MODELLING CREDIT GRADE MIGRATION IN LARGE PORTFOLIOS

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Background

- A credit rating system is an ordinal classification reflecting the probability of default of a given obligor
- Forecasting the process by which individual credit grades (including default) migrate over time allows us to forecast the evolution of default risk in a portfolio
- The grade migration process between two time points, t and u, is described by the transition matrix $P(t, u)$, with elements $p_{ij}(t, u)$, where

$$p_{ij}(t, u) = \mathrm{Prob}(\text{grade at time } u = j | \text{ grade at time } t = i)$$

The transition matrix

$$P(t,u) = \begin{pmatrix} p_{11}(t,u) & p_{12}(t,u) & p_{13}(t,u) & \cdots & p_{1D}(t,u) \\ p_{21}(t,u) & p_{22}(t,u) & p_{23}(t,u) & \cdots & p_{2D}(t,u) \\ \vdots & \vdots & \vdots & & \vdots \\ p_{D-1\,1}(t,u) & p_{D-1\,2}(t,u) & p_{D-1\,3}(t,u) & \cdots & p_{D-1\,D}(t,u) \end{pmatrix}$$

Each row of $P(t,u)$ is an ordinal probability distribution

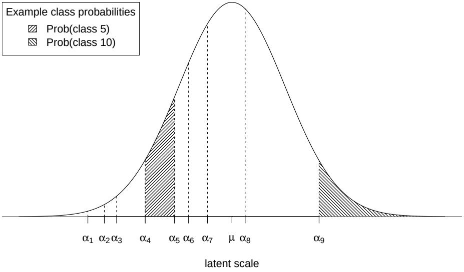
We aim to model $P(t,u)$ parsimoniously, based on the corresponding matrix of observed transiiton frequencies $X(t,u)$

Cumulative link model

For a collection of ordinal variables $\{Y_k\}$ taking values $j \in \{1, \ldots, D\}$:

$$P(Y_k \leq j) = g(\alpha_j - \mu_k)$$

Can be interpreted as latent variable with mean μ_k (potentially depending on covariates) and thresholds $-\infty = \alpha_0 < \alpha_1 < \ldots < \alpha_D = \infty$



Cumulative link model for a transition matrix

Assume that present (time t) grade is the only information available and/or relevant for predicting the next (time u) grade

A cumulative link model for $P(t, u)$ takes the form

$$q_{ij} \equiv \sum_{l=1}^{j} p_{il} = g(\alpha_j - \mu_i)$$

(dropping the (t, u) dependence)

A more general model allows *scale dependence*:

$$q_{ij} = g\left(\frac{\alpha_j - \mu_i}{\sigma_i}\right)$$

Here, the underlying latent distribution for each row can be ‘stretched’ as well as shifted, relative to the common thresholds

Which link?

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- Normal (probit)
  - Nickell et al (2000) – μ_i can depend on obligor-level characteristics
  - Hu et al (2002) – μ_i can depend on obligor-level characteristics
  - Feng et al (2008) – scale-varying
- Logistic (proportional odds)
  - McNeil and Wendin (2006)
  - Malik and Thomas (2012) – μ_i depends on on the previous grade
- Heavy-tailed? e.g. t_ν
- Skew?

Exploratory analysis

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Artificially created corporate portfolio constructed from Moody's DRD

$D = 8$ and the non-default grades are Aaa, Aa, A, Baa, Ba, B and C

We have

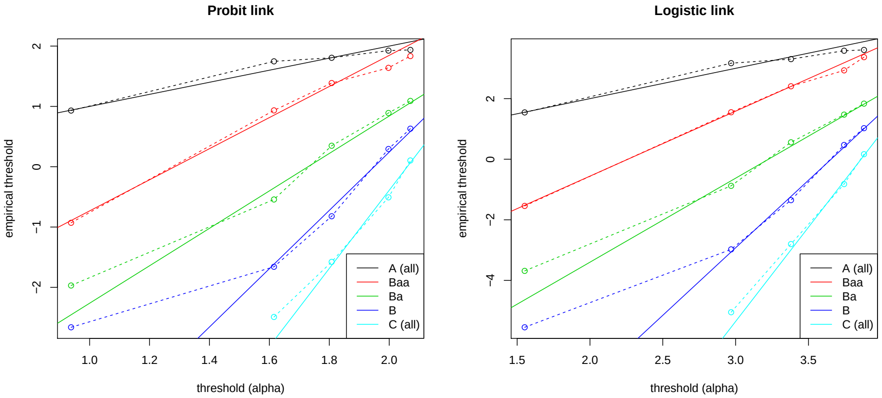
$$g^{-1}(q_{ij}) = \frac{1}{\sigma_i}\alpha_j - \frac{\mu_i}{\sigma_i}$$

