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When to Rebuild or When to Recalibrate Scorecards

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Ageing of Scorecards

- Scorecards: main way of assessing consumer credit risk

- Scorecards "age" over time
  - Changes in population
  - Changes in economic and commercial environment
  - Changes in users' policies

- Lenders rebuild from time to time if recent scorecard performance unsatisfactory

Partial Cause of Sub Prime Mortgage Crisis

- Not rebuilding or recalibrating scorecards one of causes of subprime mortgage crisis
- SEC report identified bureau scorecards had aged between 2001 and 2006.
  - Not rebuilt until 2008.
  - Analysis confirmed this:
    - Demyanyk and van Hemert, (2008)
    - Ashcraft and Schuerman (2008)
    - Rona-tas and Hiss (2008)

Model Needed of When to Rebuild and When to Recalibrate Scorecard

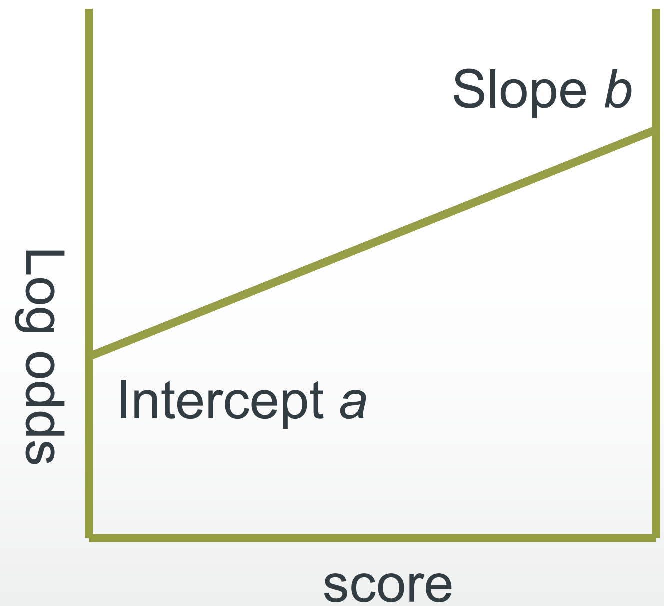
- Model would help with
    - Forecasting future ageing
    - Better sequencing of rebuild of different scorecards
    - Manpower planning
    - Maximising overall profit
- Large literature on maintenance and replacement of equipment
- None on replacement of scorecards
    - No concise description of “state” of scorecards
    - Hence No model of dynamics of movements between “states”

Linearly Transformed Log Odds Score

- Logistic and linear regression lead to scorecards which are linearly transformed log odds scores

$$\log\left(\frac{P(G \mid s(\mathbf{x}))}{P(B \mid s(\mathbf{x}))}\right) = a + bs(\mathbf{x})$$

- Graph of this used to monitor scorecard



$$a + bs(\mathbf{x}) = \log\left(\frac{P(G \mid \mathbf{x})}{P(B \mid \mathbf{x})}\right) \Leftrightarrow P(G \mid \mathbf{x}) = \frac{1}{1 + e^{-a-bs(\mathbf{x})}}$$

Cut-off Score

- Decision made by cut-off score, *c*
- If application score:accept if score above *c* reject if below *c*
- If behavioural score, above *c* increase credit limit
- Changing *c* and fixing *a* is equivalent to fixing *c* and changing *a*
- We think of lender decision as changing *c* and not able to control *a* and *b*. This is recalibration in Basel context if we require a given score s, say 100 always to have P(G|s)=0.5.

$$\log\left(\frac{P(G\,|\,s,t)}{P(B\,|\,s,t)}\right) = a(t) + b(t)(s - c(t))$$

$$with(P(G\,|\,100) = 0.5 \Rightarrow c(t) = 100 + (a(t)/b(t))$$

State Space of Scorecard

- State of scorecard at time t is $(a(t), b(t), c(t))$
- Profits from new applicants each period(month)
  - D is default cost if they default
  - L is profit to lender if they repay
  - N is number of new applicants
  - $f(s)$ is density function of score distribution
    - Assume stationary to simplify model
  - $E(a,b,c)$ is expected profit from new applicants this period

$$E(a,b,c) = N\left(-D\int_c^\infty P(B\,|\,s)f(s)ds + L\int_c^\infty P(G\,|\,s)f(s)ds\right)$$

$$= N\left(-D\int_c^\infty \frac{e^{-a-bs}f(s)}{1+e^{-a-bs}}ds + L\int_c^\infty \frac{f(s)}{1+e^{-a-bs}}ds\right)$$

