

# Mortgage refinancing as individual decision under uncertainty: how to build a reliable prepayment model

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August 2011

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# Topics of presentation

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- **Mortgage prepayment model: formulation and business objectives**
- **Modeling experiments and prediction quality**
- **Using prospect theory to improve prediction quality of mortgage prepayment model**

What kind of reasoning is behind decision under uncertainty

Prospect theory value function formulation

Using prospect theory to improve out-of-period prediction

- **Rational prepayment versus actual prepayment**
- **Conclusion and future research**

## Predictive models for individual mortgage prepayment: business objectives

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- Prepayment prediction on individual level can be an important tool for different financial applications
- It can be used for more accurate pricing, and can also help to build pools of mortgages with precisely tailored characteristics of interest rate sensitivity
- Prepayment is an individual financial decision under uncertainty and a very natural area for application mental accounting methods like prospect theory

# Predictive models for individual mortgage prepayment: general considerations

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- Prepayment can be driven by a number of different reasons, most important: refinancing and relocation.
- These two events have different patterns and are influenced by different predictive characteristics. We will concentrate on refinancing events, for which mental accounting considerations can be applied.
- In order to filter out the relocation driven records we removed the cases of prepayment, for which refinancing could not be based on a reasonable decision, for example when original rate of the mortgage is lower than current mortgage rate offered by market or current rate is above the NPV break-even threshold. The NPV break-even level is evaluated based on the condition that benefit of lower mortgage rate is precisely offset by cost of refinancing.

# Predictive models for individual mortgage prepayment: general considerations

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- **How to evaluate model predictive power?**

  - Model as classifier: quality is based on KS or GINI index

  - Model as predictor: probability versus actual rate, is model properly calibrated?

- **Out-of-sample and out-of-period prediction quality**

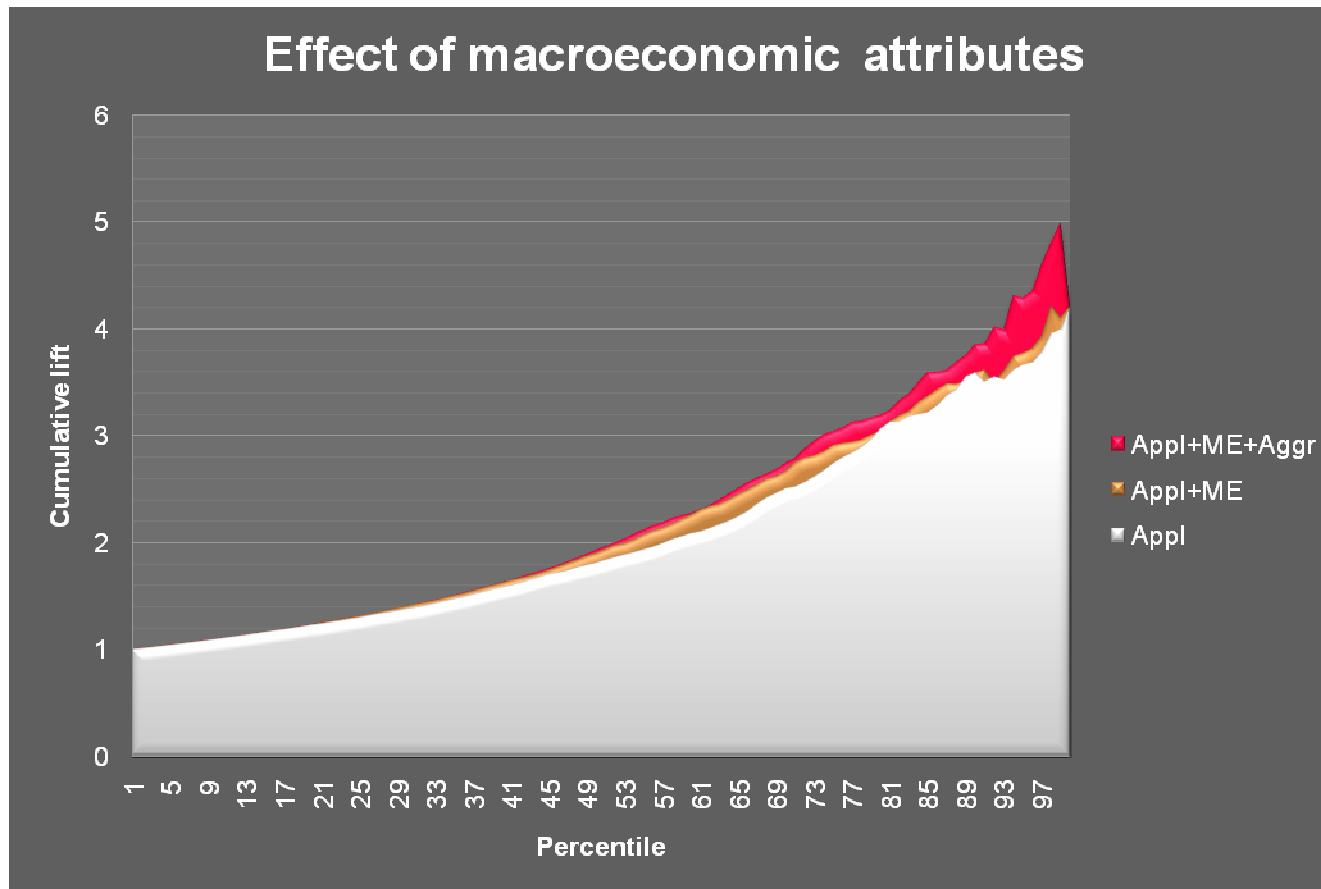
  - Since behavior pattern may change as economic environment conditions evolve, building reliable out-of-period model is especially tricky.

