Exposure at Default models with and without the Credit Conversion Factor

Edward Tong, Christophe Mues, Iain Brown, Lyn Thomas

edward.n.tong@bankofamerica.com

Credit Scoring and Credit Control XIV
Edinburgh, August 26-28 2015

Disclaimer

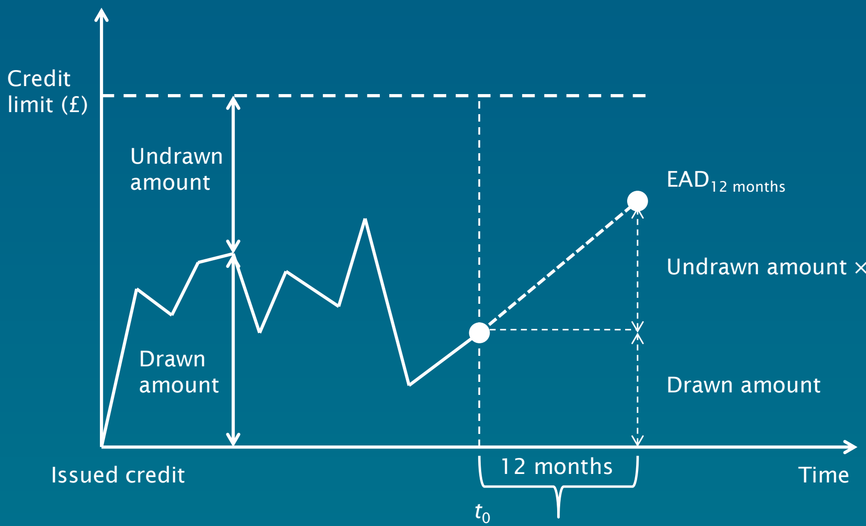
The opinions expressed in this document are solely the authors' and do not necessarily reflect those of the Bank of America or any of its subsidiaries.

EAD (Exposure at Default)

- Basel II/III - requirement for Internal Ratings Based (IRB) Advanced approach for calculating minimum capital requirements, CCAR stress testing
- EAD defined as gross exposure in the event of obligor default, typically in 12 months
- This study: EAD for credit cards (revolving exposures)

EAD model approaches

Notation
$E(t_d)$ = EAD
$E(t_r)$ = balance at time r
$L(t_r)$ = limit at time r



Credit conversion factor (Taplin et al. 2007, Jacobs 2010, Qi 2009)

$$CCF = \frac{E(t_d) - E(t_r)}{L(t_r) - E(t_r)}$$

$$EAD = Current\ \ Drawn + (CCF \times Current\ \ Undrawn)$$

EAD model approaches (cont'd)

Notation
$E(t_d)$ = EAD
$E(t_r)$ = balance at time r
$L(t_r)$ = limit at time r

Utilization change (Yang & Tkachenko, 2012)

$$util_{ch} = \frac{E(t_d) - E(t_r)}{L(t_r)}$$

Direct EAD based on OLS (Taplin et al., 2007)

Mixture models (Witzany, 2011, Leow & Crook, 2013)

Research objectives

- To directly model the EAD amount
- Compare performance of direct EAD models with known industry models – Credit Conversion Factor (CCF) and Utilization Change
- Consider segmentation by credit usage to combine direct EAD and CCF models
- Model time to default as a predictive variable using weighted PD approach