Plotting $g^{-1}(\hat{q}_{ij})$ where

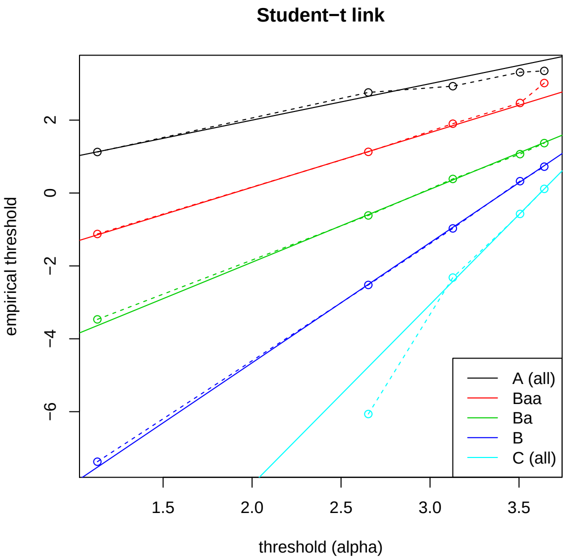
$$\hat{q}_{ij} = \frac{\sum_{k=1}^{j} X_{ik}}{\sum_{k=1}^{D} X_{ik}}$$

against $\hat{\alpha}_j$ should be approximately straight line for the 'correct' g

Exploratory analysis – normal (LH) and logistic (RH)



Exploratory analysis – t-link ($\nu = 2.65$)



Maximum likelihood estimation

- Likelihood

$$\ell(\alpha, \mu, \sigma, \nu) = \sum_{ij} X_{ij} \log \left(F_\nu \left(\frac{\alpha_j - \mu_i}{\sigma_i} \right) - F_\nu \left(\frac{\alpha_{j-1} - \mu_i}{\sigma_i} \right) \right)$$

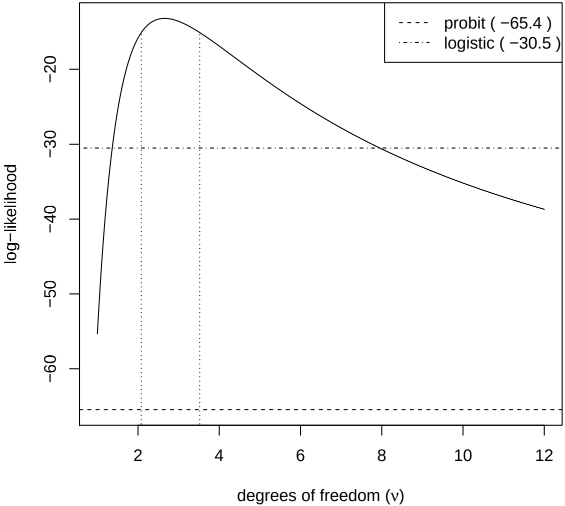
- Profile log-likelihood for ν,

$$\ell_p(\nu) \equiv \ell(\hat{\alpha}(\nu), \hat{\mu}(\nu), \hat{\sigma}(\nu), \nu)$$

- 95% confidence interval for ν

$$\nu \in C = \{\nu : 2\left(\ell_p(\hat{\nu}) - \ell_p(\nu)\right) < 3.84\}$$

Profile log-likelihood for ν



Why heavy-tailed?

