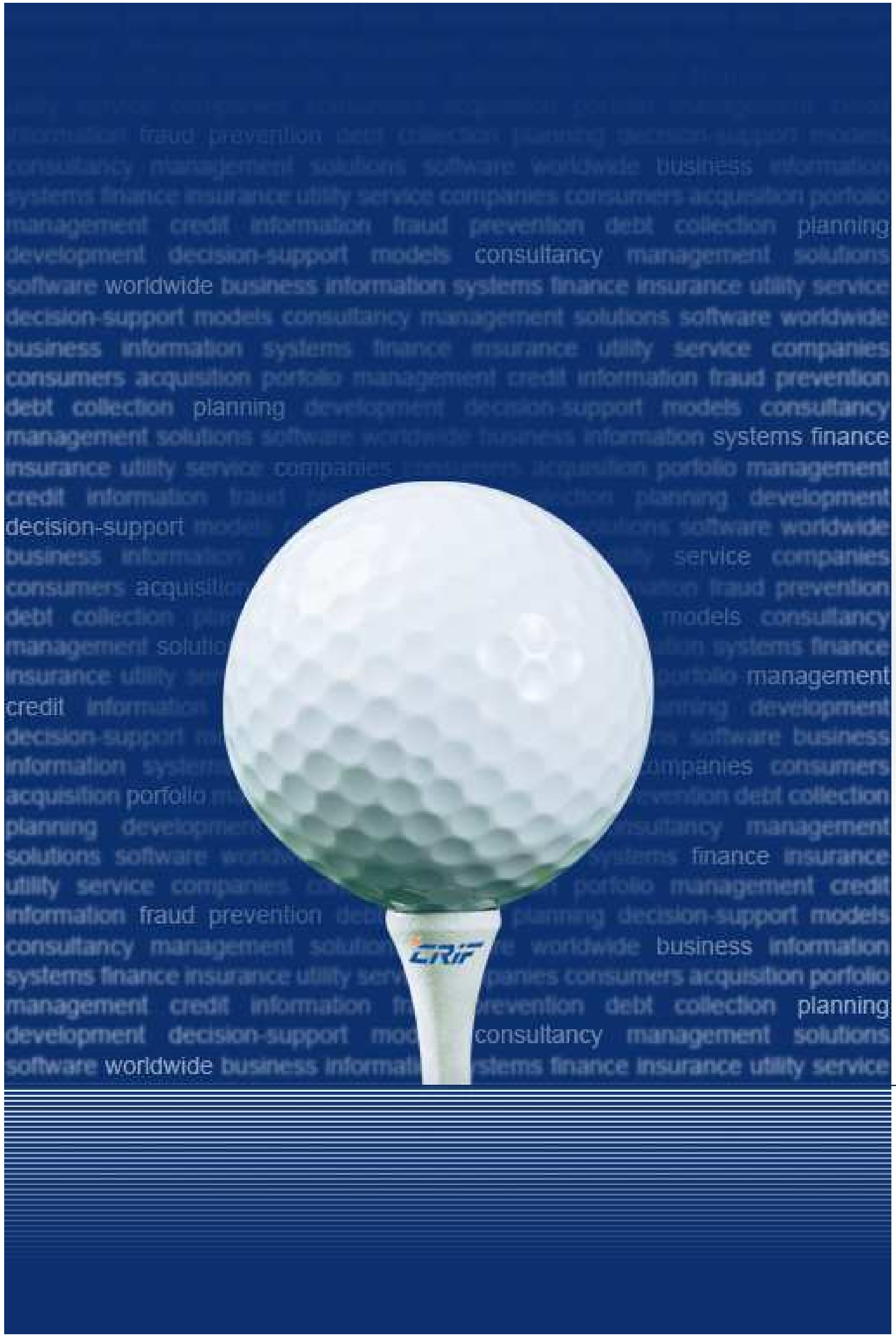


CRIF DECISION SOLUTIONS

Estimating LGD on Zero-Default Corporate Portfolios

A Market-implied Approach

Edinburgh – 24-26/08/2011

ESTIMATING LGD ON ZERO-DEFAULT CORPORATE PORTFOLIOS: A MARKET-IMPLIED APPROACH

LGD ESTIMATION

WORKOUT	➢ Computation based on analytical recovery / cost flows ➢ Discounted for: time value of money, volatility of recoveries ➢ Modeled as function of counterparty-specific and exposure-specific characteristics
MARKET	➢ Reliance on market data for direct LGD estimation (e.g. bond prices)

