ENHANCING BASEL METHOD VIA CONDITIONAL DISTRIBUTIONS THAT CAPTURE STRONGER CONNECTION AMONG CREDIT LOSSES IN DOWNTURNS

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Main objective: To estimate the probability of simultaneous *high* credit losses without assuming normally-distributed variables and normal (Gaussian) dependence

Motivation: Evidence that asset returns (loans included) are not normally distributed (neither univariate nor multivariate). Estimations based on the assumption of normality may result in probabilities of joint extreme events lower than what is observed in real portfolios.

A practical implication: The computation of the capital to cover unexpected credit losses in financial institutions is based on the assumption of normality. They are therefore subject to the underestimation of the capital necessary to face credit losses in downturns.

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COPULAS AND CONDITIONAL DISTRIBUTIONS

- Functions that link marginal distributions to joint distributions :

$$F_{1\ldots n}(y_1,\ldots, y_n) = C(F_1(y_1),\ldots, F_n(y_n))$$

- Cumulative distribution of a random variable Y at y conditional on other variables (vector $\mathbf{E}$), Joe (1996):

$$F_{Y|E}(y \mid \mathbf{E}) = \frac{\partial C_{YE_j|\mathbf{E}_{-j}}(F_{Y|E_{-j}}(y \mid \mathbf{E}_{-j}), F_{E_jE_{-j}}(E_j \mid \mathbf{E}_{-j}))}{\partial F_{E_jE_{-j}}(E_j \mid \mathbf{E}_{-j})}$$

- For one conditioning variable $E = e$:

$$F_{Y|E}(y \mid E = e) = C_{Y|E}(F_Y(y) \mid F_E(e)) = \frac{\partial C_{YE}(F_Y(y), F_E(e))}{\partial F_E(e)}$$

BASEL ACCORDS (CAPITAL REQUIREMENT - CREDIT)

$$K = \underbrace{LGD * K_V}_{\text{Total potential Losses (with probb of 99.9\%)}} - \underbrace{LGD * PD}_{\text{Expected losses}} = LGD * \boxed{(K_V - PD)}$$

$$\boxed{K_v = \Phi\left(\frac{\Phi^{-1}(PD) + \sqrt{\rho_{ij}}\,\Phi^{-1}(0.999)}{\sqrt{1-\rho_{ij}}}\right)}$$

(K_V - PD = unexpected credit losses)

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BASEL FORMULA: TWO DERIVATIONS

1) From Factor Models

- Default (Structural Models): latent variable < cutoff, i.e. $y_i < y_c$

- Joint defaults (Factor Models): $$Y_i = E\sqrt{\rho_{ij}} + \varepsilon_i\sqrt{1-\rho_{ij}}$$

Variables are N~(0,1)

ρ_{ij} : *linear correlation between* y_i *and* y_j

ρ_{YE} : *linear correlation between any* y *and* E

Essential relationship: $\rho_{YE} = \sqrt{\rho_{ij}}$

used to derive

- Potential extreme credit losses (in the calculation of capital requirement, Basel Accords)

$$K_v = \Phi\left(\frac{\Phi^{-1}(PD) + \sqrt{\rho_{ij}}\,\Phi^{-1}(0.999)}{\sqrt{1-\rho_{ij}}}\right)$$

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BASEL FORMULA: TWO DERIVATIONS

2) From the Gaussian Copula

- $F(y|E = e)$ when $C(F(y),F(e))$ is a Gaussian Copula is given by (Joe, 1997, Aas et al., 2009, Bouyé and Salmon, 2009):

$$F(y \mid E = e) = \Phi\left(\frac{\Phi^{-1}(F_Y(y)) - \rho_{YE}\Phi^{-1}(F_E(e))}{\sqrt{1 - \rho_{YE}^2}}\right)$$

- After some simple manipulation, I show that the formula above is equivalent to the Basel Formula (for v = 0.999)

$$K_v = \Phi\left(\frac{\Phi^{-1}(PD) + \sqrt{\rho_{ij}}\Phi^{-1}(0.999)}{\sqrt{1 - \rho_{ij}}}\right)$$

Note that $\rho_{YE} = \sqrt{\rho_{ij}}$

AN ALTERNATIVE STRUCTURE (FROM THE CLAYTON COPULA)

- If we assume that low values of the latent variables (i.e. when defaults happen) are more connected in downturns, we can use the Clayton Copula to represent the dependence between Y and E.

- In this case, $F(y|E = e)$ will be (Joe, 1997, Aas et al., 2009, Bouyé and Salmon, 2009):

$$F(y_c \mid E = e) = \{F_E(e)^{\theta_{YE}}[F_Y(y_c)^{-\theta_{YE}} - 1] + 1\}^{(-1-\theta_{YE})/\theta_{YE}}$$

where y_c is the default cutoff of the latent variable. The main challenge is to estimate θ_{YE}.