## Predictive models for individual mortgage prepayment: general considerations

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- Prediction quality can be significantly improved if econometric attributes (like inflation, unemployment, etc.) are taken into account
- History of mortgage rates and their volatility could be significant attributes in prediction of prepayment events. To use economic history some sort of aggregation should be applied.
- As an alternative of using common sense to decide how to aggregate historical data we can use a neural network transactional modeling approach and leave for training algorithm to decide which form of aggregation should be used to maximize predictive power.

# Adding macroeconomic attributes



## Development sample

June, 1994 – Nov., 1997

Training : 43,147 records

Validation : 18,463 records

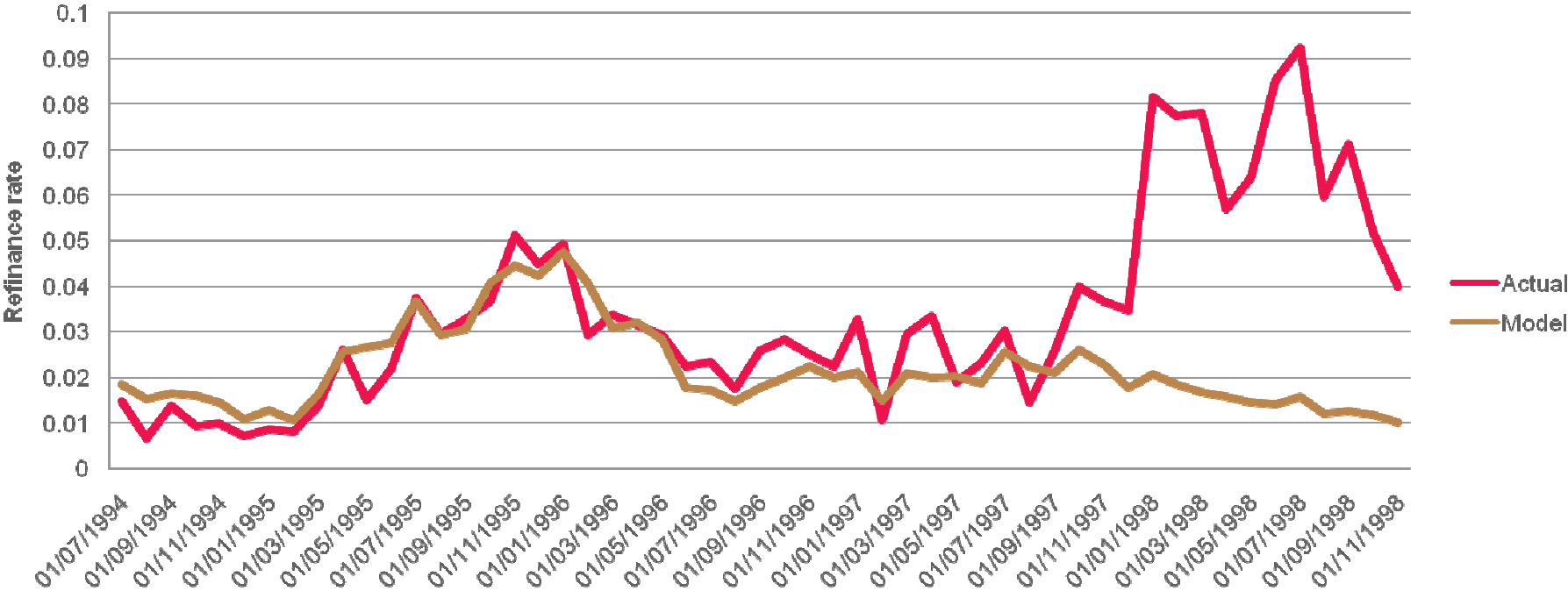
## Out-of-period sample

Dec., 1997 – Nov., 1998

8,364 records

# Model vs. Actual – out-of-period discrepancy

## Monthly Actual vs Modeled Refinance Rate



Out-of-period: Dec.1997-Dec.1998  
 Actual refinance rate 0.0635    Modeled refinance rate 0.0142

# Using mental accounting ideas to for improvement of prepayment models: introduction

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- Simplest assumption is that individuals act to maximize expected value of profit. The fact that this is not a valid assumption became clear more than two centuries ago.
- Next fundamental result is expected utility theory. Assumption: individuals behave rationally in order to maximize subjective utility function which combines risk and return consideration. Since individuals are assumed to behave rationally the theory is supposed to be both descriptive and normative.
- Mental accounting is using different approach. Most important result in mental accounting is cumulative prospect theory.
