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THE DRIFT TO DEFAULT :

A Structural Extension of the Role of House Price in determining the early termination of UK based Residential Mortgages

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Motivation

All option theory based mortgage valuation models in the literature have assumed that house prices follow a geometric Brownian motion process, (Random Walk)

however, ...

Empirical evidence suggests that house prices are poorly approximated by a geometric Brownian motion and instead are more consistent with a mean reverting process. Meese and Wallace, (1997, 1998).

The Mortgage's Economic Environment

Stochastic Processes:

Traditional Model

Interest Rate: Mean reverting square root diffusion process - Cox Ingersoll and Ross (1985):

$$dr = \gamma(\theta - r)dt + \sigma_r \sqrt{r} dz_r$$

$\gamma \equiv$ The rate of adjustment in the mean reverting process

$\theta \equiv$ The long term mean of the short-term interest rate, *r(t)* (Steady state spot rate)

$\sigma_r \equiv$ Instantaneous standard deviation of the interest rate disturbance

$z_r \equiv$ Standard Wiener process

House Price: Log Normal Diffusion process - Merton (1973)

$$\frac{dH}{H} = (\alpha - \phi)dt + \sigma_H dz_H$$

$\alpha \equiv$ The instantaneous average rate of house price appreciation

$\phi \equiv$ 'dividend type' per unit service flow provided by the house

$\sigma_H \equiv$ Instantaneous standard deviation of the house price

$z_H \equiv$ Standard Wiener process

AssetValuation Equation:

$$\frac{\partial V}{\partial t} + \frac{1}{2}H^2\sigma_H^2\frac{\partial^2 V}{\partial H^2} + \frac{1}{2}r\sigma_r^2\frac{\partial^2 V}{\partial r^2} + \gamma(\theta - r)\frac{\partial V}{\partial r} + (r - \emptyset)H\frac{\partial V}{\partial H} + \rho H\sigma_H\sigma_r\sqrt{r}\frac{\partial^2 V}{\partial r \partial H} = rV$$

The Mortgage's Economic Environment

Stochastic Processes:

Mean Reverting Model

Interest Rate: Mean reverting square root diffusion process - Cox Ingersoll and Ross (1985):

$$dr = \gamma(\theta - r)dt + \sigma_r \sqrt{r}\, dz_r$$

House Price: Mean reversion process

$$\frac{dH}{H} = \eta(\bar{H} - H)dt + \sigma_H dz_H$$

η = the speed of mean reversion in house prices

$\bar{H}$ = the long run mean of house prices

AssetValuation Equation:

$$\frac{\partial V}{\partial t} + \frac{1}{2}H^2\sigma_H^2\frac{\partial^2 V}{\partial H^2} + \frac{1}{2}r\sigma_r^2\frac{\partial^2 V}{\partial r^2} + \gamma(\theta - r)\frac{\partial V}{\partial r} + (r - \mu + \eta(\bar{H} - H))H\frac{\partial V}{\partial H} + \rho H\sigma_H\sigma_r\sqrt{r}\frac{\partial^2 V}{\partial r\partial H} = rV$$

Stochastic Disturbances: Volatility and Correlation and Service Flow (Implicit Rent)

σ_H House Price volatility parameter

σ_r Interest Rate volatility parameter

$dz_r dz_H = \rho dt$ Standard Wiener processes are such that E[dz]=0 and E[dz^2]=dt.

$\rho(r, H, t)$ is the correlation between the disturbances to the term structure and house price.

The required return on the investment in the house μ
is composed of the service flow, $\emptyset$, and the expected capital gains

$\mu = \emptyset + \eta(\bar{H} - H)$ Rearranging this gives $\emptyset = \mu - \eta(\bar{H} - H)$

Simulation: Base Parameters

LIST OF PARAMETERS	
CONTRACT TYPE :	**25YR Repayment Mortgage**
ECONOMIC EVIRONMENT	
Spot interest rate, r(0) :	6%
Long term average of the interest rate (steady state rate), θ :	7%
Speed of reversion, γ :	25%
Volatility of interest rate σ_r:	5%
House service flow, ϕ :	7.50%
Volatility of House Prices, σ_H:	5%
Mean of house Prices H-bar:	£150,000
Speed of reversion of house Price η:	0.0003
Correlation coefficient, ρ :	0
CONTRACT PROVISIONS	
Maturity, T :	300 months
Value of house at origination, H(0) :	£100, 000
Arrangement Fee, ξ :	0.05%
Early Termination Penalty, π :	1%

