Default and prepayment: an NPV analysis under a Markovian dynamics of the credit market

Credit Scoring and Credit Control XIII

Edinburgh August 28-30, 2013

Lorenzo Quirini
Consum.it – Monte dei Paschi di Siena
Head of "Servizio Sistemi Decisionali, Monitoraggio e Scoring"

Luigi Vannucci
University of Florence

Fabrizio Cipollini
University of Florence

Aim of the presentation

How to evaluate the customer's credit relation when the payment process depends on changing credit market conditions (CMCs)

The main topic is to analyze the Net Present Value (NPV) taking into account default risk, prepayment risk and CMCs for a single transaction that is representative of a given population of customers, for example accepted and rejected ones

The model will be generalized at a portfolio level, showing how to evaluate the first two moments of its NPV

The application of the model is valuable to accept or reject loans in the credit granting process

Steps of the analysis

Input 1: Credit market conditions

(given by a Markov Chain)

↓

Input 2: Customer's Payment Process

(default and prepayment random times)

↓

Output: NPV as a random variable

MPS CONSUM.IT GRUPPOMONTEPASCHI

Input 1

Over the loan term, *n*, at every period the CMCs take a distinct state in a given finite set; the time period can be assumed varying on a monthly base

For instance, with two states G (Good status) and B (Bad one), a possible scenario, given by a sequence of states, is

(B, B, G, …, B, …, G, G)

States change according to a homogeneous Markov Chain

Example: Initial state B with transition probabilities matrix

	CMC Bad	CMC Good
CMC Bad	0.92	0.08
CMC Good	0.04	0.96

MPS CONSUM.IT GRUPPOMONTEPASCHI

The local probabilities of default and prepayment depend on the CMCs and represent the second input for the model

Input 2

The local probabilities of default and prepayment are for $i = 1, \ldots, n$

$$\lambda_i \in \{\lambda_i(B), \lambda_i(G)\} \qquad \mu_i \in \{\mu_i(B), \mu_i(G)\}$$

The probability distributions of the default time and prepayment time are

$$d_1 = \lambda_1 \qquad e_1 = \mu_1$$

$$d_h = \prod_{i=1}^{h-1}(1-\lambda_i-\mu_i)\cdot\lambda_h \qquad e_h = \prod_{i=1}^{h-1}(1-\lambda_i-\mu_i)\cdot\mu_h$$

for $h = 2, \ldots, n$ for $h = 2, \ldots, n-1$

MPS CONSUM.IT GRUPPOMONTEPASCHI

Other parameters as inputs for the model

The other parameters of the model

- **Loan information: granted amount f, term n, instalments r_i (i = 1, …, n)**
- **Deterministic recovery rate in case of default: t_i (i = 1, …, n)**
- **Evaluation rate and cost of funding: y constant over the term of the loan**

NPV is a mixture random variable

The NPV variable is a mixture of the conditional ones in which the conditioning events are all the possible scenarios (2^n)

In order to manage the huge number of possible scenarios, an approximation of their distribution can be obtained by simulation

For any given scenario H, the evaluation of the first two moments of the conditional NPV, $E(\text{NPV}^k \mid H)$, $k = 1, 2$, is done

MPS
CONSUM.IT
GRUPPOMONTEPASCHI

The relation that links the *k*-th moment of the conditional NPV to the inputs show the dependency on all the risk parameters

First two moments of NPV conditioned to a given scenario *H*

$$E\left[NPV^k \mid H\right] = P(E) \cdot \left(\sum_{j=1}^{n} r_j \cdot (1+y)^{-j} - f \right)^k$$

$$+ \sum_{h=1}^{n} d_h \cdot \left(\sum_{j=1}^{h-1} r_j \cdot (1+y)^{-j} + t_h \cdot (1+y)^{-h} \cdot \left(\sum_{j=h}^{n} r_j \cdot (1+x)^{-(j-h)} \right) - f \right)^k$$

$$+ \sum_{h=1}^{n-1} e_h \cdot \left(\sum_{j=1}^{h-1} r_j \cdot (1+y)^{-j} + (1+y)^{-h} \cdot \left(\sum_{j=h}^{n} r_j \cdot (1+x)^{-(j-h)} \right) - f \right)^k$$

