Efficient Quantification of Model Risk

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Examples, graphs and tables shown are based on mock data and are for illustrative purposes only.

Overview

Model Risk Background

- Model risk – an emerging and influential idea in bank regulation and credit risk model management.

- Model risk assessment needs quick and quantified sensitivity analysis.

Geometry and Model Sensitivity

- Model specification and model sensitivity analysis can be presented geometrically within a classical, mathematically rich theory.

Efficient Sensitivity Analysis

- This point of view gives practical, quick and quantified strategies for managing sensitivity analysis and model risk.

Model Risk Background

The US Regulator (Fed / OCC 2011-12a)

- *“The use of models invariably presents model risk, which is the potential for adverse consequences from decisions based on incorrect or misused model outputs and reports.”*

FSA - Turner Review - March 2009

- *“Misplaced reliance on sophisticated maths”*
  - The assumptions and limitations of the models were not communicated adequately to the pricing and lending decision-makers.

BoE - The Dog and the Frisbee – Haldane, August 2012

- *“… opacity and complexity… It is close to impossible to tell whether results from [internal risk models] are prudent.”*
  - If we cannot say why we trust a model, are we right to use it?

Model Risk Background

Fed / OCC 2011-12a

- *"Model Risk should be managed like other types of risk."*

Identify model risks
Quantify model risk impact
Act to mitigate and manage model risks
Monitor model risks

This talk will focus on

- Specification risk – the part of model risk connected with model selection;
- Quantification of specification risk and its impact on the model.

Example Model Risk

A PD model is proposed for use in IRB compliance and capital calculation

- The model includes a factor W that has 20% missing values. The missing values have been filled in, all with the same "mean" value, and the preferred model has been built with this imputed factor. Missing values tend to be associated with older accounts.

Identify	The missingness and its treatment risks that the model includes or weighs the factor incorrectly for future use, or the model is weakened by other inappropriate factor selections or de-selections. The model could then predict and rank PDs incorrectly now or in future.
Quantify	Hypothesise other ways of treating or distributing the missing values: a/ missing value as a separate class; b/ modelled imputation (MAR); c/ introduce missing values into newer accounts at same rate as older accounts, etc. How differently could the model be built? What variation in PD and RWA?
Act	For capital purposes, propose a conservative uplift scalar ??% to PD
	Set up monitoring so that, if the missing rate rises above ??% or below ??% , conservatism level will be reassessed.
	Assessing the conservatism or model risk as sufficiently material, back-fill the development data and recalibrate the model when sufficiently complete.

Example Model Risk

Hypothetical sensitivities for factor with 20% missing values

- Which are worth exploring, and what is their impact on model choice?

Model Sensitivity

Specification Risk and Sensitivity Analysis

- How different would the model be if … ?
- And how different might be the decisions that result?

Sensitivity Analysis is the key to quantitative specification risk

- But it requires development and comparison of many alternative models.
  - Is this work out of proportion to the benefits?
- Can we assess model sensitivities quickly?
  - Without refitting models?
  - Without reference to particular structures or methodologies?