Data

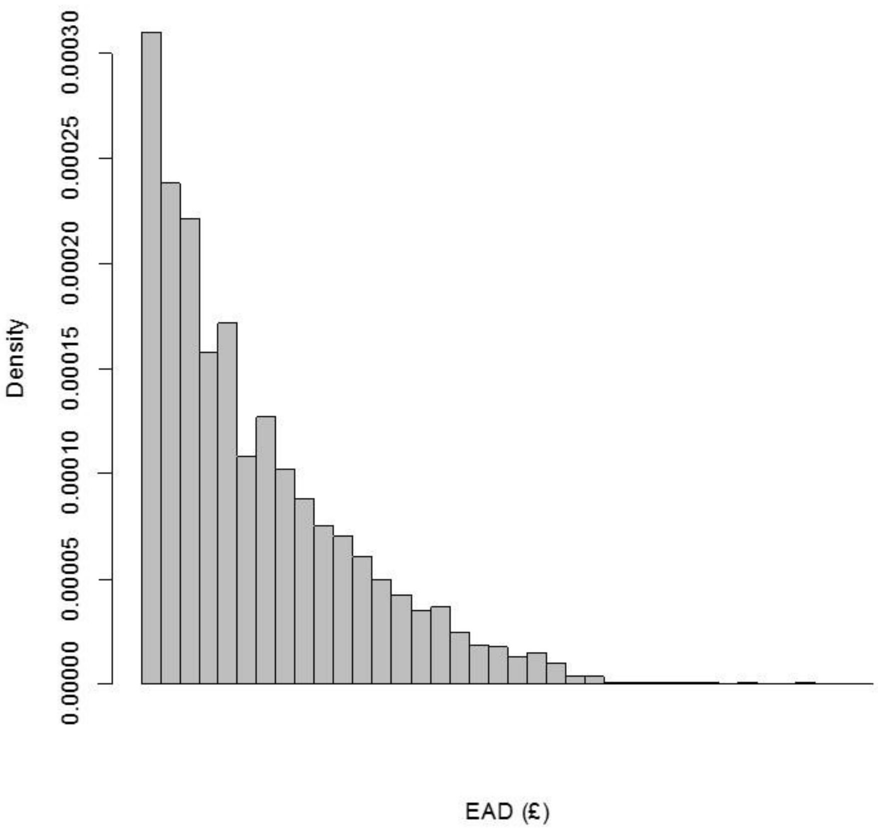
- UK bank
- Credit card portfolio
- 3 years of data (2001 to 2004)
- All observations are defaulted accounts
- >10k defaults in total sample

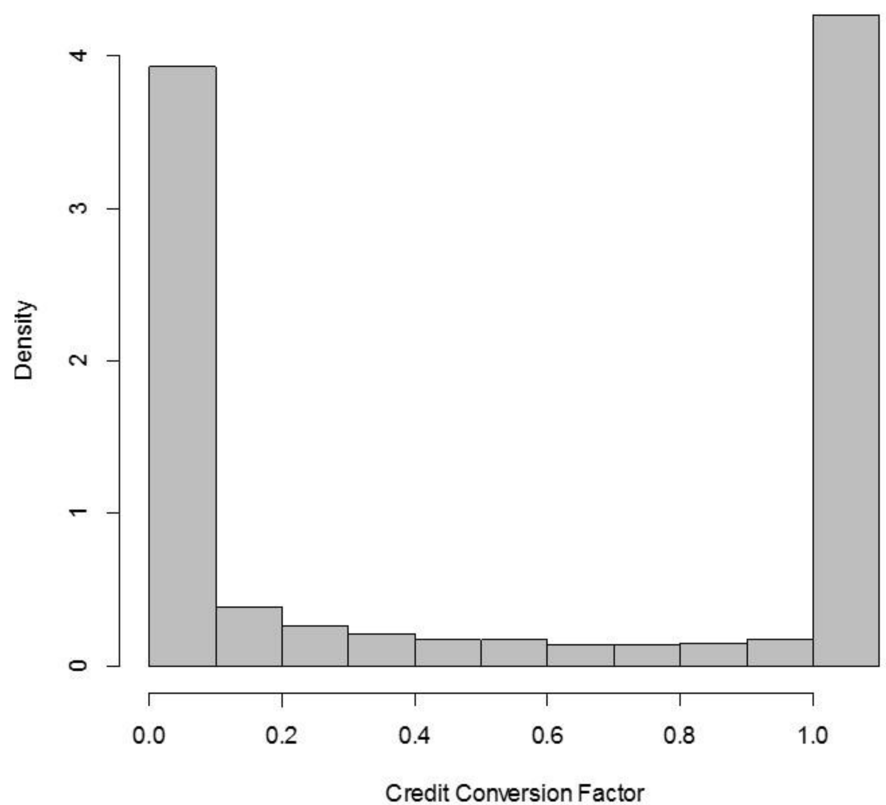
Data (cont'd)