The first term occurs when there are no risks

The second term occurs in case of default

The third term occurs in case of prepayment

$$P(E) = 1 - \sum_{h=1}^{n} d_h - \sum_{h=1}^{n-1} e_h$$

k **=1, 2**

x **is the contractual internal rate of the operation**

MPS CONSUM.IT GRUPPOMONTEPASCHI

In the model development several choices have to be made

Some practitioner's tips

1) The Markov Chain and the initial probability distribution of the states can be estimated, for example, by a Hidden Markov Model using firm's own data. This approach can be used if one supposes that the CMCs can be interpreted as latent variables

2) The local probabilities of default and prepayment have to be estimated with reference to the entire population (i.e considering rejected applicants also); if one is interested in applying any reject inference technique, a Heckman model type is summarized in the appendix of this presentation where it is shown how it works for a continuous outcome variable (see Conference Papers)

3) Constant local probabilities of default and prepayment depending on the CMCs can be used as a first approximation

For a given loan, two different simulations are performed depending on the probability distribution over the initial state

An example

- f = € 8,500, n = 60 months, r_i = 190 euros
- Constant recovery rate, t_i = 30%
- y = 500 *bps* (yearly base)
- Constant λ_i in {0.006, 0.003}
- Constant μ_i in {0.008, 0.010}
- Probabilities for the initial state = (1, 0) B as starting value
- Probabilities for the initial state = (0, 1) G as starting value
- Number of simulated scenarios for both the initial states = 1000
- Transition probabilities