Describing Data and Models

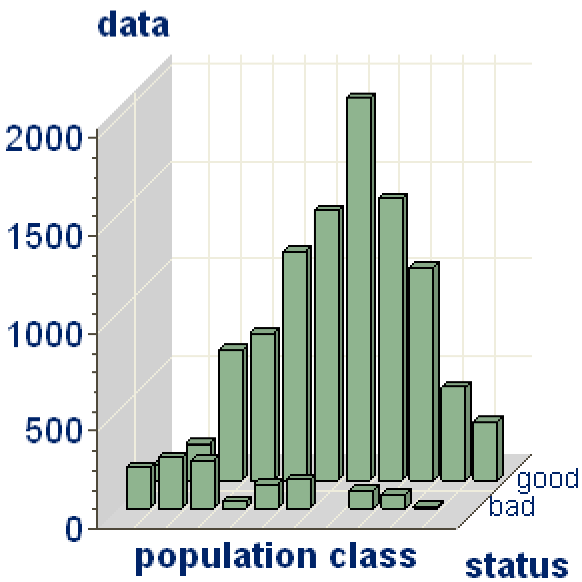
Models: preferred descriptions of data

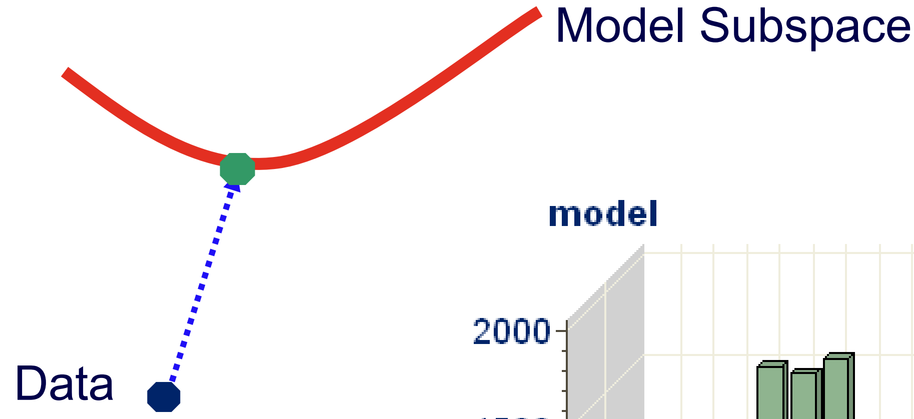
- A model is a description of the development data.
  - Model developers chose one of these descriptions for use in a decision.
  - Specification Risk considers the degree to which this choice could influence the decision.
- This talk considers frequency histogram or contingency table descriptions of models and data (Kullback, Centsov, etc.).
- Models and Data live in the same space of possible descriptions.
  - The observed data is a single point in this space.
  - Models are also “data” points: points that are preferred for use or descriptive convenience.

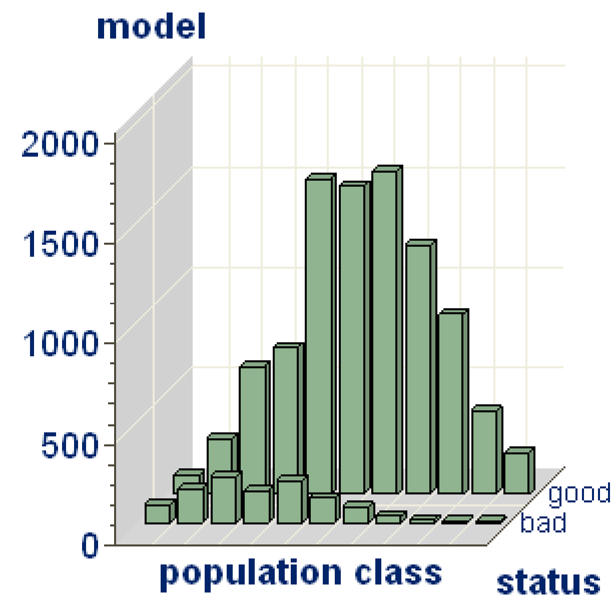
Describing Data and Models

Model Development

- Choosing the best data description from a space of preferred descriptions







Model chosen by accepted principles:

eg Maximum Likelihood Principle

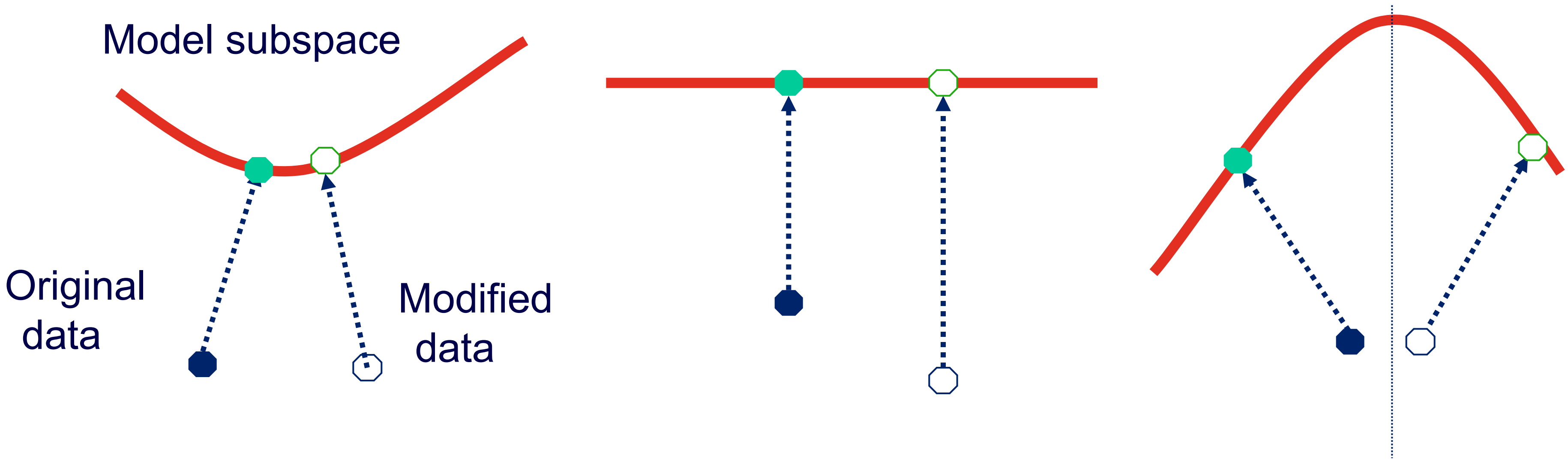
Equivalently, minimum Kullback-Leibler divergence

Geometry and Model Sensitivity

Model sensitivity analysis is a data shift problem

- The model is chosen “closest” to the data – how sensitive is this choice to data shift?

Just right



Type 3 (or type 0) Error

Over-fitting

Over-sensitive / discontinuous

Geometry and Model Sensitivity

The data shift problem is geometric and mathematically rich

- Centsov (1965, et seq.), Efron (1978 et seq.), Amari et al. (1982 et seq.), Lauritzen (1980s), Critchley et al. (1993 et seq.), etc.

- Recent developments in Machine Learning, by Kanamori, Shimodaira (2009) and others, are particularly relevant to sensitivity analysis.