MARKET APPROACHES

TRADITIONAL

	MERTONIAN MODELS	STATISTICAL MODELS
LOGICS	LGD depends on the shortfall between the simulated future value of assets and the stock of outstanding debt	Prices of comparable defaulted bonds and loans are taken as proxy for the recovery rate and modeled econometrically
EXAMPLES	Seidler (2009): Implied Market Loss Given Default in the Czech Republic	Acharya, Bharath, Srinisavan (2004): Understanding the Recovery Rates on Defaulted Securities
SHORTFALLS	Modeling risk	Data availability, sample selection bias

ESTIMATING LGD ON ZERO-DEFAULT CORPORATE PORTFOLIOS: A MARKET-IMPLIED APPROACH

CALIBRATION-BASED

FOCUS	MARKET-IMPLIED MODELS
	Extract credit risk parameters (PD, LGD) embedded in prices of traded securities

LOGICS

Hypothesize parameterized relationships between state variables, PD and LGD →

Apply risk-neutral pricing methods to obtain parameter-dependent theoretical prices →

Rely on numerical optimization techniques to calibrate the model so that the difference between observable and theoretical prices is minimized

MODEL	CALIBRATION USING:
Ahn Le (2007): Separating the Components of Default Risk: A Derivative-based Approach	OPTIONS
Bakshi, Madan, Zhang (2006): Understanding the Role of Recovery in Default Risk Models: Empirical Comparisons and Implied Recovery Rates	BONDS
Das, Hanouna (2008): Implied Recovery	CREDIT DEFAULT SWAPS

DAS / HANOUNA (2008) – IMPLIED RECOVERY

MODEL STRUCTURE

SINGLE-FACTOR MODEL

the stock price is the only state variable of the model

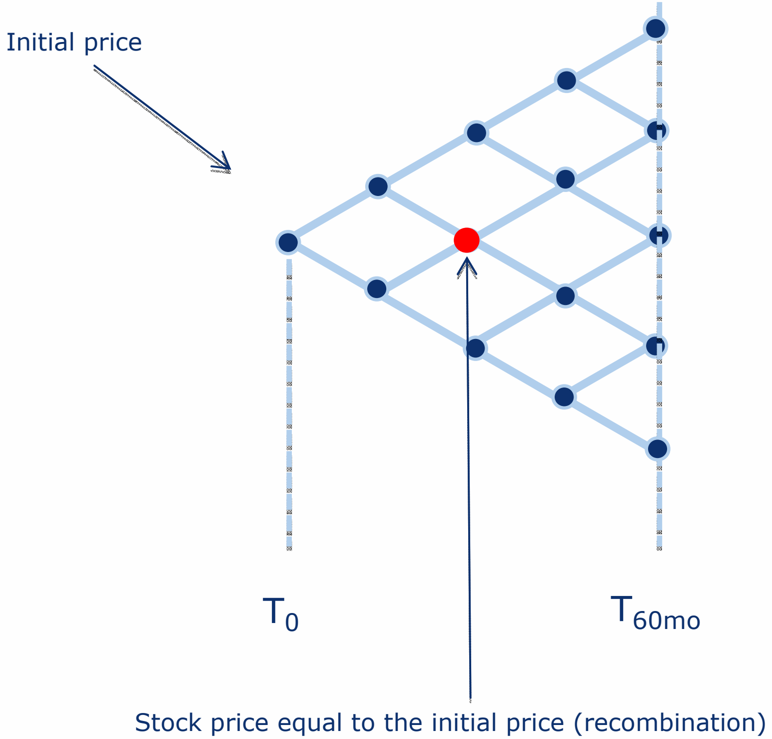
FACTOR EVOLUTION

the evolution of the stock price is simulated over a recombining tree mapping future prices in a given time interval

DYNAMICS

factor motion rules adopted are the same used in binomial trees by Cox, Ross, Rubinstein (1979):

$$S_{t+1} = \begin{Bmatrix} S_t \cdot e^{\sigma\sqrt{h}} & \textit{if up} \\ S_t \cdot e^{-\sigma\sqrt{h}} & \textit{if down} \\ 0 & \textit{if default} \end{Bmatrix}$$



The adopted, lattice-based specification for the evolution of stock prices allows to reduce the simulation complexity from exponential to polynomial

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RATIONALE

If we can hypothesize a functional relationship between equity prices, PD and LGD, then it is possible to use CDS spreads (whose theoretical pricing depends on PD and LGD) to calibrate this relationship and obtain PD and LGD term structures

HYPOTHESIZED RELATIONS

calibration!

