

Performance measures of LGD models

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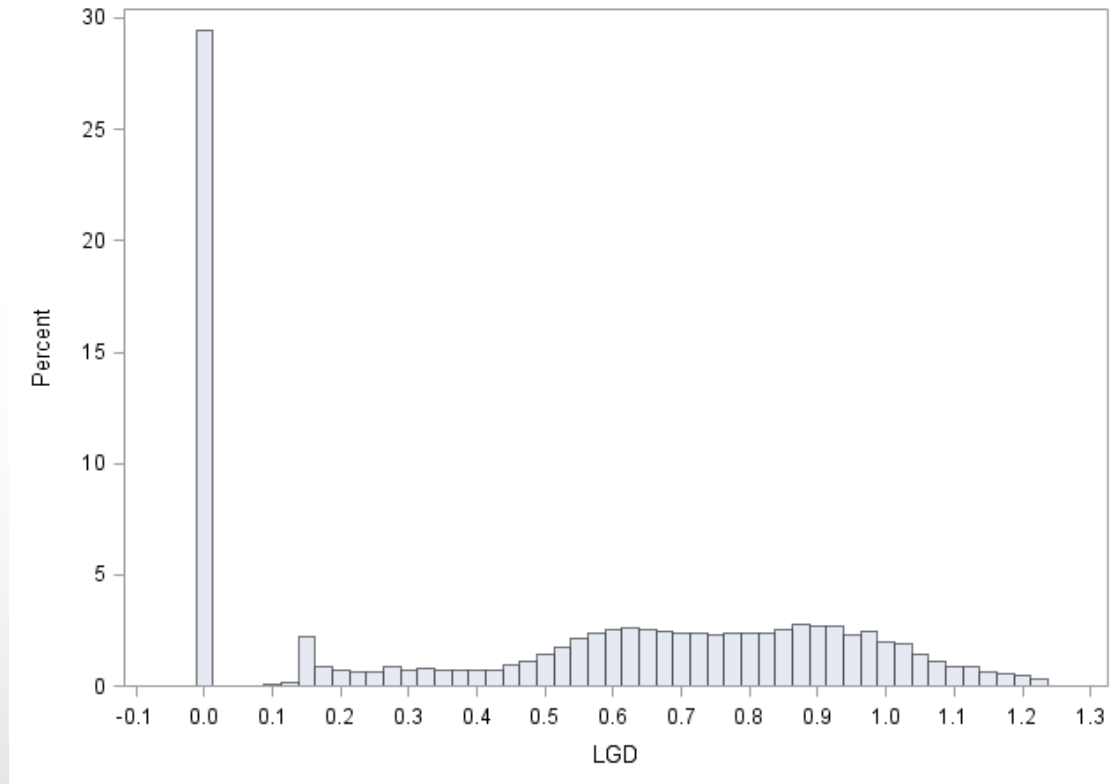
Outline

- Introduction
- LGD
- LGD models
- Performance measures of LGD models
- Example
- Conclusions

LGD

- Loss Given Default (LGD)
 - The lender's loss on a loan due to the customer's default, i.e. failure to meet the credit commitment
- Recovery Rate (RR)
 - $LGD = 1 - RR$
- Basel II and III
 - Under the Advanced Internal Ratings-Based (AIRB) approach, lenders are allowed to use their own predictions of risk parameters, including LGD

LGD distribution example



LGD models

- Unsecured loans
 - One-stage models
 - Multi-stage approaches
 - Separation of 0s (+ Separation of 1s) + Prediction
- Mortgage loans
 - One-stage models
 - Two-stage approaches
 - Repossession model + Haircut model

LGD models

- Separation stage(s)
 - Logistic regression
 - Decision trees
- Prediction stage/one-stage models
 - Regression models
 - Tobit models
 - Survival analysis
 - Classification and Regression Trees (CART)
 - Other nonlinear models

Performance measures

- PD
 - Gini coefficient
 - KS statistic
- LGD
 - ???

MSE

- Mean Square Error (MSE):

$$MSE = \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

- Sensitive to extreme values of the residuals
- E.g. Bellotti and Crook (2008)

SSE

- Sum of Squared Errors (SSE) a.k.a. residual sum of squares:

$$SSE = \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

- E.g. Qi and Zhao (2011)
- Depends on the sample size → the results are incomparable between samples of different sizes
- Useful when comparing a number of LGD models developed on the same sample

RMSE

- Root Mean Square Error (RMSE):

$$RMSE = \sqrt{MSE}$$

- Expressed in the same units as LGD
- Bastos (2010)

MAE

- Mean Absolute Error (MAE) a.k.a. Mean Absolute Deviation (MAD):

$$MAE = \frac{1}{n} \sum_{i=1}^n |y_i - \hat{y}_i|$$

- Expressed in the same units as LGD
- Compare with RMSE
- E.g. Bellotti and Crook (2008)

