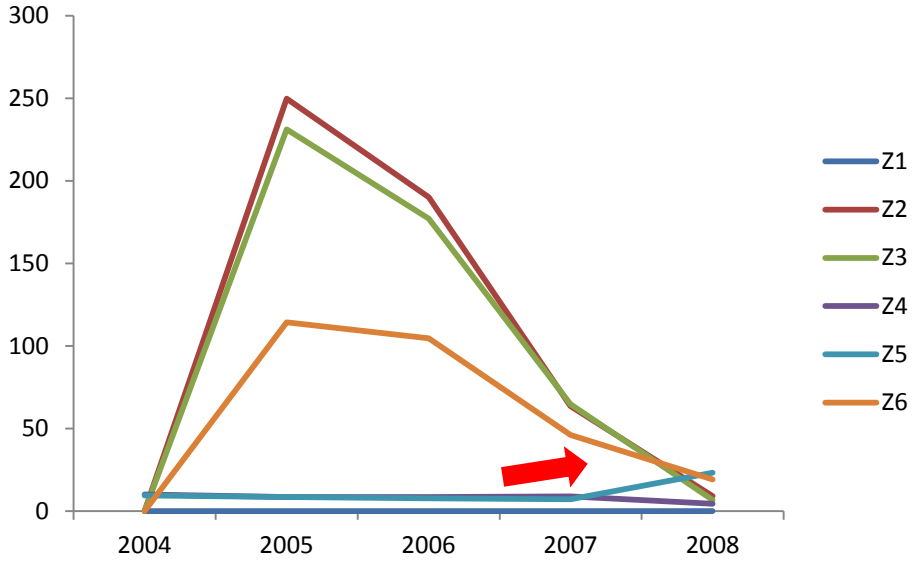


The lower the riskier

Case 3 - Lloyds

- Event: Bailout
- Dates: 7/3/2009 and 11/3/2009
- Sources: London Lite, FT

Year	Z1	Z2	Z3	Z4	Z5	Z6
2004	0.0476	0.0000	0.0000	10.0477	9.5757	0.0000
2005	0.0403	249.8118	231.2036	8.6054	8.6358	114.4149
2006	0.0391	190.0139	177.1142	8.4743	7.7641	104.7241
2007	0.0410	63.5240	64.7404	9.0121	7.1163	46.2099
2008	0.0249	9.1938	6.8069	4.5140	23.2303	19.2817



The lower the riskier

Conclusions

- We introduce a new method to compute bank solvency (Z6) based on the coefficient of variation of the Return on Assets
- Z6 outperforms other Z measures in some cases studied
- We complement existing methods by building confidence intervals around them to reflect the degree of estimation error
- Our preliminary results show the importance of including estimation error while assessing risk exposures