- Response variables of
  EAD, CCF, Utilization Change

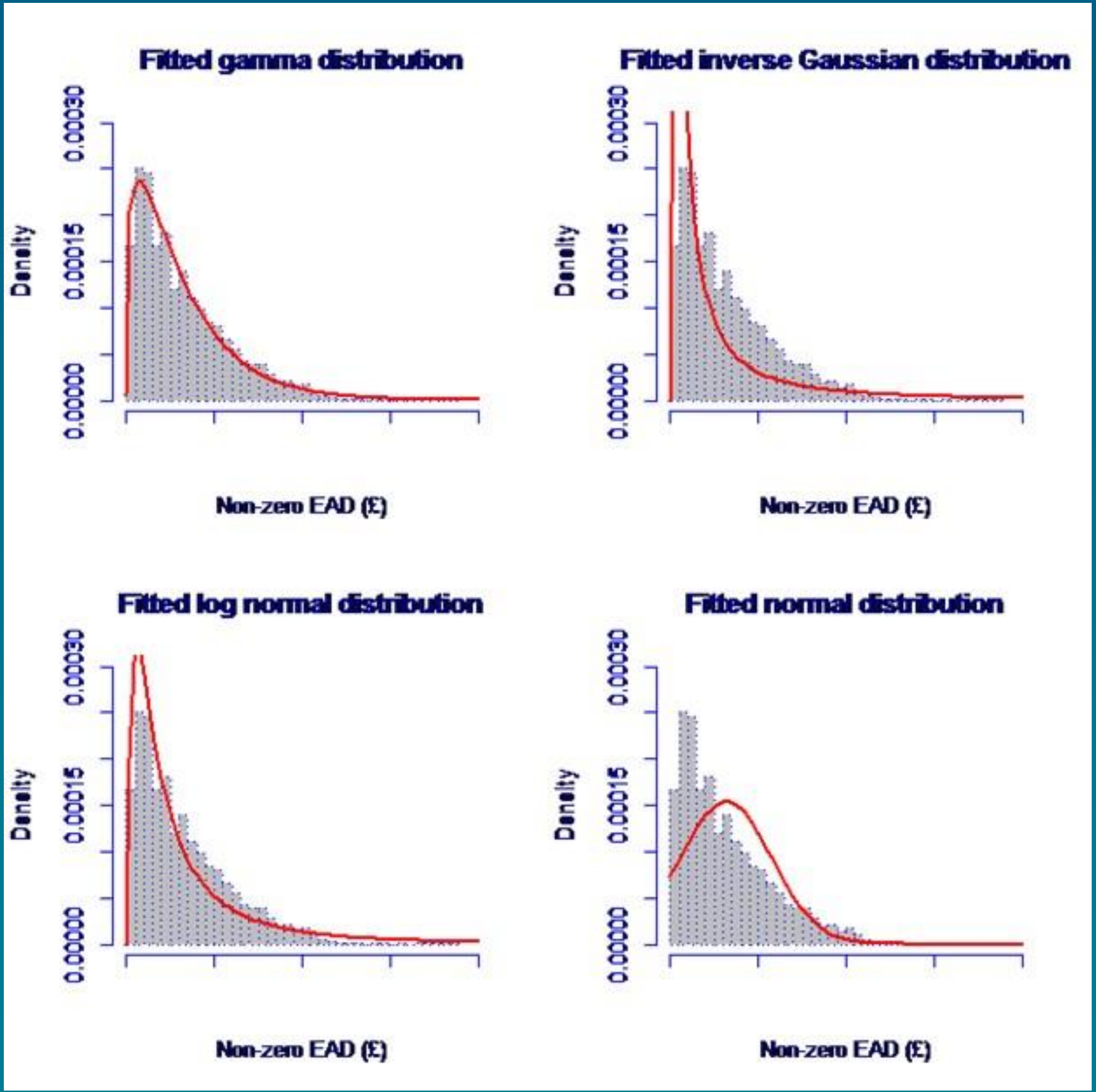
- 11 behavioural variables including
  Committed Size / Amount
  Drawn, Undrawn Amount
  Drawn Percentage (Credit Usage)
  Time to Default
  Rating Class
  Average Days Delinquent
  Absolute Change in Drawn Amount