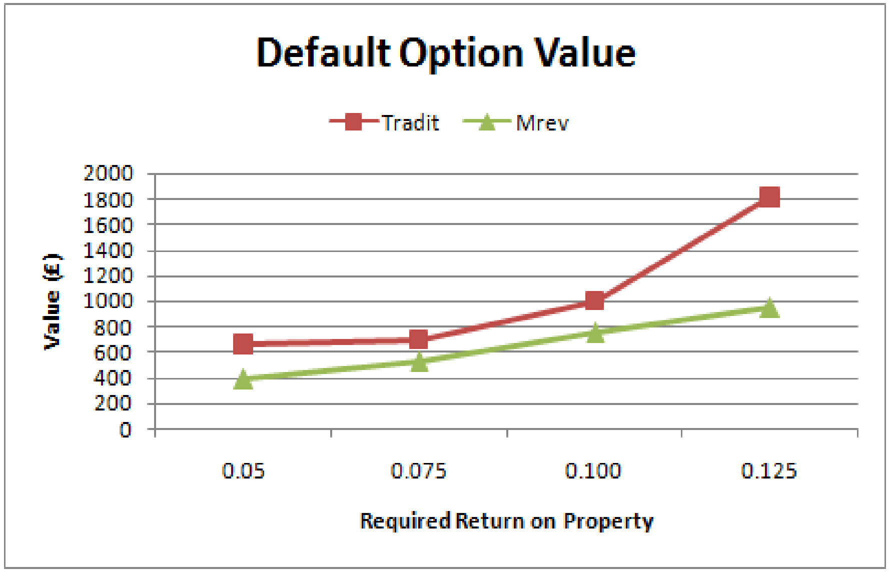
The choice of parameter values made, is informed mainly on the basis of the standard assumptions in the literature and on recent mortgage market conditions.

AFEE: Arrangement Fee

ATP: Early Termination Penalty

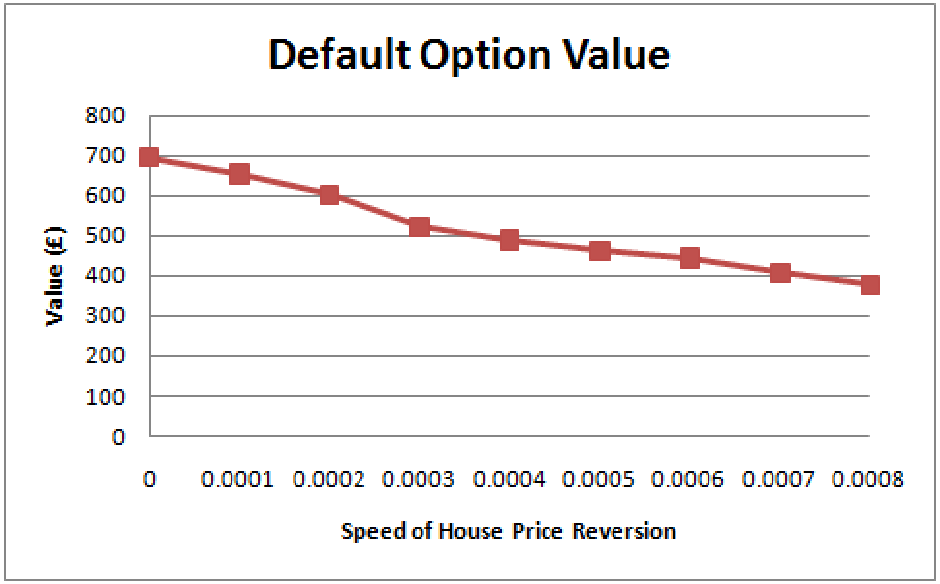
Simulation Results:
Required Return on Property
Speed of House Price Reversion

Effect on Mortgage Assets of changes in required return on property (25yr Repayment Mortgage with AFEE and ETP) (£)



Default option value increases with increase in return on asset and decreases with increase in rate of reversion

