Fitting a distribution to Value-at-Risk and Expected Shortfall, with an application to covered bonds

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[1] The opinions expressed in this presentation are those of the author and do not necessarily reflect views of the Bank of England.

Outline

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The problem

- Covered bonds are an important funding instruments for many banks.
- They are considered very safe investments.
- **No loss** due to missed payments to bondholders was ever observed.
- **No historical data** on covered bonds defaults and related losses are available.
- Estimating expected loss for covered bonds is not straightforward.
- References for suggested approach: Chan-Lau and Oura (2014), Yang (2015), Tasche (2015)

Essential features of covered bonds[2]

- The bond is issued by – or bondholders otherwise have full recourse to – a **credit institution** which is subject to public supervision and regulation;
- Bondholders have a claim against a **cover pool of financial assets** in priority to the unsecured creditors of the credit institution;
- The credit institution has the ongoing obligation to maintain sufficient assets in the cover pool to satisfy the claims of covered bondholders at all times;
- The obligations of the credit institution in respect of the cover pool are supervised by public or other independent bodies.

[2] Source: The European Covered Bond Council (http://ecbc.hypo.org)

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Two assets approach

- Bond issuer's asset values (random variables):
  - X is value of assets in cover pool (collateral for covered bonds).
  - Y is value of issuer's other assets.
- Bond issuer's debts (constants):
  - C is nominal value (principal) of covered bonds.
  - S is nominal value of senior unsecured debt.
  - U is nominal value of subordinated unsecured debt.
- **Double recourse:** In case of issuer's default,
  - covered bonds are served by proceeds from cover pool;
  - if cover pool proceeds are insufficient bond holders have a claim against the issuer's other assets, ranking pari passu with the holders of senior unsecured debt.
- L_C, L_S, L_U denote **loss rates** for covered bonds, senior unsecured debt and subordinated debt respectively.

Three loss events

- Issuer defaults, total assets sufficient for senior debt: $C + S \leq X + Y < C + S + U$

$$\Rightarrow \quad L_U = 1 - \frac{X+Y-(C+S)}{U}, \qquad L_C = L_S = 0. \qquad (1a)$$

- Issuer defaults, total assets insufficient for senior debt, cover pool sufficient for covered bonds: $X + Y < C + S, \quad X \geq C$

$$\Rightarrow \quad L_C = 0, \qquad L_S = 1 - \frac{X+Y-C}{S}, \qquad L_U = 1. \qquad (1b)$$

- Issuer defaults, total assets insufficient for senior debt, cover pool insufficient for covered bonds: $X + Y < C + S, \quad X < C$

$$\Rightarrow \quad L_C = \frac{(C-X)\,(S+C-X-Y)}{(S+C-X)\,C}, \; L_S = \frac{S+C-X-Y}{S+C-X}, \; L_U = 1. \qquad (1c)$$

Observations

- Covered bonds holders only suffer loss if
  - the issuer defaults ($X + Y < C + S + U$) and
  - the total assets value falls below the amount of senior debt ($X + Y < C + S$) and
  - the value of the cover pool assets falls below the nominal value of the bonds ($X < C$).
- Issuer's PD (probability of default) $\mathrm{P}[X + Y < C + S + U]$ is bounded from above by

$$\mathrm{P}[X + Y < C + S + U] \ \leq \ \mathrm{P}[X < C + S + U]. \qquad (2)$$

- The bound does not depend on the distribution of Y (value of issuer's other assets).

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Specifying the asset distributions

- Lognormal distribution is the most convenient choice of an asset value distribution.
- **Assumptions:**
  - Both the cover pool value X and the value of the issuer's other assets Y are lognormally distributed.
  - X and Y are linked by a normal copula.
- Parametrisation:

$$X = \exp(\mu + \sigma\,\xi), \qquad Y = \exp(\nu + \tau\,\eta), \tag{3}$$

  with $\mu, \nu \in \mathbb{R}$, $\sigma, \tau > 0$ and $(\xi, \eta) \sim \mathcal{N}\left(\left(\begin{smallmatrix}0\\0\end{smallmatrix}\right), \left(\begin{smallmatrix}1 & \varrho\\ \varrho & 1\end{smallmatrix}\right)\right)$ for some $\varrho \in [0, 1]$.
- Drawback: $X + Y$ is **not** lognormal.

