A Dynamic Theory of the Credit Union

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Motivating Scenario

- Despite their growth and relative importance to consumer banking market, little research on credit unions is being done (Bauer, 2008).
- Current theoretical models are limited to one-period and fail to take future benefit allocation into account.
- As an economy moves over time, statically optimal decision that individuals make are generally not intertemporally optimal. (See Ross).
- Current linear utility models attaches equal value to a given amount of additional benefit to both dominant and minority groups.
- We extend the credit union theory to an intertemporal model.
- Members' objectives are modeled in a framework that attaches greater value to a given amount of additional benefit to the minority member group.
- An intertemporal model has implications for the cutoff score decision in credit unions.

Theoretical Contribution

- We extend the one-period credit union model to an intertemporal model.
- We determine optimal strategies for a credit union given an initial equity holding.
- We discuss the impact of the model on the cutoff score decision for a credit union.

Credit Union

- Credit unions are *pure* cooperative financial intermediaries serving two potentially conflicted owner groups, borrows and savers.
- Members are joined by "common bonds of occupations or associations, or within a well-defined neighborhood, community, or rural district".
- Credit unions have historically bettered the market loan and deposit rates for two reasons.
  - After cost residuals can be distributed to members in the form of higher deposit and lower loan rates.
  - Credit unions enjoy subsidies from sponsor firms and the government in the form of free resources and tax exemption.
- Credit unions have substantial market share
  - 171 million members, $1.2 trillion in assets world wide.
  - 9.41% of the consumer installment loan in the US.

Credit Union Operation - An example

- Every depositor and borrower is a member with one vote (regardless of deposit or credit volume).
- The credit union board is democratically elected by its members.
- The board appoints management and establishes loan rates, deposit rates, equity retention and other services delivery policies.
- A dominant saver (or borrower) population has the potential to elect a saver (or borrower) dominated board.
- The interest of the dominant group may not be served by exercising their dominance.

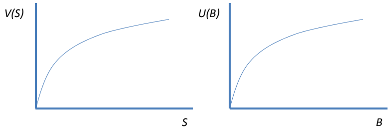
Credit Union Models

- The standard theoretical treatments of financial intermediaries and corporate enterprises cannot be directly applied to credit union behavior (Smith, 1981).
  - Members are both owners and customers and, hence, one cannot assume members seek to maximize profit generated by their transactions with the credit union.
  - The credit union intermediates between its depositors and borrowers; hence, a credit union cannot maximize the saver's (depositor's) rate while minimizing the borrower's rate.
- Smith's models maximize total benefit distribution to borrowers and savers,
  - i.e., maximize $L(l_m - l_c) + D(d_c - d_m)$, where L and D are loan and deposit volumes, l_m and l_c are market and credit union loan rates, and d_c and d_m are credit union and market loan rates.
  - See Smith (1981, 1984, 1988) and Smith *et al.* (1981).

Credit Union Models

- Smith's theoretical models are one-period models.
- These models ignore the effect of current benefit allocation and operating decisions on future credit union performance.
- Smith (1988) pointed out two decades ago that a key deficiency in the theory of credit union is the absence of intertemporal considerations (also see Overstreet and Rubin, 1991).
- We develop a intertemporal model with dynamic lending and deposit rate strategies.

Credit Union Objectives



- The objective of our stylized credit union is to maximize the present value of all future utility of borrower benefits $U(B(t))$ and saver benefits $V(S(t))$.
- Borrower (and saver) benefit is defined as $B(t) = L(t)(l_m - l_c(t))$ (and $S(t) = D(t)(d_c(t) - d_m)$).
- We model the respective utility functions as concave in borrower and saver benefits.
    - In this framework, the minority group attaches greater value to a given amount of additional benefit than the dominant group.

Discount Rate

- Multiperiod models require present value calculations; here the discount rate is that of a typical credit union member $p(E, L)$
- $p(E, L)$ is a function of equity E and loan volume L.
- When the equity to loan ratio is low, members perceive a greater chance of credit union failure and subsequently discount future benefits at a relatively higher rate.
- As equity increases, improved credit union safety leads to a lower discount rate.
- Equity allocations affect credit union value in two ways.
  - They can smooth member benefit distributions over time.
  - Increase in equity leads to a decreased discount rate making future benefits more valuable.