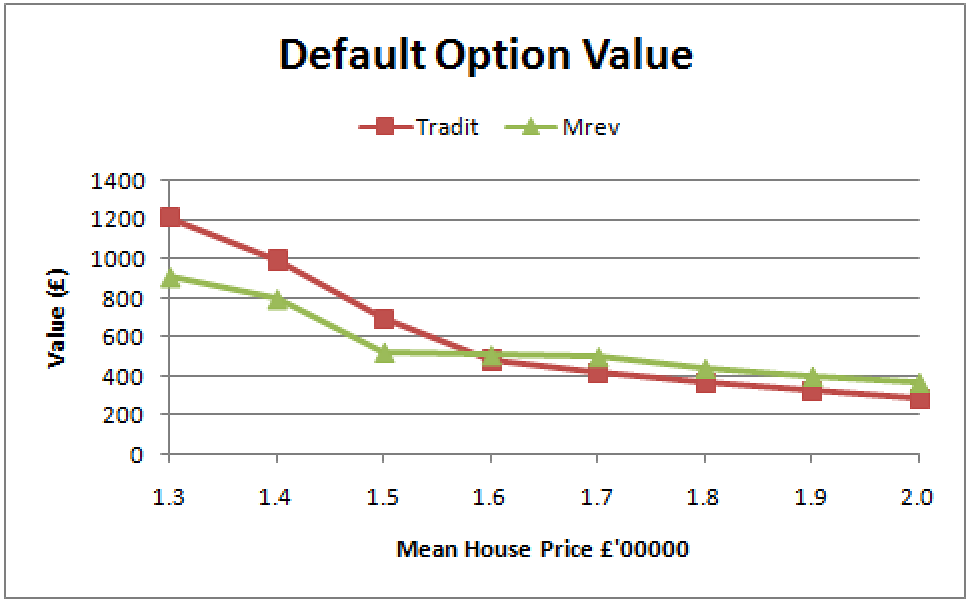
Effect on Mortgage Assets of changes in Speed of House Price Reversion (25yr Repayment Mortgage with AFEE and ETP) (£)



If the speed of the reversion in house prices differs across regions, then it is the local rate of reversion that matters.

Simulation Results: Long Term Mean House Price

Effect on Mortgage Assets of changes in long term mean House Prices (25yr Repayment Mortgage with AFEE and ETP) (£)

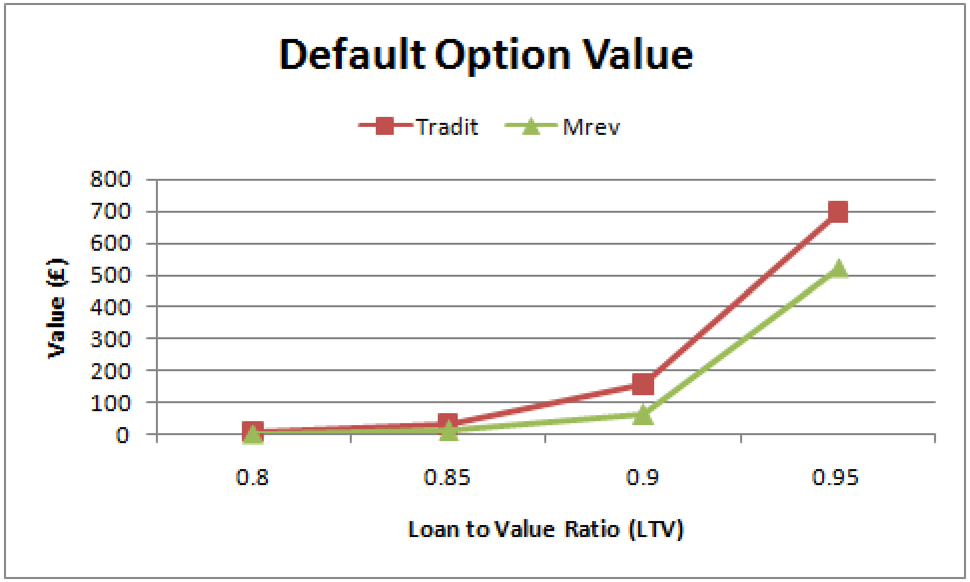


If the house value is below its long term expected value, then the value of the house is expected to rise in the mean reversion model, thus making the default option less valuable.

Likewise if the house value is above its long term expected value, then the value of the house is expected to fall in the mean reversion model, thus making the default option more valuable.

Simulation Results: LTV

Effect on Mortgage Assets of changes in Loan to Value Ratio : LTV (25yr Repayment Mortgage with AFEE and ETP) (£)



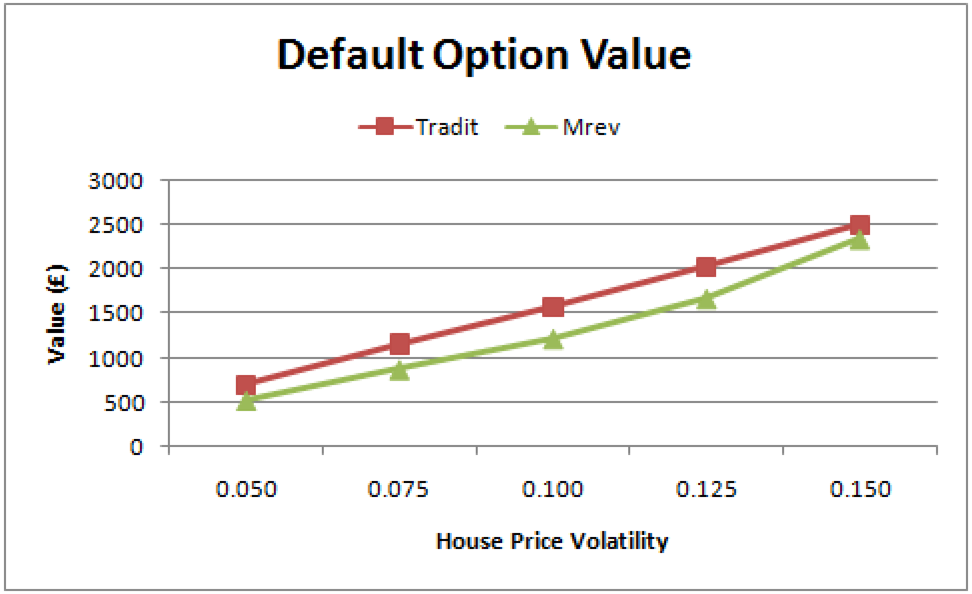
The likelihood of default tends to Increase with a rise in the relation between the original house price and the amount of loan.