	B	G
B	0.92	0.08
G	0.04	0.96

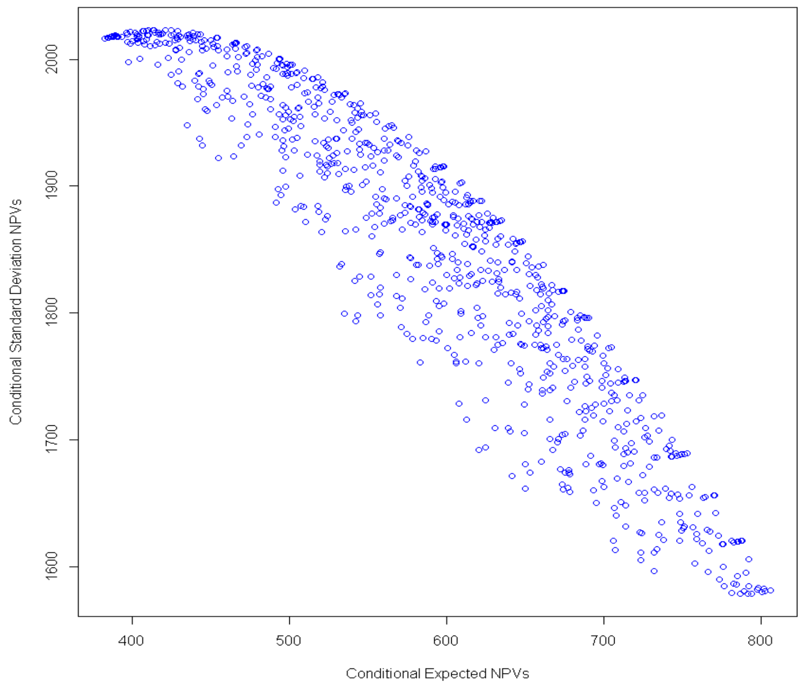
Cumulative Probabilities

	B	G
Default	0.18	0.16
Prepayment	0.37	0.39

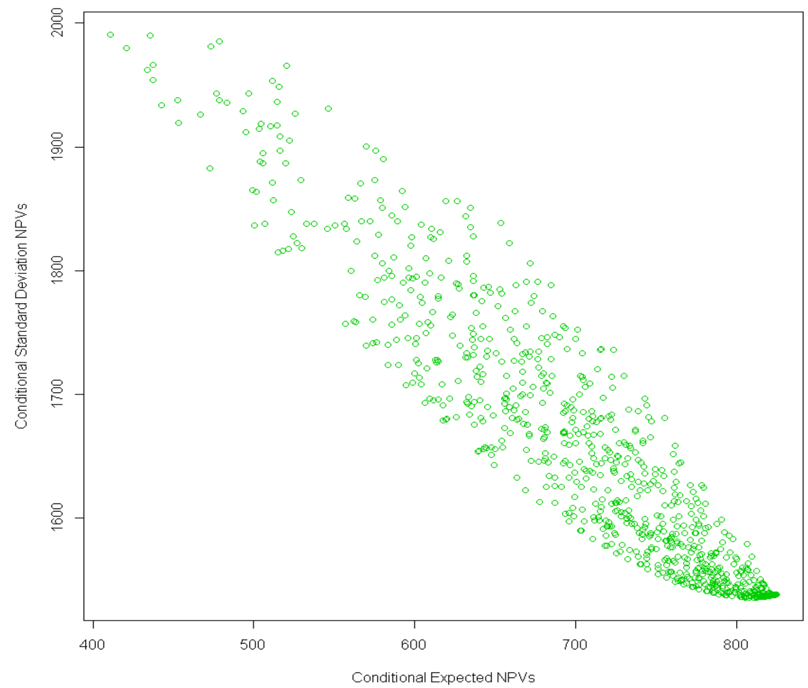
MPS CONSUM.IT GRUPPOMONTEPASCHI

The simulations show how the initial CMCs affect the conditional first two moments

Conditional Mean Vs Conditional Standard Deviation NPVs



Initial state = B

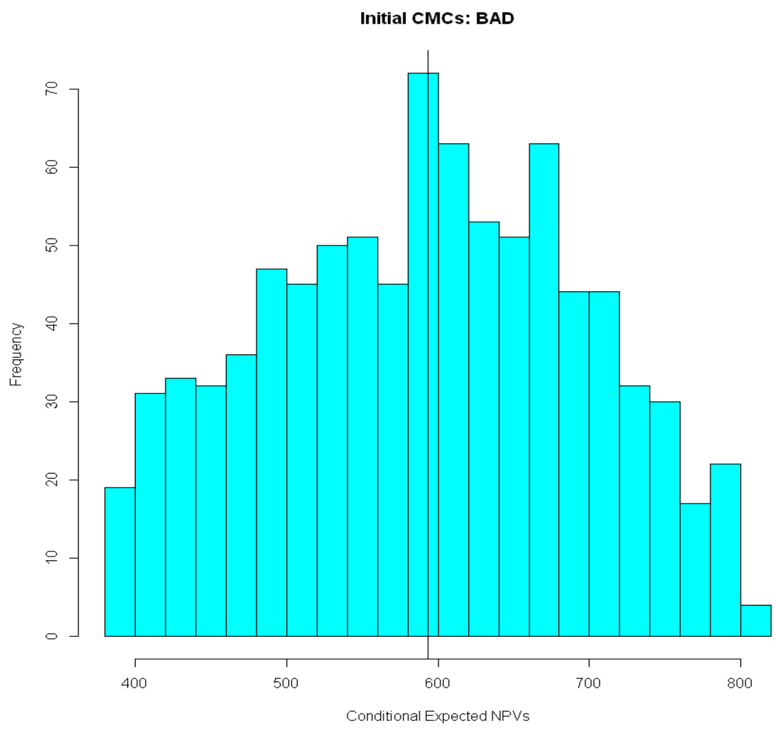


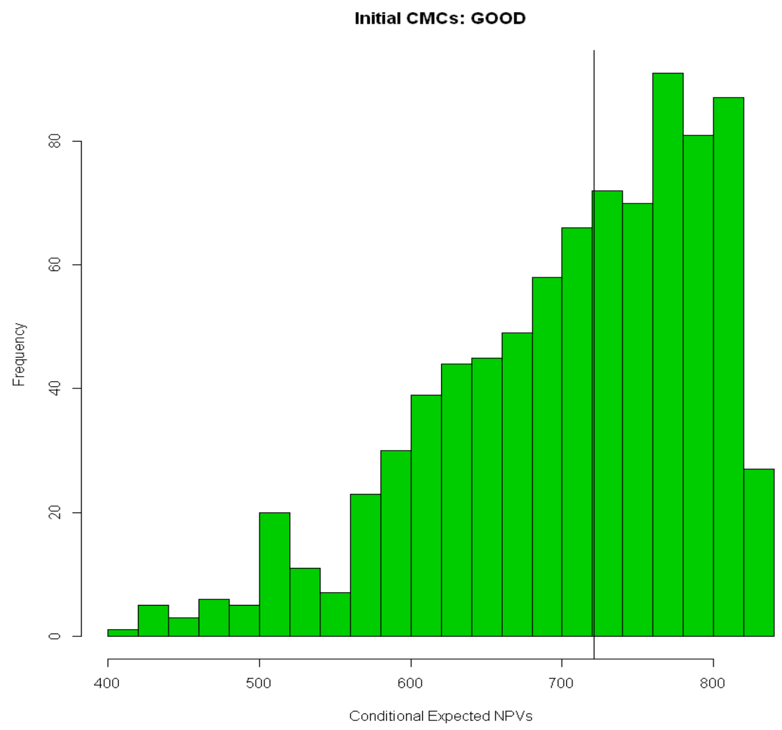
Initial state = G

MPS CONSUM.IT GRUPPOMONTEPASCHI

The difference between the expected NPVs changing the initial state of the CMCs is €128 (~ 8% of the contractual NPV)