Balance Sheet Constraint

- Since a credit union is an ongoing entity, its assets equals its liabilities and equity at all times.
- Thus, the balance sheet constraint ensures that credit union loans plus investments equal deposits and equity.
- $L(t) + I(t) = D(t) + E(t)$.
  - $E(t)$ is the equity at time t.
  - $I(t)$ is the investment at time t.
- Credit unions make safe regulated investments.

Investments

- The investment of an efficient credit union is related to the loan volume, i.e., $I(t) = \alpha(L(t))$ with $\alpha'(L(t)) > 0$.
- Liquidity requirements, market risk, loan and deposit demand, and the operating characteristics of the sponsor firm shape the function α.
- Credit union with short term deposits and/or highly correlated risk profiles among members may need a higher ratio of investment to loans than otherwise.
- This constraint is *ad hoc* in nature and necessary for later analysis.

Income Statement Constraint

- The net change in equity results from loan and investments revenue less default losses, interest paid to depositors, operating costs and taxes.
- The revenue (and loss) from loans is $l_c\beta L(t)$ (and $(1-\beta)L(t)$), where $1-\beta$ is the expected default rate of the population.
- The revenue from the investments is $rI(t)$, where r is the investment rate of return.
- Interest paid to depositors are $d_c(t)D(t)$.
- The cost of operation $c(L(t), D(t))$ is a function of loan and deposit volume.
  - We model the operating cost as increasing and concave in both the loan and deposit volumes, i.e. increasing return to scale. See Fry *et al.* (1982) and Murray and White (1983).
- Thus, the net change in equity at time t is $\dot{E} = \frac{\delta E}{\delta t} = [(l_c(t)\beta - (1-\beta))L(t) + rI(t) - d_c(t)D(t) - c(L(t), D(t))]\tau.$
  - $\tau = 1-$ the credit union tax rate. Currently, the tax rate is zero.

Assumptions

- There are inherent assumptions in the model:
  - Only one type of loan and deposit are modeled. However, the model can easily be extended to multiple products.
  - Multiple member utility functions, various non-pecuniary credit union activities, employee motivation, and deposit insurance are not considered.
  - The partition of equity into regular reserves, special reserves, and undivided earnings is ignored.
  - We ignore the effect of loan rates on default rates.
  - We assume that response to a credit offer does not indicate higher risk levels.
  - Regulatory equity (capital) constraints are disregarded.
- Explicitly adding these characteristics would greatly complicate the model and obscure the basic results.

Model Summary

- Maximize $\int_0^\infty [U(B(t)) + V(S(t))]e^{-p(E(t),L(t))t}dt$
- Subject to:
  - Balance sheet constraint, $D(t) + E(t) = L(t) + I(t)$, and
  - Income statement constraint, $\dot{E} = [(l_c(t)\beta - (1-\beta))L(t) + rI(t) - d_c(t)D(t) - c(L(t), D(t))]\tau$
  - Initial equity $E(0) = E_0$, where E is the state variable.
- Control variables:
  - Credit union loan rate $l_c(t)$ and credit union deposit rate $d_c(t)$.

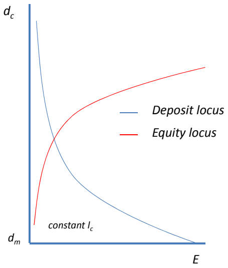
Model Simplification

- We substitute $I(t) = D(t) + E(t) - L(t)$ from the balance sheet constraint into the income statement constraint.
- The optimization model is:
  - Maximize $\int_0^\infty [U(B(t)) + V(S(t))]e^{-p(E(t),L(t))t}dt$
  - State equation, $\dot{E} = [(l_c(t)\beta - (1-\beta) - r)L(t) + rE(t) - (d_c(t) - r)D(t) - c(L(t), D(t))]\tau$
  - Initial equity $E(0) = E_0$.
  - Control variables: $l_c(t)$ and $d_c(t)$.

Solution Method

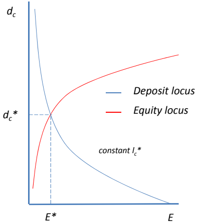
- The format of the optimization model allows us to determine the control policies using optimal control methods.
- Optimal control deals with the problem of finding a control law for a given system maximizing a given objective.
- This method results in a set of differential equations describing the system's evolution.
- We would like to observe the evolution of the lending and deposit rates with equity.
- As presented in the following phase diagram slides, we reduce the system to two differential equations for analysis.
- Steps in deriving the final equations can be found in Appendix A.