For high LTV levels the likelihood of default and the default option value increases.

The figure shows that the default option values under the mean reversion model, are lower in magnitude for the given values of the LTV ratio used.

Simulation Results:
House Price Volatility

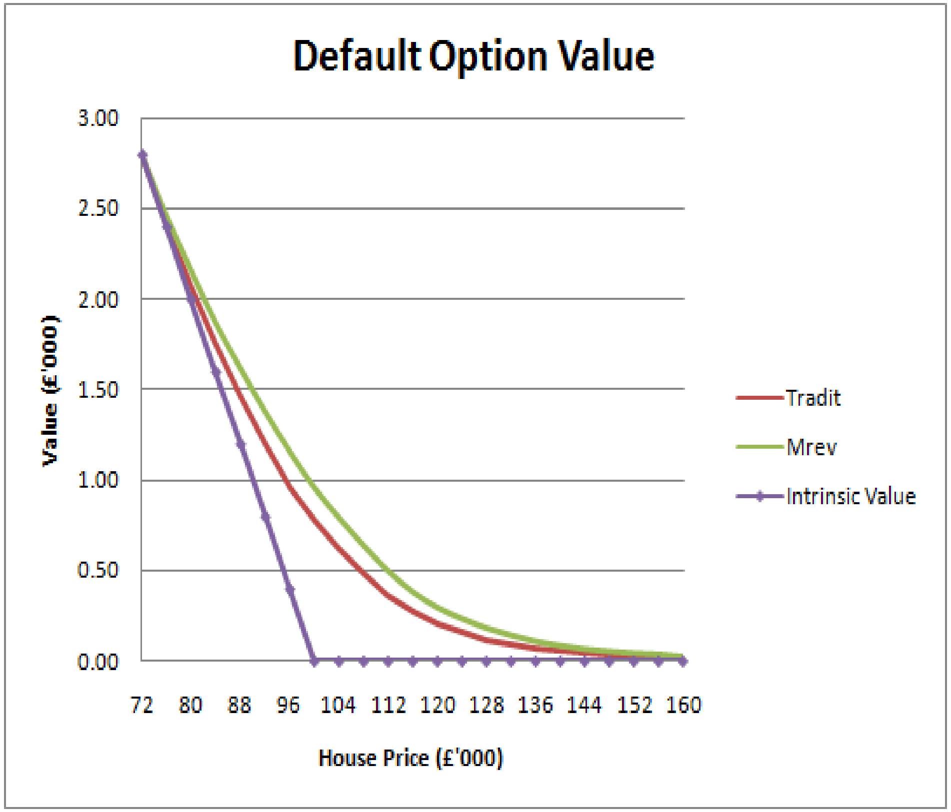
Effect on Mortgage Assets of changes in House Price Volatility
(25yr Repayment Mortgage with AFEE and ETP) (£)



In the case of default, there is evidence of a direct relationship with house price volatility.

Increase in house price volatility tends to preferentially increase likelihood of default. The effect of this is a reduction in Mortgage value.

Simulation Results: Default (Put) Option



Intuition for why the option values differ across the two models comes from thinking about what happens if the house value declines below the value of the outstanding balance.

If house prices follow a random walk, then the house value is just as likely to fall as to rise.

In the mean reversion model the option value depends upon whether or not the current house value is above or below the long term trend in house value.

The default option is more valuable under the mean reversion model than the standard random walk model especially if house prices are substantially below the long term trend.

Conclusions

The model presented suggests that additional factors relating to the local housing market – in particular deviation from the long run expected house value and the rate of reversion toward the mean house price in the local housing market – may help better forecast mortgage termination and create a more realistic mortgage valuation model.

Future work

The true house price process is undoubtedly far more complex than the one modelled here. House prices may behave more like an ARMA process and exhibit both short term serial correlation and long term mean reversion and so to explore this issue in future work an empirical investigation is planned