- Prospect theory conclusion: descriptive theory and normative theory cannot be combined. No matter how individual utility function is chosen, people systematically violate the expected utility principle.

## Most important finding of prospect theory

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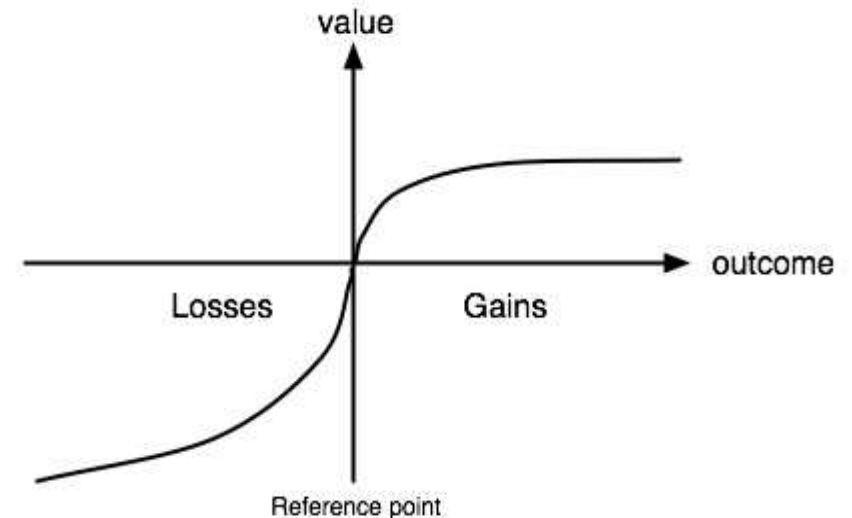
- People evaluate the value of outcome with respect to a reference point, incremental value rather than absolute wealth is important (framing effect)
- People are less willing to gamble with profit than with loss . Value function of prospect theory, which replaces classical expected utility function, is steeper for losses than for gains.
- There is an asymmetry in how people perceive gains and losses (loss aversion effect).
- Contribution of possible outcomes to value function is proportional to decision weights which are not equal to probabilities. Based on prospect theory people tend to overweight small probability and underweight high probability.

# Prospect theory value function formulation

In accordance with prospect theory the value function  $V(W)$  for each outcome with wealth  $W$  and reference level  $R$  can be expressed as following S – shaped function

$$V(W) = \frac{(W - R)^{1-\gamma}}{1-\gamma} \quad \text{if} \quad (W - R) \geq 0$$

$$V(W) = -\lambda \frac{(R - W)^{1-\gamma}}{1-\gamma} \quad \text{if} \quad (W - R) < 0$$



# Prospect theory probability weighting function

If gamble can be subjectively evaluated as paying

$$x_{-m} < x_{-m+1} < \dots < x_{-1} < x_0 = 0 < x_1 < \dots < x_n$$

with corresponding probabilities  $p_{-m}, p_{-m+1}, \dots, p_{-1}, p_0, p_1, \dots, p_n$

The value function for the gamble will have the following form:

$$U = \sum_{i=-m}^n \pi_i V(x_i) \quad \text{where } V(x_i) \text{ is S-shaped function as above}$$