The histograms for the NPVs conditional means under different initial CMCs





Contractual NPV = €1,607

NPV Mean

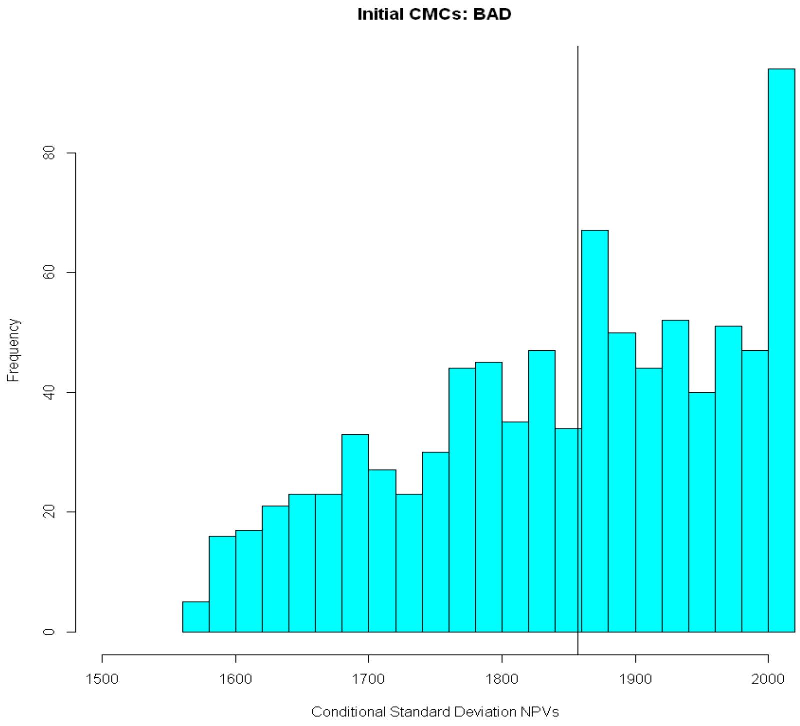
Initial CMCs

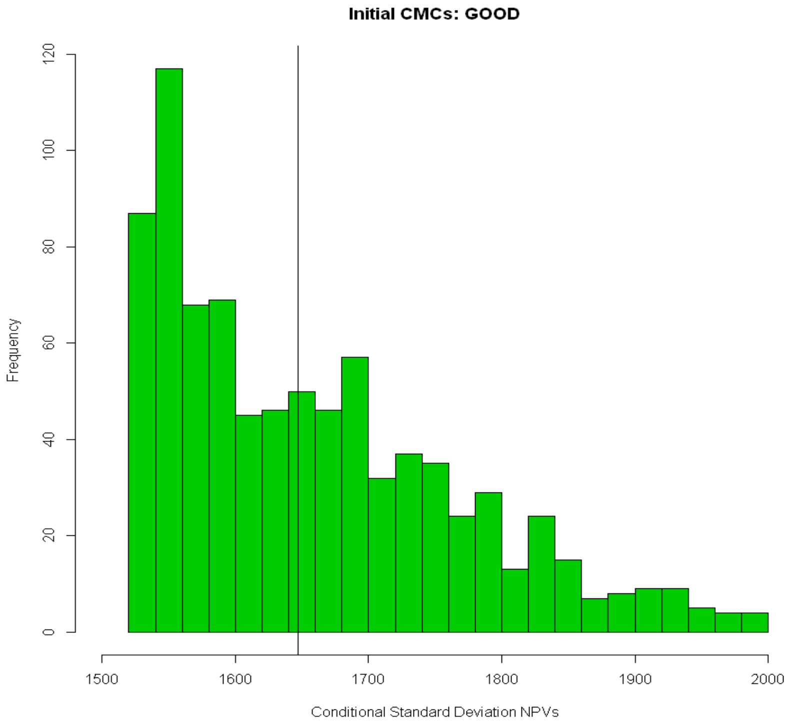
Bad = €593 Good = €721

MPS CONSUM.IT GRUPPOMONTEPASCHI

The NPV Standard Deviation is also affected by the CMCs initial state

The histograms for the NPVs conditional standard deviations under different initial CMCs