Markov Decision Model

- R is the cost of rebuilding scorecard
- B is the cost of recalibrating scorecard
- β is monthly discount factor
- (a_B, b_B, c_B) is "state" after rebuilding
- $c_R(a, b)$ is cut-off after recalibration
- $V(a, b, c)$ is expected discounted profit when in state (a, b, c)
- Satisfies optimality equation:

$$V(a,b,c) = \max \begin{cases} E(a,b,c) + \int \beta V(x,y,c)\, p(x,y \mid a,b)\, dx\, dy & \textbf{Do nothing} \\ \max_{c_R} \left(-R + V(a,b,c_R(a,b))\right) & \textbf{Recalibrate} \\ \max_{c_B} \left(-B + V(a_B, b_B, c_B)\right) & \textbf{Rebuild} \end{cases}$$

Optimal Recalibration and Rebuilding

- If interested in optimal immediate profitability
- Choose cut-off $c_M(a, b)$ where marginal profit is zero

$$\frac{p(G \mid s(x)=c_M)}{p(B \mid s(x)=c_M)} = \frac{D}{L} \Leftrightarrow a + bc_M(a,b) = \log\left(\frac{D}{L}\right)$$

- Only optimal if this remains marginal zero profit cut-off
- If scorecard drifts. Then choose fixed cut-off fixed, which allows for drift
- D=L=1, N=200, β =0.99; f(1)=f(-1)=0.5; c_M(0,1)=0
- $P(a+1,1|a,1) = 1$, then those with scores of -1 are profitable over 4 periods.

So cut-off of -2 better than cut-off of 0.

Rebuilding Decision

- Dynamics of scorecard has stochastic ordering property
  - if $\bar{P}(x, y|a, b) = \int_y^\infty dw \int_x^\infty dv p(v, w|a, b)$,

    Then if $a \geq \acute{a}$, $\bar{P}(x, y|a, b) \geq \bar{P}(x, y|\acute{a}, b)$

    and if $b \geq \acute{b}$, $\bar{P}(x, y|a, b) \geq \bar{P}\left(x, y\middle|a, \acute{b}\right)$

- Then there exists functions $a^*(b)$ and $b^*(a)$ so one rebuilds the scorecard provided for(a, b),

$$a \leq a^*(b) \text{ or } b \leq b^*(a)$$

Dynamics of Score to Log Odds Graph

- At time t, $a(t)+b(t)s(\mathbf{x},t)=\log\left(\dfrac{P_t(G\mid\mathbf{x})}{P_t(B\mid\mathbf{x})}\right)$
- $a(t), b(t)$ both follow Brownian motion around a linear trend

$$da(t)=\mu_a dt+\sigma_a dw_1$$
$$db(t)=\mu_b dt+\sigma_b dw_2$$

- only calculated every month – use discrete time process

$$a(t)=a(t-1)+\mu_a+\sigma_a W_1 \qquad P\{W_1=+1,W_2=+1\}=P\{W_1=-1,W_2=-1\}=\frac{1+\rho}{4}$$
$$b(t)=b(t-1)+\mu_b+\sigma_b W_2 \qquad P\{W_1=+1,W_2=-1\}=P\{W_1=-1,W_2=+1\}=\frac{1-\rho}{4}$$

- $E\{W_i\}=0,\ Var\{W_i\}=1,$ and correlation coefficient is

$$\rho(W_1,W_2)=E\{W_1W_2\}=\rho$$

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Discretizing the State Space

- Only finite number of scores and so cut offs c
- Let $g(a)$ be the greatest common denominator of μ_a and σ_a
  so $\mu_a = k_a g(a)$ and $\sigma_a = l_a g(a)$ then
  $$a(t) = a(t-1) + (k_a \pm l_a)\text{g(a)}$$
  Thus, we can identify $a(t)$ with the integer $n(t)$ where
  $$a(t) = a_B + n(t)g(a)$$
- Similarly, let $g(b) = \text{gcd}(\mu_b, \sigma_b)$, so $\mu_b = k_b g(b)$ and $\sigma_b = l_b g(b)$, then $b(t) = b(t-1) + (k_b \pm l_b)\text{g(b)}$ where $\text{b}(t) = b_B + m(t)g(b)$

$$v(n,m,c) = \max \begin{cases} E(n,m,c) + \beta \begin{pmatrix} ((1+\rho)/4)(v(n+k_a+l_a, m+k_b+l_b, c) + v(n+k_a-l_a, m+k_b-l_b, c) + \\ ((1-\rho)/4)(v(n+k_a-l_a, m+k_b+l_b, c) + v(n+k_a+l_a, m+k_b-l_b, c) \end{pmatrix} \\ \max_{c_R}(-R + v(n,m,c_R(n,m)) \\ \max_{a_B}(-B + v(n_B, m_B, c_B)) \end{cases}$$