MAPE

- Mean Absolute Percentage Error (MAPE)
a.k.a. Mean Absolute Percentage Deviation (MAPD):

$$MAPE = \frac{1}{n} \sum_{i=1}^n \frac{|y_i - \hat{y}_i|}{y_i}$$

- Zero observations → the division-by-zero problem
- SAS (2012) unconventionally uses the predicted LGD as the denominator → low errors when the predicted values are e.g. ten or more times larger than the observed values
- **We do not recommend using MAPE for LGD**

RSE

- Relative Square Error (RSE):

$$RSE = \frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{\sum_{i=1}^n (y_i - \bar{y})^2}$$

- Ratio of MSE of the model and MSE of a simple predictor
- E.g. Bastos (2014)

RAE

- Relative Absolute Error (RAE):

$$RAE = \frac{\sum_{i=1}^n |y_i - \hat{y}_i|}{\sum_{i=1}^n |y_i - \bar{y}|}$$

- Ratio of MAE of the model and MAE of a simple predictor
- E.g. Bastos (2010)

AOC

- Regression Error Characteristic (REC) curve estimates the CDF of the squared or absolute residual
- Area Over the REC Curve (AOC) estimates the expected regression error (Bi and Bennett, 2003)
- If the REC curve is derived using the squared residuals, then $AOC \rightarrow MSE$ as the sample size $\rightarrow \infty$
- If the REC curve is derived using the absolute residuals, then $AOC \rightarrow MAE$ as the sample size $\rightarrow \infty$

AOC

- Loterman *et al.* (2012) calculated both RMSE and AOC (based on the squared residuals)
 - LGD models: 24 various techniques and six datasets
 - Differences between AOC and the squared RMSE:
 - < 0.001 for five larger datasets
 - < 0.01 for the smallest dataset (test: ca. 1100 loans)
- **We recommend applying either AOC or MSE/MAE in order to avoid information redundancy**

R-squared

- Coefficient of determination (R-squared):

$$R^2 = 1 - \frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{\sum_{i=1}^n (y_i - \bar{y})^2}$$

- E.g. Loterman *et al.* (2012)
- In an OLS regression model with a constant term, R-squared can be interpreted as the proportion of variation in LGD that is explained by variation in the regressors
- **We only recommend using R-squared in OLS models**

Out-of-sample R-squared

- To assess the out-of-sample performance, the out-of-sample mean is normally used
- Out-of-sample R-squared is calculated using the in-sample mean instead of the out-of-sample mean
- E.g. Gürtler and Hibbeln (2013)
- In-sample and out-of-sample means are often similar → no difference between out-of-sample and “normal” R-squared
- Useful?

Adjusted R-squared

- Adjusted coefficient of determination (adjusted R-squared):

$$\bar{R}^2 = 1 - (1 - R^2) \frac{n - 1}{n - k - 1}$$

- Corrected for the number of regressors (k)
- E.g. Caselli *et al.* (2008)
- Useful when comparing a number of linear LGD models

Pseudo R-squared

- For example, McFadden's pseudo R-squared a.k.a. likelihood ratio index
 - for models estimated using the ML method

$$Pseudo R^2 = 1 - \frac{\ln L(M_1)}{\ln L(M_0)}$$

- E.g. Dermine and Neto de Carvalho (2006)
- Useful when applying non-linear transformations of LGD

Correlation coefficients

- Measure correlation between the observed and predicted LGD (Loterman *et al.*, 2012)
- Pearson's correlation coefficient:

$$r = \frac{\sum_{i=1}^n (y_i - \bar{y})(\hat{y}_i - \hat{\bar{y}})}{\sqrt{\sum_{i=1}^n (y_i - \bar{y})^2 \sum_{i=1}^n (\hat{y}_i - \hat{\bar{y}})^2}}$$

- Measures the strength of the linear relationship between the observed and predicted LGD ($r^2 = R^2$ in OLS models)
- Spearman's and Kendall's correlation coefficients

AUC

- Loans need to be classified into two groups based on the observed LGD, e.g. below-the-mean and over-the-mean
 - Analogy with PD: the predicted LGD – the estimated probability; the predicted RR – a score
- CDFs of the predicted RR are computed for the groups
- Receiver Operating Characteristic (ROC) curve is drawn by plotting the CDFs against each other
- Area Under the ROC Curve (AUC) measures how well the model separates loans belonging to the two groups
- E.g. Gupton and Stein (2005)

AR

- Accuracy Ratio (AR) a.k.a. Gini coefficient

$$AR = 2AUC - 1$$

- E.g. Chalupka and Kopecsni (2008)
- The probability that a randomly selected “Good” will have a lower predicted LGD value than a randomly selected “Bad”

Proposed measure: MAUC

- AUC has a drawback when applied to LGD as it requires an arbitrary classification of the dependent variable
- m – the number of unique values of the observed LGD
- Mean AUC (MAUC) is calculated as the average of AUC for all possible divisions into two groups:

$$MAUC = \frac{1}{m-1} \sum_{j=1}^{m-1} AUC_j$$

- MAUC takes values from the interval $[0.5, 1]$ like AUC

Example

- Two-stage model applied to the data on personal loans granted by a large UK bank