NPV Standard Deviation
Initial CMCs
Bad = €1,857
Good = €1,647

MPS CONSUM.IT GRUPPOMONTEPASCHI

The variance of the NPV for a single operation is mainly specific

Variance decomposition of the NPV for a single transaction

$$\mathrm{var}(NPV) = E_H(\mathrm{var}(NPV \mid H)) + \mathrm{var}_H(E(NPV \mid H))$$

$E_H(\mathrm{var}(NPV \mid H))$ **is the expected value, under the scenarios' probability distribution, of the conditional variances (specific component)**

$\mathrm{var}_H(E(NPV \mid H))$ **is the variance, under the scenarios' probability distribution, of the conditional means (systematic component)**

For this example the variance is mainly specific

€3,448,976 = €3,437,527 + €11,448 (for initial state = B)

The mean and the variance of a homogeneous portfolio of size *s* in term of the conditional first two moments and the probability distribution of the *M* scenarios q_m

The NPV mean and variance for a portfolio of *s* identical transactions

$$PV_s = \sum_{l=1}^{s} NPV_l$$

$$E(PV_s) = E_H(E(PV_s \mid H)) = s \cdot \sum_{m=1}^{M} q_m \cdot E(NPV_1 \mid H_m)$$

$$\mathrm{var}(PV_s) = E_H(\mathrm{var}(PV_s \mid H)) + \mathrm{var}_H(E(PV_s \mid H))$$

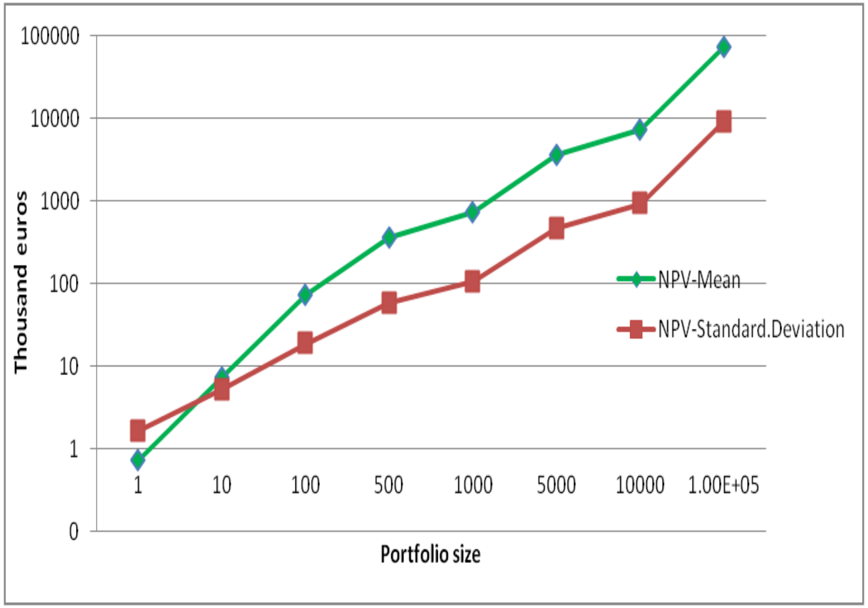
$$E_H(\mathrm{var}(PV_s \mid H)) = \sum_{m=1}^{M} q_m \cdot \left[\begin{array}{l} \left(s \cdot E(NPV_1^2 \mid H_m\right) + \\ + s \cdot (s-1) \cdot E^2(NPV_1 \mid H_m) - (s \cdot E(NPV_1 \mid H_m)^2) \end{array} \right]$$