Formulae for covered bonds expected loss I

- Derive formulae by conditioning on X or Y.
- Case $\varrho < 1$:

$$C\,\mathrm{E}[L_C] = \int_{-\infty}^{\frac{\log(C)-\mu}{\sigma}} \left(C - e^{\mu+\sigma\,x}\right)\varphi(x)\,\Phi\left(\frac{\log(C+S-e^{\mu+\sigma\,x})-(\nu+\tau\,\varrho\,x)}{\tau\,\sqrt{1-\varrho^2}}\right)dx$$

$$-\,e^{\nu+\tau^2\,(1-\varrho^2)/2}\int_{-\infty}^{\frac{\log(C)-\mu}{\sigma}} \frac{\left(C - e^{\mu+\sigma\,x}\right)e^{\tau\,\varrho\,x}}{C + S - e^{\mu+\sigma\,x}}\,\varphi(x)$$

$$\Phi\left(\frac{\log(C+S-e^{\mu+\sigma\,x})-(\nu+\tau\,\varrho\,x)}{\tau\,\sqrt{1-\varrho^2}} - \tau\,\sqrt{1-\varrho^2}\right)dx. \quad (4)$$

Formulae for covered bonds expected loss II

- Consider also **strongest dependence** between X and Y, i.e. comonotonicity.
- Comonotonic case, $\varrho = 1$:

$$C\,\mathrm{E}[L_C] = \int_{-\infty}^{\min\left(\frac{\log(C)-\mu}{\sigma},\, x(C+S)\right)} \varphi(x)\,\frac{\left(C-e^{\mu+\sigma\,x}\right)\left(C+S-e^{\mu+\sigma\,x}-e^{\nu+\tau\,x}\right)}{C+S-e^{\mu+\sigma\,x}}\,dx. \tag{5a}$$

- For $a > 0$, $x(a)$ denotes the unique solution of

$$a \;=\; e^{\mu+\sigma\,x} + e^{\nu+\tau\,x}. \tag{5b}$$

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The comonotonic case

- So far, a complete solution is known only in the comonotonic case.
- **Assumption:** We know
  - the issuer's PD_{issuer} and expected loss EL_{issuer}, and
  - the PD_{cover} and EL_{cover} of the cover pool.
- Fitting the distribution (3) of X to PD_{cover} and EL_{cover} is equivalent to fitting a lognormal distribution to given Value-at-Risk and Expected Shortfall.
- A unique lognormal distribution for X can be fitted as long as we have $0 < EL_{\text{cover}} < PD_{\text{cover}} < 1$.

Fitting the cover pool asset value distribution

- Denote by v the **level of over-collateralisation** of the covered bonds.
- First, determine σ in (3) for the distribution of X by solving

$$0 = \Phi\big(\Phi^{-1}(PD_{\text{cover}}) - \sigma\big) - \big(PD_{\text{cover}} - EL_{\text{cover}}\big)\,\exp\big(\sigma\,\Phi^{-1}(PD_{\text{cover}}) - \sigma^2/2\big). \quad \text{(6a)}$$

- Then calculate μ:

$$\mu = \log((1+v)\,C) - \sigma\,\Phi^{-1}(PD_{\text{cover}}), \quad \text{(6b)}$$

- Factor $1+v$ reflects the fact that PD and EL refer to the entire pool including the assets for over-collateralisation.

Fitting the distribution of the value of other assets I

- In the comonotonic case, we need to solve this equation system for parameters ν and τ for Y (see (5b) for $x(a)$):

$$PD_{\text{issuer}} = \Phi\big(x(C+S+U)\big), \tag{7a}$$

$$\begin{aligned}(C+S+U)\, PD_{\text{issuer}}\, (1 - LGD_{\text{issuer}}) \\ = e^{\mu+\sigma^2/2}\, \Phi\big(x(C+S+U) - \sigma\big) \\ + e^{\nu+\tau^2/2}\, \Phi\big(x(C+S+U) - \tau\big).\end{aligned} \tag{7b}$$

- Due to (2) and other dependence issues, there is not always a solution (ν, τ) of (7a), (7b).