Response variables
EAD and CCF





Fitted EAD distribution Training set



Zero-adjusted gamma distribution

The probability function of the ZAGA is defined by

$$f_Y(y \mid \mu,\sigma,\pi) = \begin{cases} \pi & \text{if } y = 0 \\ (1-\pi)\left[\dfrac{1}{(\sigma^2\mu)^{1/\sigma^2}} \dfrac{y^{\frac{1}{\sigma^2}-1} e^{-y/(\sigma^2\mu)}}{\Gamma(1/\sigma^2)}\right] & \text{if } y > 0 \end{cases}$$

for $0 \le y < \infty$, $0 < \pi < 1$, $\mu > 0$, $\sigma > 0$

where μ denotes mean, σ scale,

π probability of zero EAD

$$E(Y) = (1-\pi)\mu \text{ and } Var(Y) = (1-\pi)\mu^2(\pi+\sigma^2)$$

Generalized Additive Model for Location, Scale & Shape

- GAMLSS (Rigby & Stasinopoulos, 2005, 2007, 2015) implemented in `gamlss` package in R
- General framework for regression models
- Response variable $y \sim D(y \mid \mu, \sigma, \nu, \tau)$ where $D()$ can be any distribution (over 70 different types including highly skew and kurtotic distributions)
- Mortgage LGD model (Tong et al., 2013)

ZAGA model setup

$$\log(\mu) = \eta_1 = \mathrm{X}_1\beta_1 + \sum_{j=1}^{J_1} h_{j1}(x_{j1})$$

$$\log(\sigma) = \eta_2 = \mathrm{X}_2\beta_2 + \sum_{j=1}^{J_2} h_{j2}(x_{j2})$$

$$\operatorname{logit}(\pi) = \eta_3 = \mathrm{X}_3\beta_3 + \sum_{j=1}^{J_3} h_{j3}(x_{j3})$$

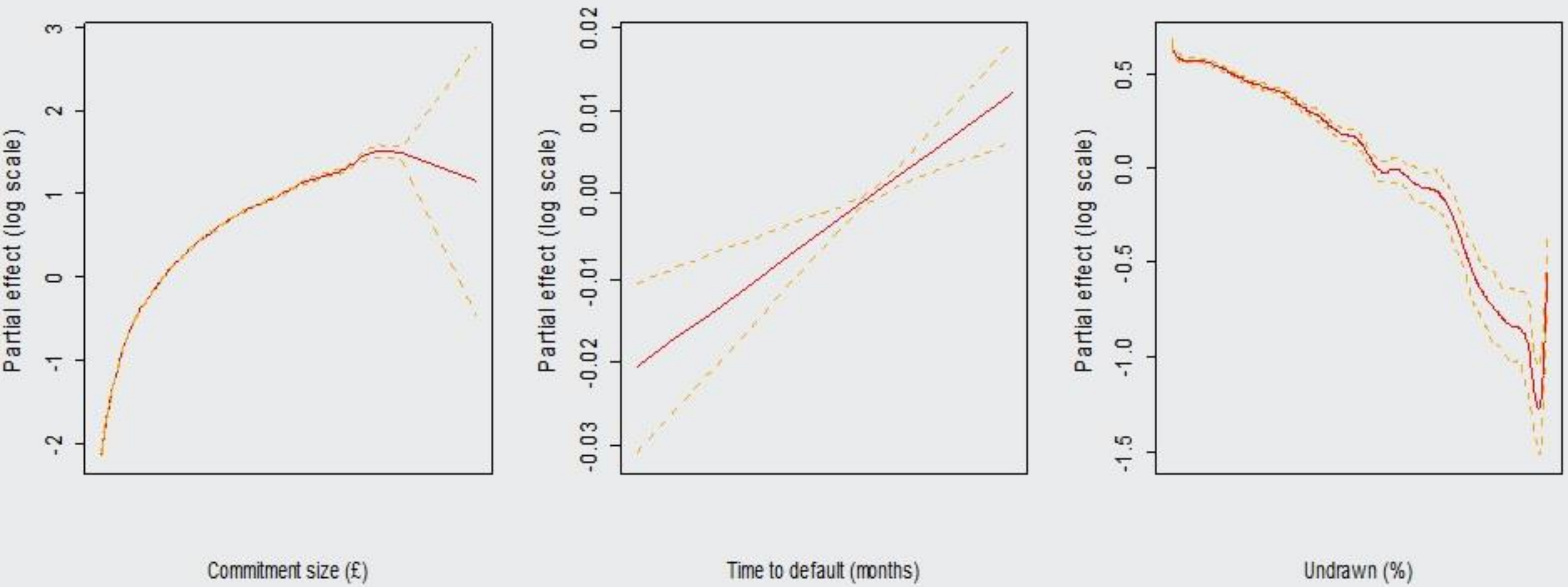
where $\mathrm{X}_k\beta_k$ denote parametric linear terms,