Results: Phase Diagram



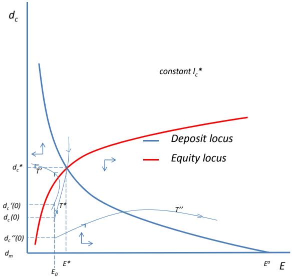
- The result of our analysis is plotted in (E, d_c) space (for ease of analysis, we chose E and d_c).
- Along the deposit (or equity) locus the deposit rate (or equity level) does not change, i.e., $\dot{d}_c = 0$ (or $\dot{E} = 0$).
- The diagram is plotted for a given l_c.

Phase Diagram



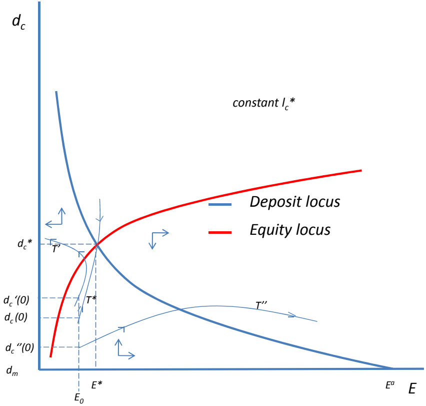
- The equilibrium point (E^{*}, d_c^{*}) formed at the intersection of the deposit rate and equity equilibrium loci represents the long run, steady state rate for any given l_c and parameter set.
- We can also determine the long run, steady state loan rate l_c^{*} (see Appendix A).
- Hence, we may draw the above for l_c^{*}
- The above diagram illustrates the long run deposit rate, loan rate and equity level.

Phase Diagram



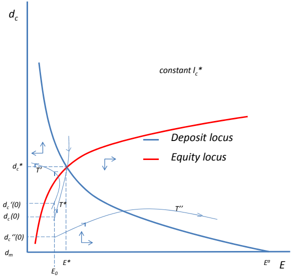
- Now, let us consider the optimal path from the initial equity level E_0 to steady state rates and equity levels.
- Given constant environmental factors, the optimal path is unique (see Blanchard and Fischer, 1989).

Phase Diagram



- Phase diagram illustrating the optimal path.

Phase Diagram



- Given an initial equity of E_0, $d_c(0)$ and $l_c(0)$ is chosen resulting in the optimal path.
- If d_c' is chosen, the credit union approaches the vertical axis resulting in zero equity level resulting in unbounded benefits.
- If d_c'' is chosen, the credit union approaches d_m in finite time, resulting in zero saver benefit and finite equity level.

Operational Considerations

- The optimal path depends on environmental factors.
- Shifting environmental conditions change parameter values and, hence, the optimal path of evolution.
- After such a change, the credit union may find that what once was an optimal rate policy is now sub-optimal.
- For example, an increase in borrower-oriented membership induced by an influx of young members revises the equilibrium deposit rate upward requiring an update long-term rate strategy.
- By resetting, $t = 0$, the credit union chooses optimal rates for the new conditions.
- Our intertemporal model also has implications for scoring decisions.

Cutoff Score Decision - Credit Union

- Credit unions are largely categorized by the NCUA into two groups based on their lending behavior.
  - Those offering risk-based pricing and those with one-price fits all.
- The latter group generally applies a cutoff score and then offers the credit product to approved applicants irrespective of their profitability.

Cutoff Score Decision

- Given initial conditions, the portfolio manager is faced with deciding on a cutoff score as well as lending and deposit rate paths.
- Given a cutoff score s_c, we may denote the expected default rate of the accepted population as $1 - \beta(s_c)$.
- For any given population with default rate $1 - \beta(s_c)$, the optimal path is unique as illustrated previously.
- The intertemporal model provides a framework to select the cutoff score that maximizes benefits to both member groups.
- However, shifting environmental conditions change parameter values and hence, the credit union may find that what once was an optimal cutoff score is now sub-optimal.
- Importantly, intertemporality provides a more meaningful framework to assess the impact of environmental changes on cutoff score.
- Keep in mind, the current model does not consider suboptimal account attrition.
- A profit-maximizing financial institution's (bank's) objectives are shown in Appendix B.