Fitting the distribution of the value of other assets II

- **Proposition.** Assume that $\mu \in \mathbb{R}$ and $\sigma > 0$ are fixed. Then there is a solution (ν, τ) of (7a), (7b) if and only if

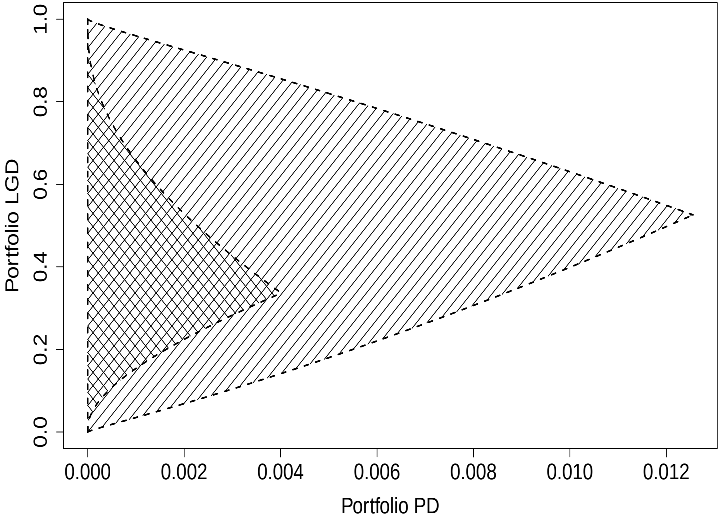
$$0 < PD_{\text{issuer}} < \Phi\left(\frac{\log(C+S+U)-\mu}{\sigma}\right) \quad \text{and} \tag{8a}$$

$$\frac{PD_{\text{issuer}}\, e^{\mu+\sigma\,\Phi^{-1}(PD_{\text{issuer}})} - e^{\mu+\sigma^2/2}\,\Phi\left(\Phi^{-1}(PD_{\text{issuer}})-\sigma\right)}{PD_{\text{issuer}}\,(C+S+U)} < LGD_{\text{issuer}}$$
$$< 1 - \frac{e^{\mu+\sigma^2/2}\,\Phi\left(\Phi^{-1}(PD_{\text{issuer}})-\sigma\right)}{PD_{\text{issuer}}\,(C+S+U)}. \tag{8b}$$

  If there is a solution (ν, τ) of (7a), (7b) it is unique.

- **Next slide:** Illustration for $PD_{\text{cover}} = 0.05\%$, $LGD_{\text{cover}} = 30\%$ (small range) and $PD_{\text{cover}} = 0.5\%$, $LGD_{\text{cover}} = 50\%$ (large range).

Illustration of feasible PD and LGD range for issuer



Outline

- We have presented an approach to the calculation of expected loss for covered bonds, based an a structural approach with two asset values.
- A number of open issues remains to be solved:
  - How to estimate the asset correlation?
  - How to define 'default' of the cover pool? As 'losses exceed loss reserve'?
  - How to better reflect different horizons (one year for issuer's default, – say – ten years covered bonds maturity)?
  - What to do in the case where there is no solution for a combination of issuer's PD and LGD as well as cover pool PD and LGD?

Outline

J. Chan-Lau and H. Oura. Bail-in Clause, Depositor Preference, and Asset Encumbrance: The End of Cheap Senior Unsecured Debt? A Structural Pricing Prespective. Working paper, 2014. URL http://www.researchgate.net/publication/262047985.

D. Tasche. Fitting a distribution to Value-at-Risk and Expected Shortfall, with an application to covered bonds. Working paper, 2015. URL http://arxiv.org/abs/1505.07484.

H.Z. Yang. A volatility-based single parameter Loss Given Default model. *Journal of Risk Management in Financial Institutions*, 8(2): 196–210, 2015.