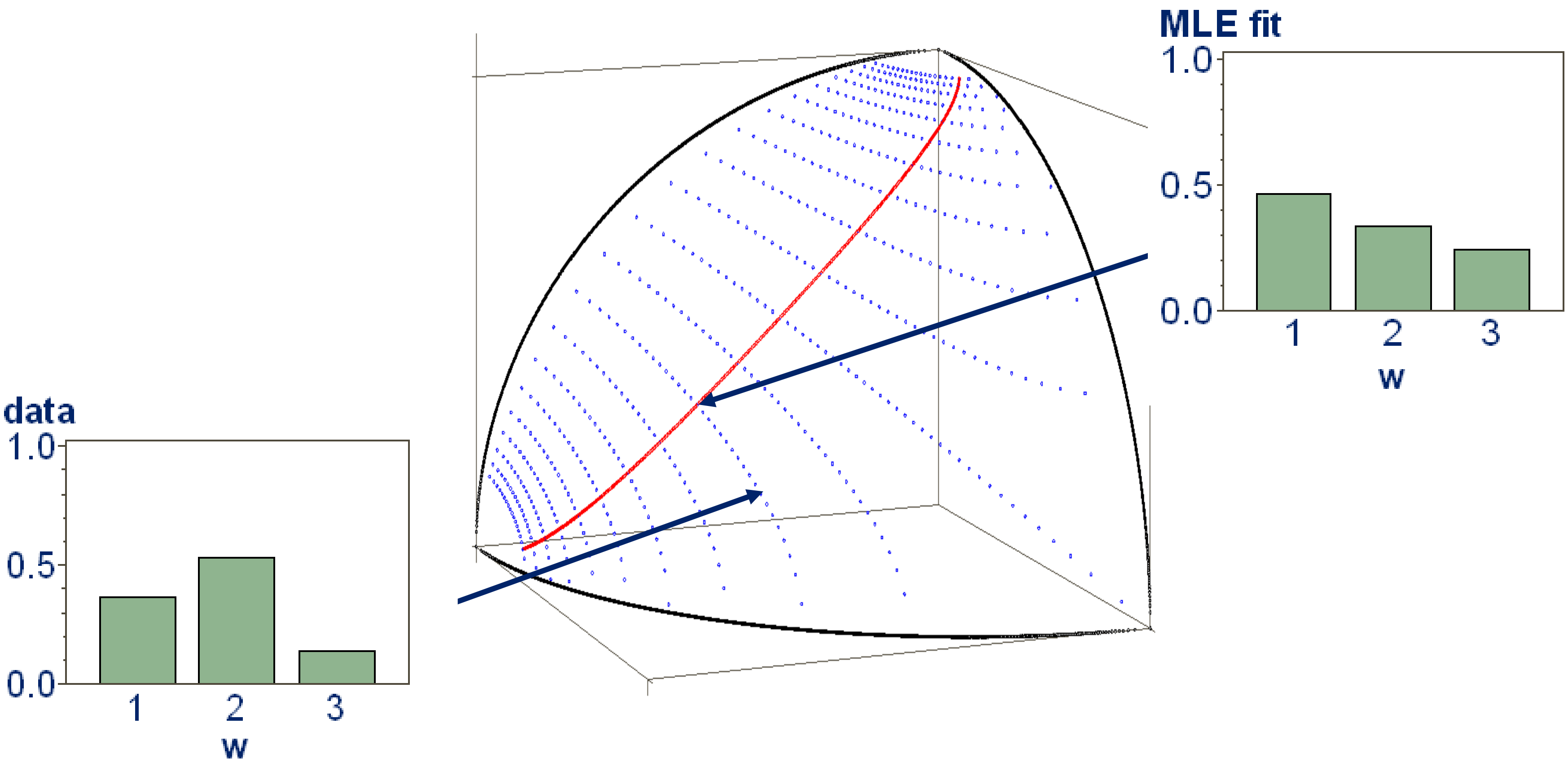
- Hellinger distance $s^2(x,x') = \Sigma_w (SQRT(x_w) - SQRT(x'_w))^2$
  where x_w , x'_w are cell frequencies.

- This metric is naturally connected with
  - Kullback-Leibler -divergence : $\delta s^2 = KL(x,x+\delta x) = KL(x+\delta x,x)$ up to second order;
  - Chi-squared : $\delta s^2 = \Sigma_w \delta x_w^2 / x_w$;
  - Spherical geometry : $4x = u^2$;
  - Bootstrapping variation.

- In this metric, the model space curvature is true and reflects model specification sensitivity accurately.