$h_{jk}(x_{jk})$ denote additive smoothers

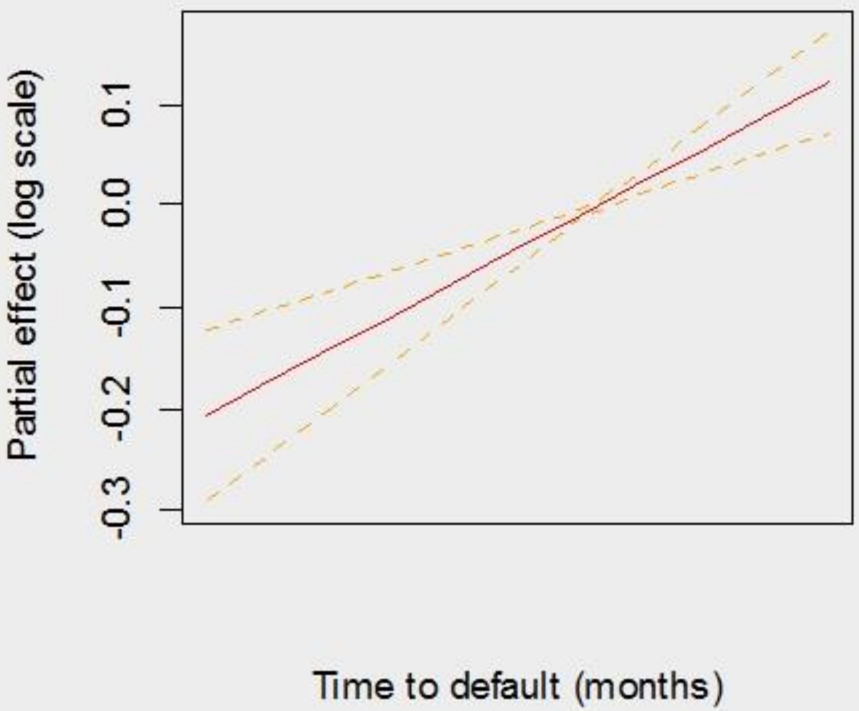
ZAGA model development

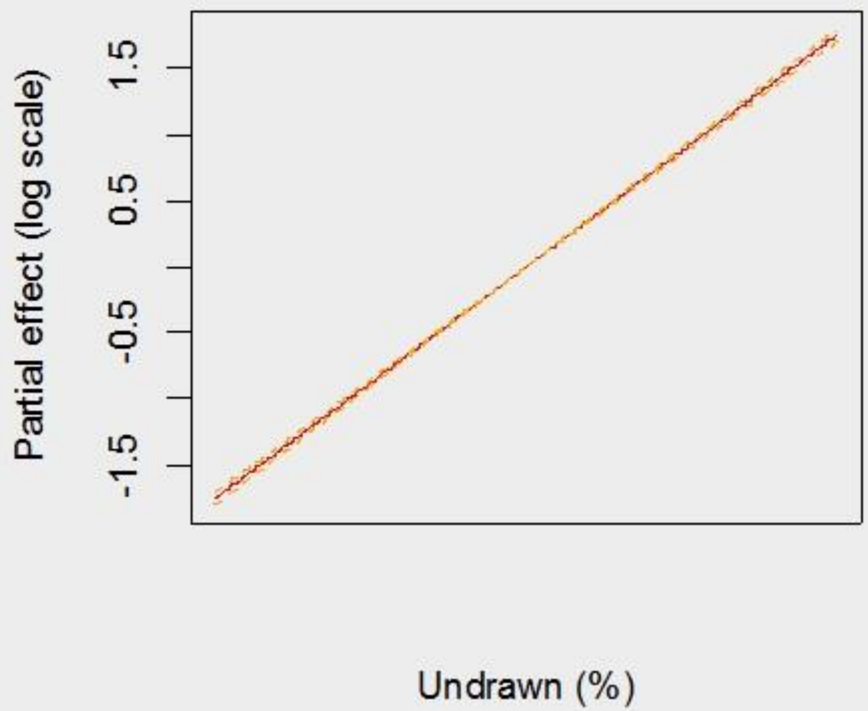
- Separate model components estimated for μ, σ and π components
- Developed with stepwise selection based on Akaike Information Criteria (AIC)
- Continuous variables fitted with smoothers based on penalized *B*-splines (Eilers & Marx, 1996)