Hazard rate – Instantaneous, (annualized) probability of default

$$\xi[i,j]=\frac{1}{S^{b}}$$

Probability of default

$$\lambda[i,j]=1-e^{-\xi[i,j]\cdot h}$$

Recovery rate (1-LGD)

$$\phi[i,j]=N(a_0+a_1\cdot\lambda[i,j])$$

Fair CDS spreads (fair)

$$CDS^{fair}=f(\lambda,\phi)$$

Stock price is the only driver

- The calibration process is directed at optimally choosing parameters a_0, a_1 and b, that is, at identifying parameters that minimize the difference between theoretical and observable spreads

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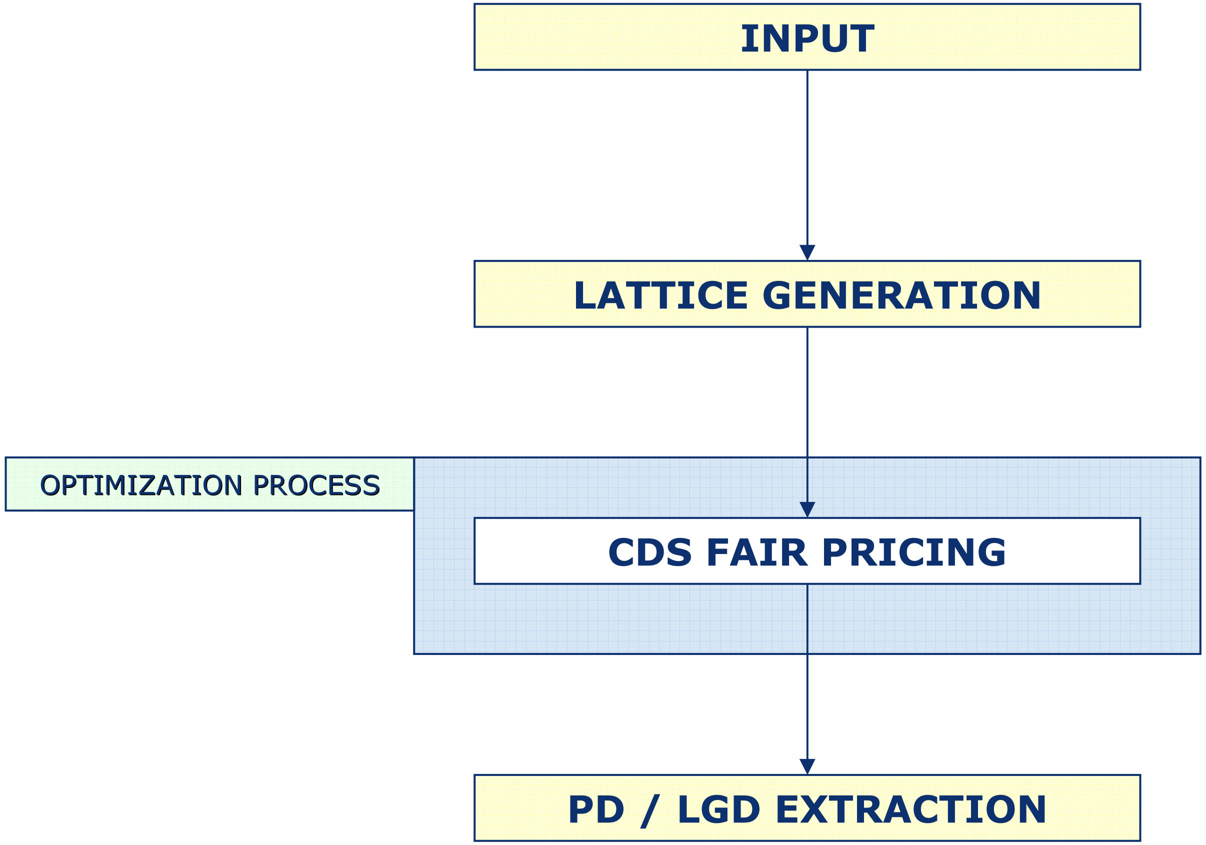
MODEL DEVELOPMENT