Credit Card Example

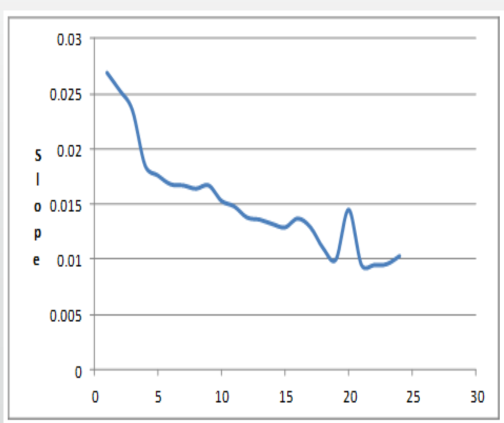
- 2,000,000 cards; 2 years history; Scorecard range: 24 to 307
- Split into 40 bands

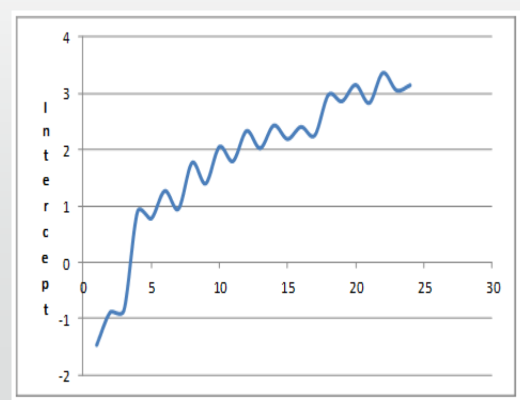
Band	Scoreband Lower limit	Scoreband Upper limit	% in Band	Band	Scoreband Lower limit	Scoreband Upper limit	% in Band
1	34	144	1.71	21	250	252	2.47
2	145	158	1.85	22	253	255	2.40
3	159	168	2.04	23	256	257	1.77
4	169	176	2.12	24	258	260	3.23
5	177	182	1.94	25	261	261	1.23
6	183	188	2.18	26	262	263	3.07
7	189	193	1.98	27	264	266	3.18
8	194	197	1.59	28	267	268	2.68
9	198	203	2.60	29	269	271	3.67
10	204	206	1.35	30	272	272	2.13
11	207	212	2.97	31	273	273	2.47
12	213	217	2.35	32	274	274	0.99
13	218	222	2.92	33	275	277	3.35
14	223	227	2.88	34	278	279	0.86
15	228	230	1.58	35	280	280	1.73
16	231	235	2.78	36	281	282	3.90
17	236	239	2.19	37	283	285	3.73
18	240	243	2.64	38	286	289	4.55
19	244	247	2.75	39	290	294	3.79
20	248	249	1.35	40	295	307	5.03

Estimates of $a(t)$ and $b(t)$

Time Period	a(t):Intercept	b(t):Slope	Time Period	a(t):Intercept	b(t):Slope
1 (JAN 2002)	-1.4715	0.0269	13 (JAN 2003)	2.0228	0.0136
2 (FEB 2002)	-0.8866	0.0253	14 (FEB 2003)	2.4301	0.0132
3 (MAR 2002)	-0.8567	0.0235	15 (MAR 2003)	2.1875	0.0129
4 (APR 2002)	0.8989	0.0185	16 (APR 2003)	2.4029	0.0137
5 (MAY 2002)	0.7725	0.0176	17 (MAY 2003)	2.2516	0.0129
6 (JUN 2002)	1.2673	0.0168	18 (JUN 2003)	2.9714	0.011
7 (JUL 2002)	0.9466	0.0167	19 (JUL 2003)	2.8581	0.01
8 (AUG 2002)	1.7658	0.0164	20 (AUG 2003)	3.1495	0.0145
9 (SEP 2002)	1.3969	0.0167	21 (SEP 2003)	2.8273	0.00951
10 (OCT 2002)	2.0485	0.0153	22 (OCT 2003)	3.3631	0.00948
11 (NOV 2002)	1.7942	0.0148	23 (NOV 2003)	3.0513	0.00957
12 (DEC 2002)	2.3302	0.0138	24 (DEC 2003)	3.1424	0.0103