Measure	Value	Measure	Value
MSE	0.143	AOC (s. r.)	0.143
MAE	0.329	AOC (a. r.)	0.329
RMSE	0.378	RSE	0.928
MAPE	N/A	RAE	0.940

Example -- continued

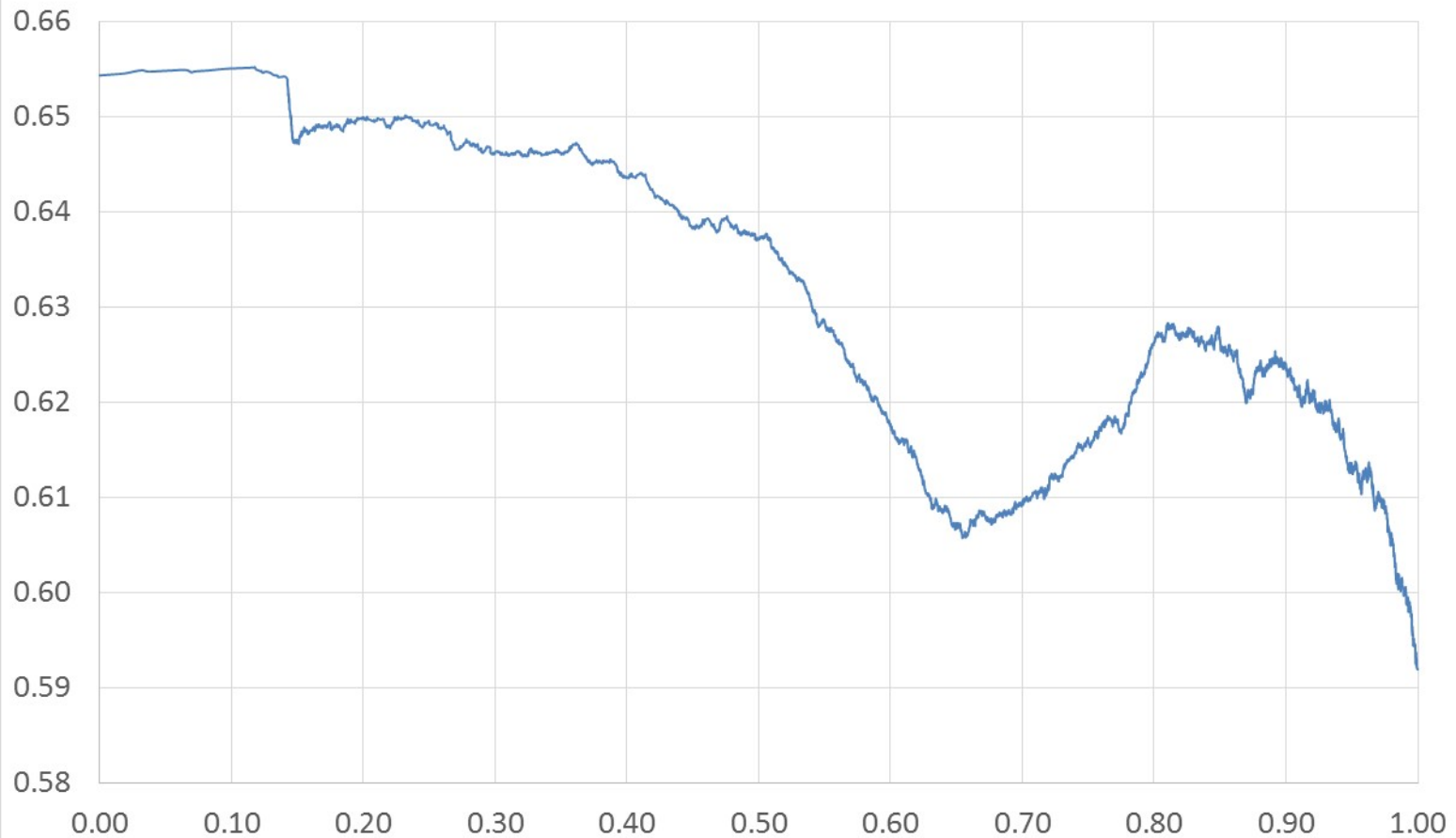
- Two-stage model applied to the data on personal loans granted by a large UK bank

Measure	Value	Measure	Value
R-squared	0.072	Pearson	0.268
Adjust. R-sq.	N/A	Spearman	0.255
Pseudo R-sq.	N/A	Kendall	0.179
AUC	0.637	AR	0.272

MAUC = 0.616

Example -- continued

AUC for different divisions into two groups



New ideas

- Loterman *et al.* (2014)
 - Discussed statistical tests suitable for the central tendency and dispersion of the errors
 - Proposed four test-statistics-based measures
- Fischer and Pfeuffer (2014)
 - Focused on measures related to discriminatory power
 - Proposed new measures

Conclusions

- Recommendations for LGD model developers/users
 - Do not use MAPE
 - Apply either AOC or MSE/MAE
 - Only use R-squared in OLS models
 - Look for alternatives to AUC

- Do you prefer any of the discussed performance measures? Why?
- Do you use any other performance measures of LGD models?

Thank you!

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