INPUT

LATTICE GENERATION

OPTIMIZATION PROCESS

CDS FAIR PRICING

PD / LGD EXTRACTION

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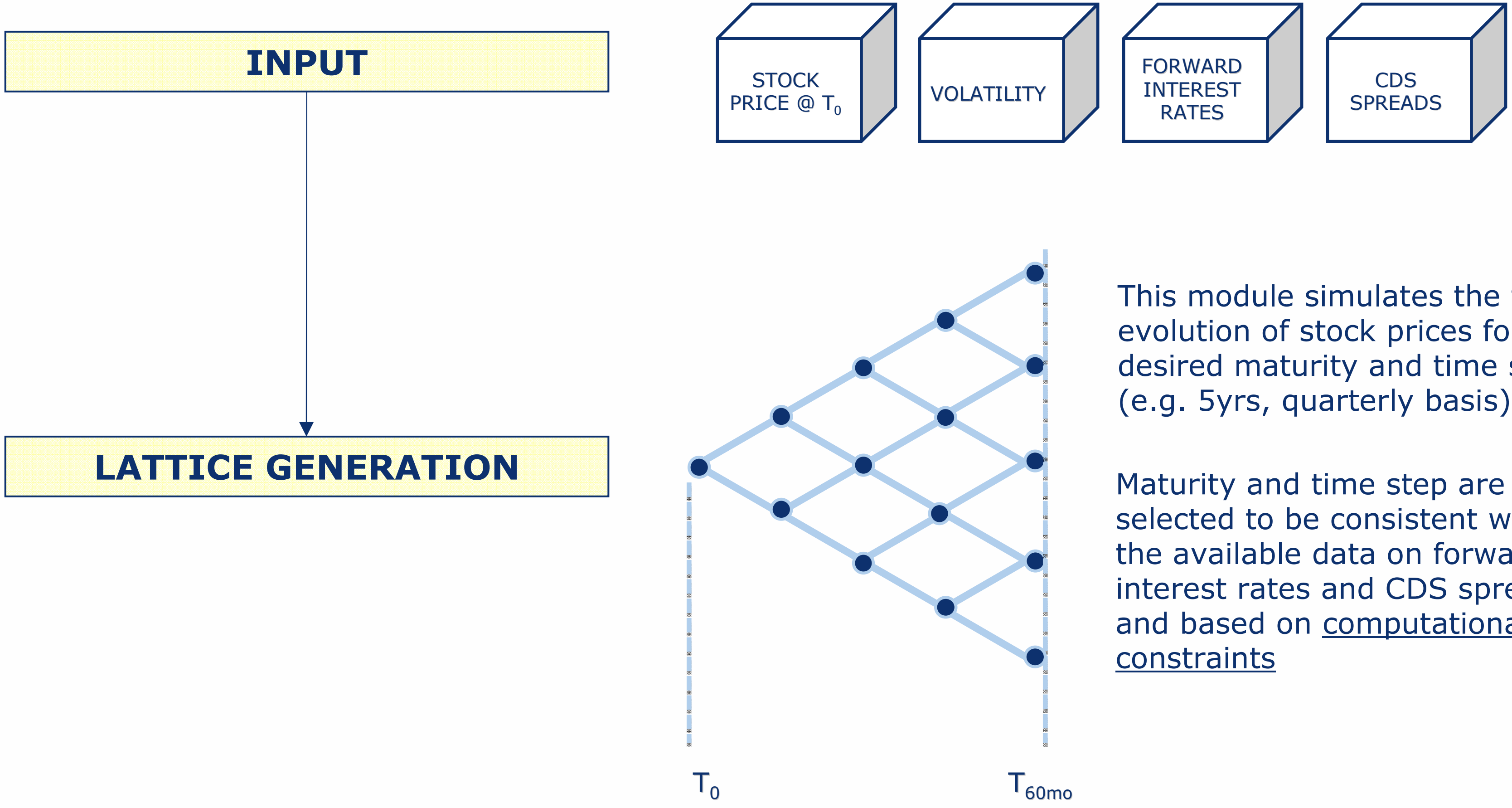
MODEL DEVELOPMENT



INPUT

LATTICE GENERATION

STOCK PRICE @ T_0

VOLATILITY

FORWARD INTEREST RATES

CDS SPREADS

This module simulates the future evolution of stock prices for the desired maturity and time step (e.g. 5yrs, quarterly basis)

Maturity and time step are selected to be consistent with the available data on forward interest rates and CDS spreads and based on <u>computational constraints</u>

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MODEL DEVELOPMENT

OPTIMIZATION PROCESS

CDS FAIR PRICING

Maximum amount a risk-neutral investor would be willing to pay in order to buy protection against a default event of the underlying company.