Contributions

- The intertemporal framework utilized in this paper yields a far richer set of implications than static, one-period modeling.
- The main difference between this model and Smith's (1984) lies in the intertemporal structure of the main choice variables.
  - By granting the credit union control over the timing and magnitude of benefit allocation, the problem of optimization becomes more intricate.
  - Theoretical control techniques, which call for continual appraisal, targeting, and corrective action, are comparable to actual credit union operation.
- The second major difference between our model and Smith's (1984) is the concavity assumption modeling the utility of benefits.
  - The concavity assumption allows us to set the slope and level of utility for diverse member groups, resulting in a richer set of utility functions.
  - Moreover, it should allow us to test the net benefits of favoritism vs. symbiotic strategies intertemporally.
- The intertemporal model allows us to test the sensitivity of cutoff score decision on the credit union's objective of maximizing member benefits.

Summary

- Theoretical control techniques, which call for continual appraisal, targeting, and corrective action, are comparable to actual credit union operations.
- Credit unions are vital entities that evolve through time. It is important and natural to model these important institutions within a more relevant intertemporal framework.
- Surprisingly, it took over two decades for the research community to answer Smith's observation of this need.
- Let us hope that future research builds on the work here more rapidly.

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Appendix A - Objective Function

- The credit union chooses functions $d_c(t)$, $l_c(t)$ and $E(t)$ to maximize $\int_0^\infty [U(B) + V(S)]e^{-p(E,L)t}dt$.
- where,
  - Borrower benefit $B(t) = l(t)L(t)$,
  - Saver benefit $S(t) = d(t)D(t)$,
  - $l(t) = l_m - l_c(t) > 0$,
  - $d(t) = d_c(t) - l_m > 0$,
  - Borrower utility function $U(B(t))$ with $U'(B) > 0$ and $U''(B) < 0$,
  - Saver utility function $V(S(t))$ with $V'(S) > 0$ and $V''(S) < 0$,
  - Credit union equity $E(t)$ with $E(t) > 0$ and $E(0) = E_0$,
  - Credit union deposit volume $D(t)$ and loan volume $L(t)$,
  - t is time.

Appendix A - Discount Rate

- The discount rate of a typical credit union member $p(E, L)$ is a function of equity E and loan volume L.
- $\frac{\delta p(E,L)}{\delta E} < 0$ and $\frac{\delta^2 p(E,L)}{\delta E^2} > 0$.
  - Higher equity value results in lower discount rate due to lower probability of the credit union defaulting.
- $\frac{\delta p(E,L)}{\delta L} > 0$ and $\frac{\delta^2 p(E,L)}{\delta L^2} < 0$.
  - Higher loan volumes result in higher discount rate due to the increase in the probability of credit union defaulting.

Appendix A - Demand

- Loan demand is an increasing, concave function of the difference between the market and credit union rates.
  - $L(t) = \epsilon(l(t))$ with $\epsilon'(l) > 0$ and $\epsilon''(l) < 0$
- Deposit demand is an increasing linear function of the difference between the credit union and market rates.
  - $D(t) = \sigma(d(t))$ with $\sigma'(d) > 0$ and $\sigma''(d) = 0$

Appendix A - Balance Sheet Constraint

- The balance sheet constraint ensures that credit union loans plus investment, $I(t)$, equal deposits plus equity.
- $L(t) + I(t) = D(t) + E(t)$.

Appendix A - Investments

- At optimal operation, the investment of a credit union is related to the loan volume, i.e., $I(t) = \alpha(L(t))$ with $\alpha'(L(t)) > 0$.
- Liquidity requirements, market risk, loan and deposit demand, and the operating characteristics of the sponsor firm shape the function α.
- Credit union with short term deposits may find a need for higher ratio of investment to loans than otherwise.
- Credit unions with highly correlated risk profiles among members, perhaps due to undiversified membership, may find a higher ratio of investment desirable.
- This constraint is *ad hoc* in nature and is necessary for later analysis.

Appendix A - Operating Costs

- Suppose $c(L(t), D(t))$ is the operating cost of the firm.
- Operating cost is increasing and concave in both the loan and deposit volumes, i.e. increasing return to scale. See Fry *et al.* (1982) and Murray and White (1983).
- $\frac{\delta c(L(t),D(t))}{\delta L(t)} > 0$ and $\frac{\delta^2 c(L(t),D(t))}{\delta L(t)^2} < 0.$
- $\frac{\delta c(L(t),D(t))}{\delta D(t)} > 0$ and $\frac{\delta^2 c(L(t),D(t))}{\delta D(t)^2} < 0.$

Appendix A - Income Statement Constraint

- The net change in equity results from loan and investments revenue less default losses, interest paid to depositors, operating costs and tax.
- The revenue (and loss) from the operations is $l_c \beta L(t)$ (and $(1-\beta)L(t)$), where $1-\beta$ is the expected default rate of the population.
- The revenue from the investments is $rI(t)$, where r is the investment rate of return.
- Interest paid to depositors are $d_c(t)D(t)$.
- Thus, the net change in equity at time t is $\dot{E} = \frac{\delta E}{\delta t} = [l_c(t)L(t) + rI(t) - d_c(t)D(t) - c(L(t), D(t))]\tau$.
    - Currently, the tax rate $1 - \tau = 0$.