Results
Mean of non-zero EAD



Results
Dispersion of non-zero EAD





Dispersion increases with Undrawn (%), decreases with credit usage

Benchmark models: CCF, Util. Change, PD weighted

- 3 CCF models based on OLS, Fractional Response Regression (Papke & Woolridge, 1996) and Tobit models (Tobin, 1956)
- Utilization Change based on Tobit model
- Survival EAD model - PD weighted approach to model time to default using Cox PH regression:

$$EAD = \sum_{t=1}^{12} \left(\frac{\left[S(t-1) - S(t) \right]}{1 - S(t=12)} \times EAD(t) \right)$$

Benchmark models: Segmentation by credit usage

- Segmentation using combined Direct EAD (ZAGA) and CCF models
- CCF models perform better for low credit usage, Direct EAD model better for high credit usage
- Credit usage of 90% was optimal cut-off for segmentation based on discrimination and calibration performance measures

Validation methodology

- 10-fold cross validation (CV)
- Spearman ρ
- MAE from EAD
- MAE_{norm} from $\dfrac{\textbf{EAD}}{\textbf{Commitment Size}}$
- RMSE, RMSE_{norm}

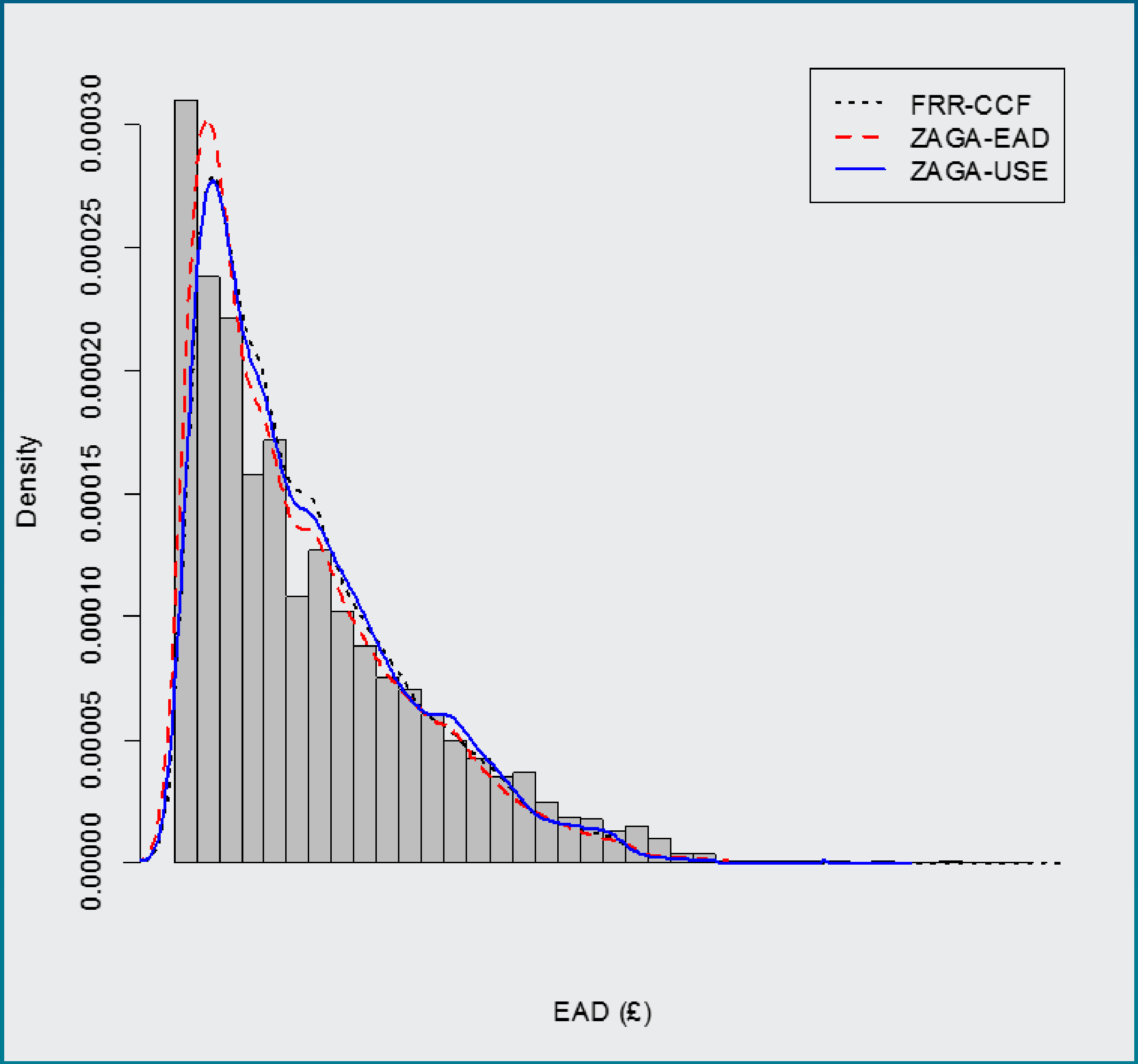
Validation with 10 fold CV

	OLS-CCF	Tobit-CCF	FRR-CCF	Tobit-UTIL	OLS-EAD	ZAGA-EAD	OLS-USE	ZAGA-USE
Spearman ρ	0.741	0.737	0.743	0.746	0.744	0.742	0.750	0.749
MAE	859.0	870.6	856.1	925.2	883.3	833.5	829.9	819.2
RMSE	1614.8	1586.3	1577.7	1654.3	1546.1	1602.5	1565.7	1571.0
MAE_{norm}	0.273	0.276	0.273	0.294	0.301	0.268	0.269	0.260
$RMSE_{norm}$	0.432	0.430	0.430	0.442	0.448	0.454	0.430	0.429

ZAGA-EAD, ZAGA-USE have lowest MAE

Survival EAD MAE=830.3, MAE_{norm}=0.266

Validation
Observed vs Fitted EAD Densities



Conclusions

- Modelling the EAD amount directly can produce competitively predictive EAD models
- Segmentation by credit utilization offers greater performance benefits - a combined approach with EAD and CCF models may work better