Consider a latent structural asset-value model where the population of obligors is heterogeneous with respect to the variance of individual asset value increments

If

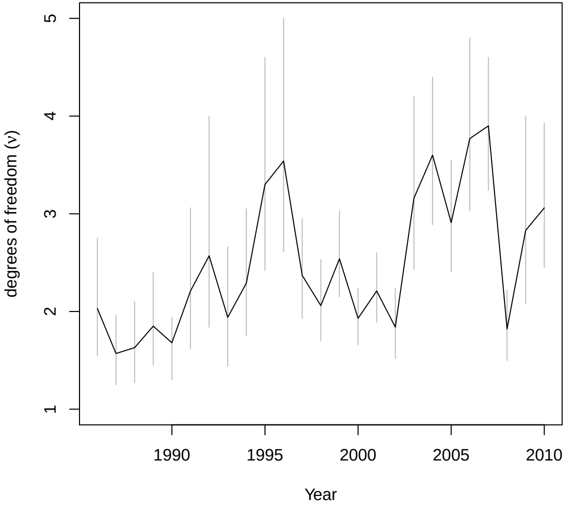
$$Z_{t+1} \,|\, Z_t = z \sim N(z + \mu', \tau^2) \qquad \text{where} \qquad \frac{\sigma^2}{\tau^2} \sim \chi^2_\nu$$

then, the marginal asset value increment process is

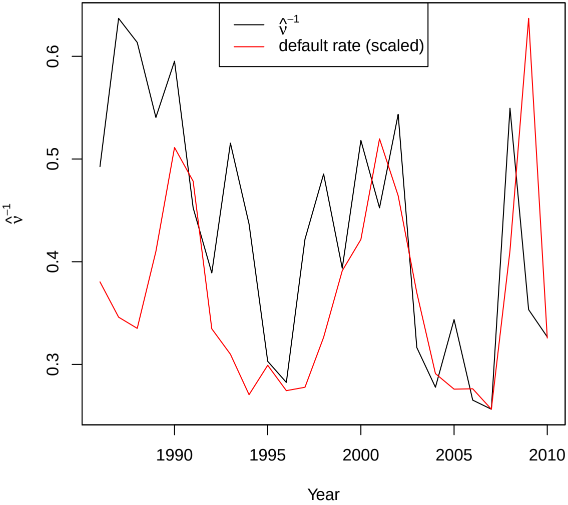
$$\frac{Z_{t+1} - (z + \mu')}{\sigma} \,|\, Z_t = z \sim t_\nu$$

Unobserved heterogeneity requires model adjustment (c.f. frailty in survival models)

Time evolution of $\hat{\nu}$ (with 95% confidence intervals)



Tail weight $1/\hat{\nu}$ and default rate, against time



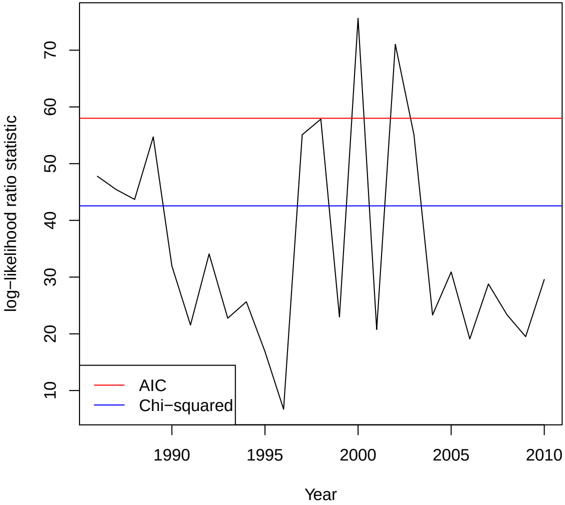
Goodness-of-fit analysis (1)

Compare against the saturated (unstructured) model using

$$L = 2\left(\ell(\hat{\alpha},\hat{\mu},\hat{\sigma},\hat{\nu}) - \sum_{ij} X_{ij}\log\left(\frac{X_{ij}}{\sum_{k=1}^{D} X_{ik}}\right)\right)$$

- LR test – under the cumulative t-link model (for an $R \times D$ transition matrix) $L \sim \chi^2_d$ where $d = (R-1)(D-3)-1$
- Akaike Information Criterion (AIC) – prefer the cumulative link model when $L < 2d$
- Bayes Information criterion (BIC) – prefer the cumulative link model when $L < d \log n$ respectively, where n is the sample size

Goodness-of-fit analysis (2)



Discussion

- For both corporate and retail portfolio data, cumulative t-link with degrees of freedom ν between 2 and 3 models tail behaviour well
- There may be scope for using a varying ν, modelled in conjunction with other cumulative link model parameters
- Overall fit is good for these corporate data – further improvements possible for large retail portfolios – skewness?
- For a complete forecasting model, combination of transitions is required. Markov property is also suspect under aggregation (Malik and Thomas, 2012, Lando and Skødeberg, 2002)