Given the future evolution of stock prices, fair CDS spreads are computed by this module until the following optimization problem is solved:

$$\min_{a_0,a_1,b}\left(\frac{1}{N}\sum_{j=1}^{N}\left[C_j(a_0,a_1,b)-C_j^0\right]^2\right)$$

where:

- a_0, a_1 and b are the control variables of the optimization problem
- C_j is the fair spread computed for the maturity j
- C_j^0 is the observable spread for the maturity j

Each optimization cycle performs the following steps:

GUESS [a_0 a_1 b]

ASSIGN THE RESULTING PD / LGD TO EACH NODE OF THE LATTICE

FAIR-PRICE SWAPS, SUCH THAT: **EXPECTED PREMIA = EXPECTED PAYOFF**

COMPUTE THE OBJECTIVE FUNCTION

PD / LGD EXTRACTION

DAS / HANOUNA (2008) – IMPLIED RECOVERY

MODEL TESTING

SAMPLE	**WESTERN EUROPEAN BANKING SECTOR**

RESULTS

GUESS-DEPENDENCY	The first tests performed on the model highlighted the dependency of the solutions obtained on the initial guesses used by the optimization algorithm, this holding true regardless of the convergence criteria adopted
MULTIPLE MINIMA	The issue above appears to be linked to the mathematical structure of the model itself, with its flexible framework incorporating an identification problem and resulting in a multitude of available local solutions
STATIC PARAMETERS	When tested on our sample, the static parameter set suggested by the authors for initial guessing (a_0 = 1,a_1 = -1, b = 1) led to performances comparable to those obtained by Das and Hanouna
FIRM-SPECIFIC PARAMETERS	However, further tests based on the adoption of firm-specific initial parameters displayed better performances, with best-performing solutions concentrating in parameter areas related to the risk profile of the company
MODEL PERFORMANCE	Based on this evidence, the results obtained by the authors (better model performance on hi-risk companies, poorer model performance on lo-risk companies) can be therefore be interpreted as function of the initial parameters chosen, that is, induced by the static "suggestion" provided to the optimization algorithm with respect to the parameter area to be searched for solutions.

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MODEL ENHANCEMENT

DYNAMIC RULE FOR INITIAL GUESSING

1

$$CDS^{1YR} \cong PD^{1YR} \cdot LGD$$

1-yr CDS spreads are the best market proxy for [PD x LGD], as they are less influenced by the time value factor if compared to spreads on longer maturities

a reasonable LGD guess can be then used to extract the 1-year probability of default embedded in equation (1)

Following the conclusions above, a dynamic rule for initial guessing was defined according to the principles below:

1. consistency with model specification
2. consistency with observables
3. no data snooping

2

$$\frac{CDS^{1YR}}{LGD^{guess}} \cong PD^{1YR} = \xi^{guess}$$

the PD extracted is hypothesized to be equal to the **target hazard rate**, that is, the instantaneous annualized probability of default to be used in the first optimization cycle

3

$$\xi^{guess} = \frac{1}{S^b} \qquad \boxed{b^{guess}} = \log_S \frac{1}{\xi^{guess}} = \frac{\ln \frac{1}{\xi^{guess}}}{\ln S}$$

once the target hazard rate is extracted, we can invert the model function to extract a **guess on parameter *b*** (in T_0)

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MODEL ENHANCEMENT

DYNAMIC RULE FOR INITIAL GUESSING

4

$$\lambda^{guess} = 1 - e^{-\xi^{guess} \cdot (h)}$$

The target hazard rate obtained can be used to extract the corresponding PD in the sub-period defined by the time step chosen (h)

5

$$1 - LGD^{guess} = \phi = N(a_0 + a_1 \cdot \lambda^{guess})$$

The PD can then be plugged into the functional relationship for the recovery rate and extract the rule-consistent pairs of parameters a_0 and a_1 that make the LGD^{guess} equal to the level hypothesized in step 1