Appendix A - Solution

- Denote $l_l = \beta l_c - (1 - \beta)$.
- We substitute I from the balance sheet constraint into the income sheet constraint, i.e., $\dot{E}[(l_l - r)L + rE - (d_c - r)D - c(L, D)]\tau$.
- E is the state variable, l_c and d_c are the control variables.
- The resulting Hamiltonian is $H = [U(B) + V(S)]e^{-pt} + \tau\mu(t)[(l_l - r)L + rE - (d_c - r)D - c(L, D)]$, where $\mu(t)$ is the costate variable.
- The necessary conditions are
  - (1)...$\dot{\mu}(t) = -H_E = [U(B) + V(S)]tp_E e^{-pt} - \mu\tau r$
  - (2)...$0 = H_{l_c} = \left[\frac{dU}{dB}\frac{dB}{dl_l} + \frac{dp}{dL}\epsilon'(l)[U(B) + V(S)]t\right] e^{-pt} + \tau\mu[L - \epsilon'(l)(l_l - r - \frac{\delta c}{\delta L})]$
  - (3)...$0 = H_{d_c} = \left[\frac{dV}{dS}\frac{dS}{dd_c}\right] e^{-pt} + \tau\mu[(r - d_c - \frac{\delta c}{\delta L})\sigma'(d) - D]$
  - (4)...$\dot{E} = H_\mu = \tau[(l_l - r)L + rE + (r - d_c)D - c(L, D)]$

Appendix A - Solution

- Since $U(B)$, $V(S)$, and $C(L, D)$ are all strictly concave, the following transversality condition is sufficient to show a unique maximum,
  - (5)...$\lim_{t\to\infty} \mu(t)E(t) = 0$
- We make the following substitution for notational convenience:
  - $-\frac{dp}{dL}\epsilon'(l) = P_l$
  - $\frac{dU}{dB}\frac{dB}{dl_l} = U_l$
  - $\frac{dV}{dS}\frac{dS}{dd_l} = V_d$
  - $L - (l_l - r - \frac{\delta c}{\delta L})\epsilon'(l) = \bar{L}$
  - $r - d_c - \frac{\delta c}{\delta D})\sigma'(d) - D = \bar{D}$

Appendix A - Solution

- Solve (3) for μ:
  - (6)....$\mu = -V_d e^{-pt} \tau^{-1} \bar{D}^{-1}$
- Which can be differentiated with respect to time
  - (7)... $\dot{\mu} = \tau^{-1} V_d \bar{D}^{-1} e^{-pt} [p - 2D_{bar}^{-1} \sigma'(d) \dot{(d_c]}$
- Rearranging (2):
  - (8)... $[U(B) + V(S)] t e^{-pt} = P_l^{-1} [\tau \mu \bar{L} Y_l e^{-pt}]$
- Substituting (6) into (8), then (6), (7) and (8) into (1):
  - (9)... $\tau^{-1} V_d \bar{D}^{-1} e^{-pt} [p - 2\bar{D}^{-1} \sigma'(d) \dot{(d_c]} = P_E P_l^{-1} e^{-pt} [U_l - L_{bar} \bar{D}^{-1} V_d] + r V_d e^{-pt} \bar{D}^{-1}$
- Rearranging terms gives:
  - (10)... $\dot{d}_c = \frac{1}{2} \bar{D} \sigma'(d)^{-1} [\tau p_E p_l^{-1} [\bar{L} - \bar{D} U_l V_d^{-1}] + p - \tau r]$