ESTIMATING THE COPULA PARAMETER θ_{YE}

- Copula parameters can be inferred from rank correlations such as the Kendall's tau (τ) between the variables. For some copulas, this relationship has closed forms. In the case of the Clayton Copula it is:

$$\theta_{YE} = \frac{2\tau_{YE}}{1 - \tau_{YE}}$$

- Now, the point is: how to find τ_{YE} ?

FINDING RANK CORRELATION BETWEEN Y AND E

- Recall that traditional approaches rely on the equality

$$\rho_{YE} = \sqrt{\rho_{ij}}$$

- I show in the paper that, for any copula:

$$\tau_{YE} \in [(-(\tau_{ij} + 1)) / 2, (\tau_{ij} + 1) / 2]$$

- Specifically for the Clayton Copula:

$$\tau_{YE} \in (0, (\tau_{ij} + 1) / 2] \quad \Longrightarrow \quad \theta_{YE} \in \left(0, \frac{2(\tau_{ij} + 1)}{1 - \tau_{ij}}\right]$$

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INITIAL COMPARISONS TO BASEL (SIMULATIONS)

- Losses skewed to the right (Kalyvas et al., 2006): Beta and Gamma distributions
- Right tail dependence across losses (e.g. Rosenberg and Schuermann, 2006) represented by the Gumbel Copula
- Corporate and retail credit (different correlations in Basel)
- Ten levels of PD (from 0.01 to 0.10)
- Three values for θ_{YE} : 1/3 of the maximum, the average, and the maximum possible
- In all scenarios, at least one alternative estimate (Clayton) was better that the results from Basel Formula. Example:

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Comparison between Basel and alternative estimates (retail credit, beta-distributed losses)

Avg PD	ρ_{ij}	Unexpected losses	Basel	Clayton 1/3 of max θ_{YE}	Clayton avg θ_{YE}	Clayton max θ_{YE}
0.01	0.12	0.3040	0.0825	0.1144	0.1876	**0.3626**
0.02	0.09	0.3353	0.1039	0.1831	**0.3134**	0.7248
0.03	0.08	0.3508	0.1107	0.2291	**0.3910**	0.8419
0.05	0.05	0.3756	0.1187	**0.2982**	0.4938	0.9043
0.07	0.04	0.3810	0.1205	**0.3190**	0.5206	0.9067
0.08	0.04	0.3843	0.1226	**0.3378**	0.5437	0.9057
0.09	0.04	0.4011	0.1260	**0.3548**	0.5630	0.9015
0.10	0.03	0.4199	0.1302	**0.3678**	0.5761	0.8946

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CALIBRATION OF τ_{YE} (AND θ_{YE})

- So, the higher PD is, the lower θ_{YE} will be in the interval (0, 2(τ_{ij} + 1)/(1 - τ_{ij})]

- Consequently, the higher PD is, the lower τ_{YE} must be in the interval (0, (τ_{ij} + 1)/2]

- According to the simulations, τ_{YE} may be assumed an exponential function of PD:

$$\tau_{YE} = \frac{(1 - PD)}{e^{PD*K}} * \tau_{YE\_MAX} \quad \text{or} \quad \tau_{YE} = \frac{(1 - PD)}{e^{PD*(K1 - K2*PD)}} * \tau_{YE\_MAX}$$

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Comparison between Basel and alternative estimates (retail credit , beta-distributed losses – θ_{YE} as a function of PD)

Avg PD	ρ_{ij}	Unexpected losses (UL)	Basel (BE)	Copula estimate (CE)	Abs (UL – BE)	Abs (UL – CE)
0.01	0.12	0.2953	0.0818	0.2752	0.2135	0.0201
0.02	0.09	0.3378	0.1032	0.3771	0.2346	0.0393
0.03	0.08	0.3582	0.1124	0.3630	0.2458	0.0048
0.05	0.05	0.3728	0.1183	0.3138	0.2545	0.0590
0.07	0.04	0.3844	0.1226	0.3025	0.2619	0.0819
0.08	0.04	0.4058	0.1262	0.3133	0.2796	0.0925
0.09	0.04	0.4223	0.1301	0.3362	0.2923	0.0862
0.10	0.03	0.4215	0.1342	0.3791	0.2872	0.0424

CONCLUSIONS (1)

- Basel formula is equivalent to a conditional distribution derived from the Gaussian Copula (1st derivative)
- Thus, if variables present tail dependence, we can use conditional distributions from copulas that express this property
- However there is not a closed-form relationship between τ_{ij} (observable) and τ_{YE} (non observable)
- θ_{YE} can be inferred from τ_{YE} (also non observable) by using τ_{ij} (assumed observable)

CONCLUSIONS (2)

- For the Clayton Copula: $\tau_{YE} \in (0, (\tau_{ij} + 1) / 2]$
- Alternative (Clayton) model always outperforms Basel formula for some θ_{YE}
- In principle, I am proposing an exponential function to find the "best" τ_{YE} in the range according to PD level (such that the alternative model outperforms Basel in all scenarios tested)
- I am currently trying to derive a precise relationship between τ_{YE} and PD
- Once I can do it, this approach can be extended to Factor Models related to assets in general (e.g. stocks)