Geometry and Model Sensitivity

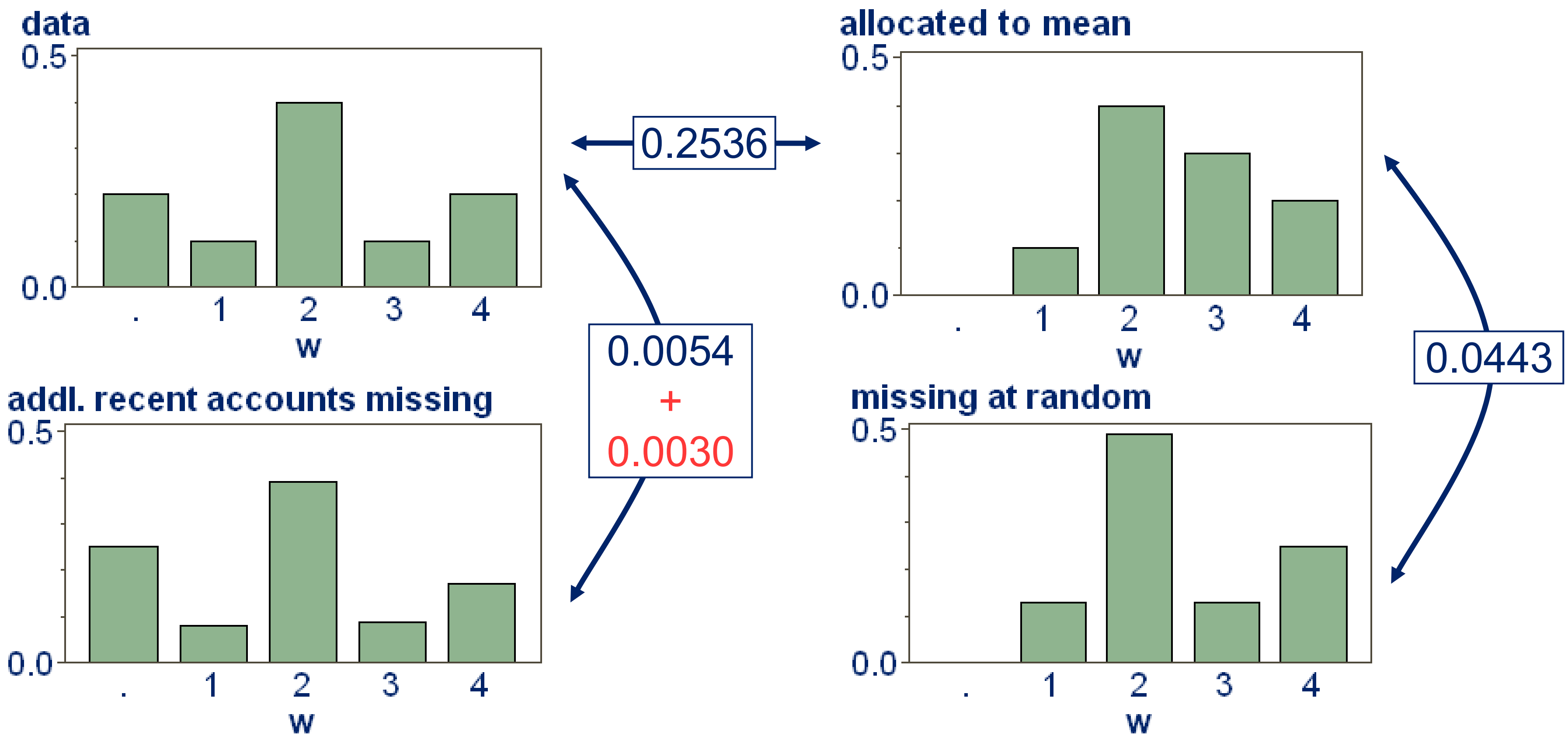
Fitting a model to data: a log-linear example $m_w = ce^{aw}$