6

$$\boxed{a_0^{guess}} = N^{-1}(1 - LGD^{guess}) - \boxed{a_1^{guess}} \cdot \lambda^{guess}$$

A rule-compliant guess on parameter a_0 can now be extracted for the a_1 parameter interval [0, -50, -100, -150, …, -2000], that is, for reasonably chosen levels of correlation between PD and recovery rate

DAS / HANOUNA (2008) – IMPLIED RECOVERY

MODEL RESULTS

Implementation software: STATA

Optimization algorithm used: Nelder-Mead

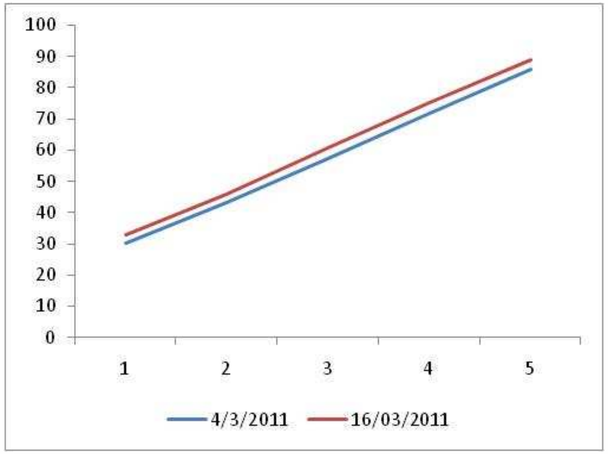
Convergence criteria: 16th decimal

Volatility input: Implied volatility

Calibration horizon: 5yrs

BANK 1

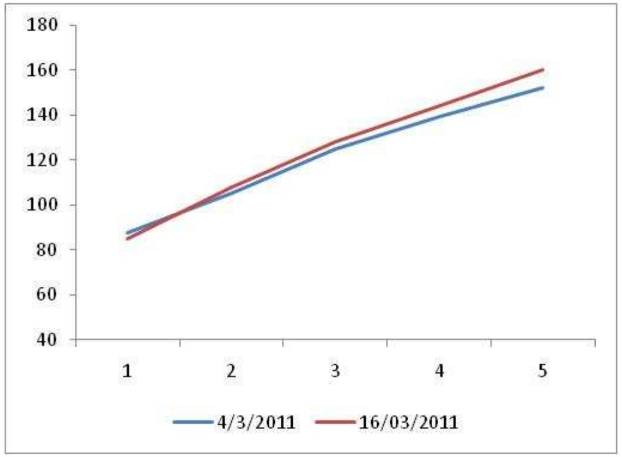
LOW RISK



	BANK 1 - LOW RISK			
	LGD - 04/03	AVG_%RMSE	LGD - 16/03	AVG_%RMSE
DYNAMIC	19%	8.4%	20%	4.1%
STATIC	13%	12.7%	73%	12.7%

BANK 2

HIGH RISK



	BANK 2 - HI-RISK			
	LGD - 04/03	AVG_%RMSE	LGD - 16/03	AVG_%RMSE
DYNAMIC	59%	1.4%	50%	0.9%
STATIC	59%	1.5%	31%	1.5%

CONCLUSIONS

CONCLUSIONS

The solution illustrated represents an equilibrium point on the trade-off line between model flexibility and identification needs. Without adding to the complexity of the overall framework, it provides a reasonable guidance in the initialization of the optimization process, making the model more tractable and credible in its outcomes.

HINTS

Calibration on shorter horizons can help reduce the noise effect induced by traders' myopia on credit risk (flattening spread curves)

IMPROVEMENT AREAS

Functional specifications for the integration between equity and credit markets, that is, the relationship between stock price and hazard rate

Functional specifications for the relation between PD and recovery rates: adapting physical relations to their risk-neutral counterparts?

Multi-calibration using other credit-related securities: bonds, options, recovery swaps, digital default swaps (these last two securities look very promising for LGD research purposes, although the trade volumes are still too thin)

Embedding different or multiple **state variables** in the framework: interest rates, macroeconomic variables (the latter can suit stress testing purposes as well)

THANKS FOR YOUR ATTENTION

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CRIF DECISION SOLUTIONS