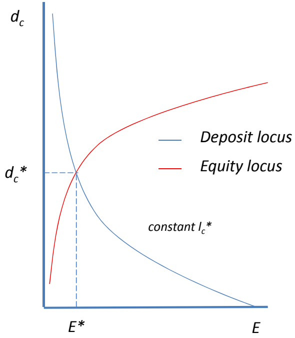
Appendix A - Steady State Solution

- The deposit rate locus is found by setting $\dot{d}_c = 0$ in (10) and solving for $p(E)$.
  - (11)... $p(E) = \tau\left[r - p_E p_l^{-1}(\bar{L} - \bar{D}U_l V_d^{-1})\right]$.
- The set of points in (E,d_c) space which satisfy this equation comprise the equilibrium locus.
  - (12a)... $\frac{dp}{dd_c} = -2\tau p_E p_l^{-1} U_l V_d^{-1} \sigma'(d) > 0$.
  - (12b)... $\frac{d^2p}{dd_c^2} = -2\tau p_E p_l^{-1} U_l V_d^{-1} \sigma''(d) = 0$.
- (12) shows that p increases linearly in the d_c direction along this locus but since p is convex and decreasing in E, results in a convex and decreasing deposit rate locus in (E,d_c).
- As $E \to 0$ in (12), $p \to \infty$ so $d_c \to \infty$.
- $d_c = d_m$ results in the locus intersecting the horizontal axis at some finite point E^a when p is positive.

Appendix A - Steady State Solution

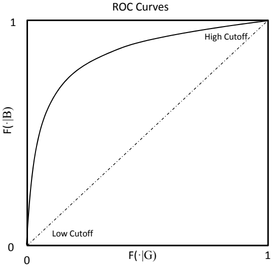
- The equity retention locus is found by setting $\dot{E}_c = 0$ in (4) and solving for E.
  - (13)... $E = r^{-1}\left[c(L,D) - (l_l - r)L + (d_c - r)D\right]$.
- The set of points in (E,d_c) space which satisfy this equation comprise the equity equilibrium locus.
  - (14a)... $\frac{dE}{dd_c} = r^{-1}\left[\left(\frac{dc}{dD} + d_c\right)\sigma'(d) + D\right] > 0$.
  - (14b)... $\frac{d^2E}{dd_c^2} = 2r^{-1}\sigma'(d) > 0$.
- Thus, the equity locus is decreasing and convex in (E,d_c) space.

Appendix A - Steady State Solution



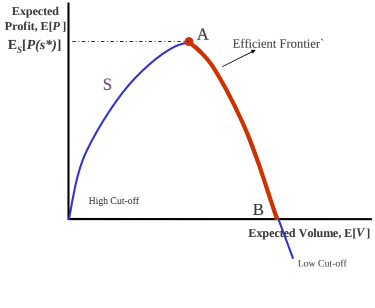
- We plot (E, d_c) for a given l_c.

Appendix B - Receiver Operating Characteristic Curve



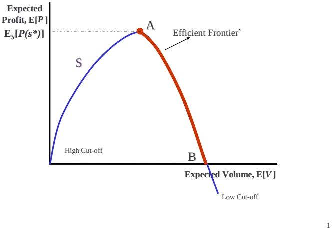
- When creating a portfolio, each applicant is scored using a scorecard.
- The ROC curve plots the cumulative proportion of bads vs. cumulative proportion of goods for a given cutoff score.
  - Each point on the receiver operating curve (ROC) curve represents a cutoff score, s_c.
  - It is hard for a portfolio manager to make a cutoff score decision from the ROC curve.

Appendix B - Profit Volume Space



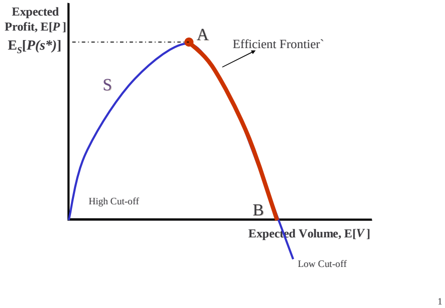
- Oliver and Wells (2001) showed how to build the Profit-Volume curve from the ROC curve given the following:
  - p_G, p_B be the priori probability of being good and bad respectively.
  - The profit l_c (loss l_b) on a good (bad) account.
- Initially, when the s_c is high, we accept no one and E$[P] = 0$ and E$[V] = 0$.

Appendix B - Efficient Frontier



- As we lower the cutoff score s_c, the expected profit increases until a maximum is reached, decreasing thereafter.
- Recall that a portfolio manager for a profit making financial institution faces the following conflicting objectives:
  - Maximize profit, maximize market share (volume) and minimize losses.
- Considering the trade-off between profit and market share, curve A to B is the efficient frontier.

Appendix B - Cutoff Score Decision for a Bank



- In contrast to an ROC curve, a portfolio manager makes a decision using the E[P] vs. E[V] curve.
    - See Oliver and Well, 2001; Beling *et al.*, 2005, Beling *et al.*, 2007.