Geometry and Model Sensitivity

Example: Sensitivities for factor with 20% missing values

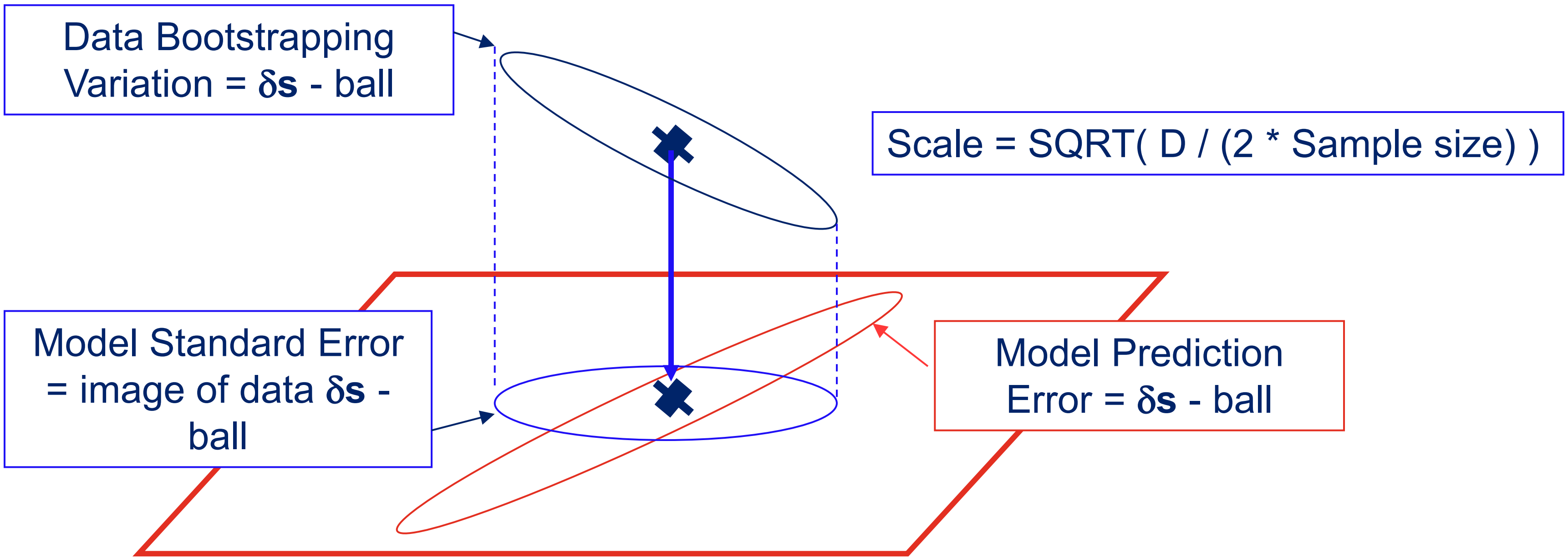
- Distances (squared) between hypothetical alternative datasets, computed in spherical metric from marginals illustrated.
- Additional distance estimated by KL information value relative to marginals.



Geometry and Model Sensitivity

A geometric principle implied by bootstrapping (for large samples)

- The Bootstrapping variation in data space is a Hellinger ball.
- The model standard error is the projection of the bootstrapping variation onto model space.
- The appropriate radius of the ball derives from a connection with Chi-squared, degrees of freedom = dimension of the data space, D, assumed large.



Managing Sensitivities

Data Shift distance helps manage sensitivity analysis:

- Filtering using the Bootstrapping Scale as a cut-off.
  - Other scales can be set to address other recognised impacts, eg capital impact.
- Data Shift distances give first view of potential impact of each model risk.
  - Especially useful when model shifts are not easy to determine in detail.
- Analysis tasks can be planned in proportion to the quantified data shift.
  - Modellers can now look in more detail at a much reduced and prioritised set of sensitivities.

Managing Sensitivities

Model Risk - 20% missing values - example revisited :

- PD has been built from a pool of 12 classed factors:
  - Dimension of the data space (roughly = number of cross-tab cells), D = 50,000 , say.
- PD model built on sample of N = 500,000 records, say.
- Bootstrap scale (squared) is D/2N = 0.05 .

Sensitivity Test	Data Shift distance-squared	Observation	Action
Force missing values among new accounts	0.008	Correlation of missingness with age of accounts is unlikely to cause significant change in model build.	No need to investigate.
Missing at Random *v.* Mean Allocation	0.045	These two imputation options are likely to result in same factor selections, but different factor weights.	Low priority
Missing as a separate category	0.254	Model build is likely to be materially different if missing is treated as a separate category.	High priority

Conclusions

Model Risk Principles

- Model risk and model specification risk are important and growing parts of banks' risk management.
- The key to quantitative specification risk assessment is sensitivity analysis, and
- The key to practical sensitivity analysis is a quick, effective method to gauge model variation without having to rebuild models.

Efficient Sensitivity Analysis and Model Risk Management

- Classical ideas of statistical geometry and information theory add insight to the quantification of model risk: sensitivity analysis is framed as a differential data-shift problem.
- The Hellinger distance is a practical metric that helps quantify, filter and prioritise sensitivity analysis without needing to rebuild models.