Slope





Intercept

Results

- 21 different scenarios

Scenarios	D/R	L/R	B/R	No. of Iterations	Change the cut-off (%)	Rebuild the scorecard (%)	No action (%)
1a	250	5	30	690	69.8	28.4	1.8
2a	400	5	30	692	69.2	28.9	1.8
3a	500	5	30	694	68.9	29.3	1.8
4a	750	5	30	696	68.1	30.1	1.8
5a	1000	5	30	698	67.0	31.2	1.7
6a	2500	5	30	720	67.7	30.5	1.8
7a	5000	5	30	731	66.3	31.8	1.9
1b	250	5	90	693	78.9	19.0	2.0
2b	400	5	90	693	77.4	20.6	2.0
3b	500	5	90	695	76.3	21.7	2.0
4b	750	5	90	696	74.8	23.2	2.0
5b	1000	5	90	698	73.9	24.2	1.9
6b	2500	5	90	720	72.1	25.9	2.0
7b	5000	5	90	732	69.9	28.0	2.1
1c	250	5	180	693	83.9	13.9	2.1
2c	400	5	180	693	81.4	16.4	2.2
3c	500	5	180	695	80.6	17.3	2.1
4c	750	5	180	696	78.3	19.6	2.1
5c	1000	5	180	699	77.6	20.4	2.1
6c	2500	5	180	721	74.1	23.8	2.1
7c	5000	5	180	733	72.6	25.1	2.3

Optimal Policy for Scenario 1a (c=3)

m

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88
2	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88
3	2	2	2	2	2	2	2	2	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88
4	2	2	2	2	2	2	2	2	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88
5	1	1	1	1	1	1	1	1	2	2	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88
6	1	1	1	1	1	1	1	1	2	2	2	2	2	88	88	88	88	88	88	88	88	88	88	88	88
7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	88	88	88	88	88	88	88	88	88
8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	88	88	88	88	88	88	88	88	88
9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	88	88	88	88	88	88	88	88	88
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	88	88	88	88	88	88	88
11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	88
12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	88
13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	88
14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	88
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
17	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
18	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
21	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

99: Do nothing

88: Rebuild

1-40 : Recalibrate and the corresponding new cut off

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Optimal Policy for Scenario 1c (c=3)

m

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	2	2	2	2	2	2	2	2	2	99	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88
2	2	2	2	2	2	2	2	2	2	2	2	2	88	88	88	88	88	88	88	88	88	88	88	88	88
3	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	88	88	88	88	88	88	88	88	88
4	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	88	88	88	88	88	88	88	88	88
5	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	88	88	88	88	88	88	88	88	88
6	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	88	88	88	88	88	88	88	88
7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	88	88	88	88	88	88
8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	88
9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	88
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	88
11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	88
12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
17	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
18	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
21	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

99: Do nothing

88: Rebuild

1-40 : Recalibrate and the corresponding new cut off

Optimal Policy for Scenario 7a (c=8)

m

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88
2	5	5	5	5	5	5	5	5	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88
3	4	4	4	4	4	4	4	4	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88
4	3	3	3	3	3	3	3	3	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88
5	3	3	3	3	3	3	3	2	99	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88
6	2	2	2	2	2	2	2	2	7	7	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88
7	2	2	2	2	2	2	2	2	5	5	5	5	5	5	5	88	88	88	88	88	88	88	88	88	88
8	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	88	88	88	88	88	88	88	88	88
9	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	88	88	88	88	88	88	88	88	88
10	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	88	88	88	88	88	88	88	88	88
11	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	99	88	88	88	88	88	88	88	88
12	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	5	5	88	88	88	88	88	88	88
13	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	3	3	3	3	3	88	88	88	88
14	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	88
15	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	88
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	88
17	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	88
18	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	1	3
19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2
21	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

99: Do nothing

88: Rebuild

1-40 : Recalibrate and the corresponding new cut off

Optimal Policy for Scenario 7c (c=8)

m

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	6	6	6	6	6	6	6	6	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88
2	5	5	5	5	5	5	5	5	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88
3	4	4	4	4	4	4	4	4	13	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88
4	3	3	3	3	3	3	3	3	11	11	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88
5	3	3	3	3	3	3	3	2	99	99	99	99	88	88	88	88	88	88	88	88	88	88	88	88	88
6	2	2	2	2	2	2	2	2	7	7	7	7	7	7	7	7	88	88	88	88	88	88	88	88	88
7	2	2	2	2	2	2	2	2	5	5	5	5	5	5	5	5	88	88	88	88	88	88	88	88	88
8	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	88	88	88	88	88	88	88	88	88
9	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	14	88	88	88	88	88	88	88	88
10	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	11	11	88	88	88	88	88	88	88
11	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	99	99	99	88	88	88	88	88	88
12	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	5	5	5	5	5	5	5	88	88
13	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	88
14	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	88
15	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	88
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	88
17	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	5
18	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	1	3
19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2
21	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

99: Do nothing

88: Rebuild

1-40 : Recalibrate and the corresponding new cut off

Conclusions

- Using a, b, c from log odd score graph and cut-off gives concise state description of scorecard
- Allows estimates of dynamics of scorecard performance. Surely worth doing
- Allows rebuild and recalibrate decisions
- Optimal cut-offs allow for future dynamics
- Optimal Rebuilding is a control limit policy