- The time to default variable can be used a priori within a PD weighted model

References

Djennad, A.D., Rigby R., Stasinopoulos, D., Voudouris, V., Eilers, P.H.C. (2015). Beyond location and dispersion models: the Generalized Structural Time Series Model with Applications. http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2577330

Eilers, P. H. C., & Marx, B. D. (1996). Flexible smoothing with B-splines and penalties. *Statistical Science*, 11(2), 89-102.

Jacobs, M., Jr. (2010). An Empirical Study of Exposure at Default. *Journal of Advanced Studies in Finance*, 1, 31-59.

Leow, M., Crook, J. (2013). A Two Stage Mixture Model for Predicting EAD. *Credit Scoring & Credit Control XIII*. Edinburgh, UK.

Qi, M. (2009). Exposure at Default of Unsecured Credit Cards. *Economics Working Paper 2009-2*. Office of the Comptroller of the Currency.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized Additive Models for Location, Scale and Shape. *Applied Statistics*, 54, 507-554.

Rigby, R. A. & Stasinopoulos, D. M. (2007). Generalized Additive Models for Location Scale and Shape (GAMLSS) in R. *Journal of Statistical Software*, 23.

Taplin, R., Minh To, H. & Hee, J. 2007. Modeling exposure at default, credit conversion factors and the Basel II Accord. *The Journal of Credit Risk*, 3, 75-84.

Tong, E. N. C., Mues, C. & Thomas, L. (2013). A zero-adjusted gamma model for mortgage loan loss given default. *International Journal of Forecasting*, 29, 548-562.

Yang, B. H. & Tkachenko, M. 2012. Modeling exposure at default and loss given default: empirical approaches and technical implementation. *The Journal of Credit Risk*, 8, 81-102.

Witzany, J. 2011. Exposure at Default Modeling with Default Intensities. *European Financial and Accounting Journal*, 6, 20-48.