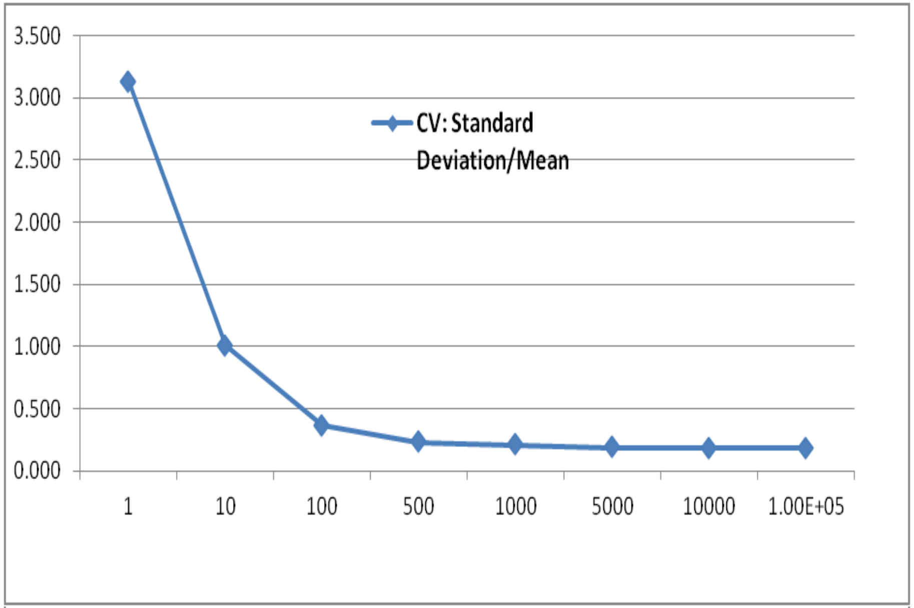
$$\mathrm{var}_H(E(PV_s \mid H)) = \sum_{m=1}^{M} q_m \cdot (s \cdot E(NPV_1 \mid H_m) - E(PV_s))^2$$

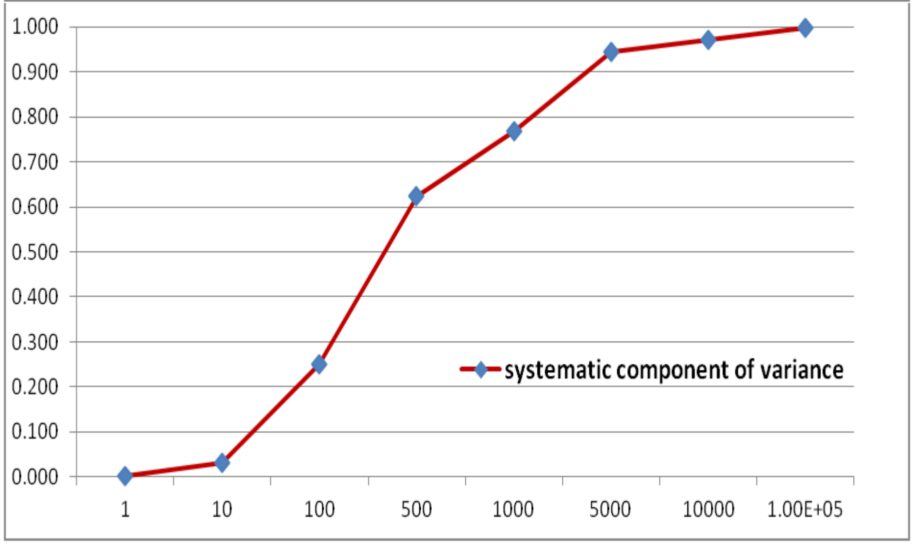
MPS
CONSUM.IT
GRUPPOMONTEPASCHI

Increasing *s* the expected NPV and the standard deviation grow, CV is asymptotically strictly positive; the systematic risk component predominates

Portfolios NPV of size *s* (initial state Bad)







With *s* = 10,000

NPV mean = €5.93 mln
NPV standard deviation = €1.08 mln
CV = 0.183
Systematic component of variance = 0.971

With *s* = 100,000

NPV mean = € 59.3 mln
NPV standard Deviation = €10,8 mln
CV = 0.180
Systematic component of variance = 0.997

MPS CONSUM.IT GRUPPOMONTEPASCHI

Summary

NPV is a random variable whose first two moments have to be evaluated taking into account default risk, prepayment risk and CMCs

CMCs may be described by a homogeneous Markov Chain

Initial state of the Chain influences NPV first two moments

NPV for a portfolio of identical loans has a Coefficient of Variation strictly positive and its variance is mainly due to the systematic component (*s* increasing)

Key references

Thomas L., D.B. Edelman and J.N. Crook (2002), *Credit Scoring and its Applications*, SIAM Monographs on Mathematical Modeling and Computation, Philadelphia, USA

Quirini L. and L. Vannucci (2010), A New Index of Creditworthiness for Retail Credit Products, *Journal of Operation Research Society*, 61, 455-461

Quirini L. and L. Vannucci (2013), Creditworthiness Dynamics and Hidden Markov Models, *Journal of Operation Research Society*, doi: 10.157/jors.2012.181

E-mail
lorenzo.quirini@consum.it

luigi.vannucci@dmd.unifi.it

fabrizio.cipollini@ds.unifi.it

MPS CONSUM.IT GRUPPOMONTEPASCHI