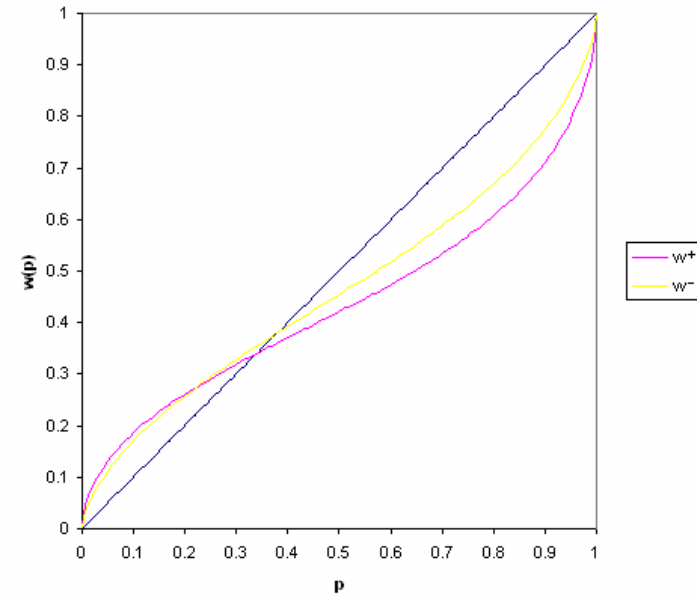
$$\pi_i = w^+(p_i + \dots + p_n) - w^+(p_{i+1} + \dots + p_n) \quad \text{if } 0 \leq i \leq n$$

$$\pi_i = w^-(p_{-m} + \dots + p_i) - w^-(p_{-m} + \dots + p_{i-1}) \quad \text{if } -m \leq i \leq 0$$

Probability weighting functions  $w^-(p)$  and  $w^+(p)$  capture subjective perception of probability and can be expressed as following:

$$w^+(p) = \frac{p^{\alpha^+}}{(p^{\alpha^+} + (1-p)^{\alpha^+})^{1/\alpha^+}}$$

$$w^-(p) = \frac{p^{\alpha^-}}{(p^{\alpha^-} + (1-p)^{\alpha^-})^{1/\alpha^-}}$$



# Applying prospect theory to predicting mortgage prepayment: Summary of mortgage accounting

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Current monthly payment

$$w = \frac{r_0}{1 - (1 + r_0)^{-T}}$$

Monthly payment after re-financing

$$v = \frac{r}{1 - (1 + r)^{-T}}$$

Outstanding principal amount

$$B_t = \frac{1 - (1 + r_0)^{-(T-t)}}{1 - (1 + r_0)^{-T}}$$

Outcome of exercising prepayment option

$$c(r, r_0, t) = [w/v - (1 + e)]B_t - C_0$$

Gain due to lower rate can be corrected to include tax considerations and limited expected life of loan until exogenous prepayment (relocation, etc)

$$c(r, r_0, t) = [(w/v - 1)(1 - \rho) + e]B_t - C_0$$

Where  $\rho$  is taxation rate

## Using prospect theory to improve out-of-period prediction

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From cumulative theory perspective the gamble can be described in a following simple way:

A mortgage holder faces the following alternative:

Do nothing which we consider a reference point

Refinance. Then if rates change, the resulting gain/loss is:

$$D = c(r, r_0, t) - \max(c(r + dr, r_0, t + dt) - \sum_{i=1}^{i=i_{\max}} (w - v)(1 - \rho)B_{t_i}, 0)$$

If rates go up the result is a gain, otherwise it can be a loss. For simplicity we considered only two outcomes: rates up and rates down. The magnitude of the changes was evaluated based on observed mortgage rate volatility.

## Segment statistics

Segm. #	# of records	Condition	Avg. appraisal value	Avg. Mtg.Amt	Avg. Home Value	Avg fam income	Prosp. Theory $\lambda$
1	10,927	M<50000 & V<138400	31,159	32,805	71,500	37,640	<b>3.9</b>
2	9,618	M<87100 & V>=138400	100,075	55,640	169,000	50,400	<b>2.25</b>
3	13,261	50000<=M<87100 & V<138400	77,857	67,436	84,870	42,860	<b>1.5</b>
4	21,522	M>=87100 & V<253500	175,511	132,933	144,720	54,990	<b>1.18</b>
5	6,265	M>=87100 & V>=253500	262,727	172,473	273,760	62,920	<b>1.33</b>

M- Mortgage Amount (from application)

V- Area Average Home Value (from associated demographic data)



# Using prospect theory to improve out-of-period prediction

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**Overall the strategy of applying prospect theory to improve model quality can be summarized as following:**

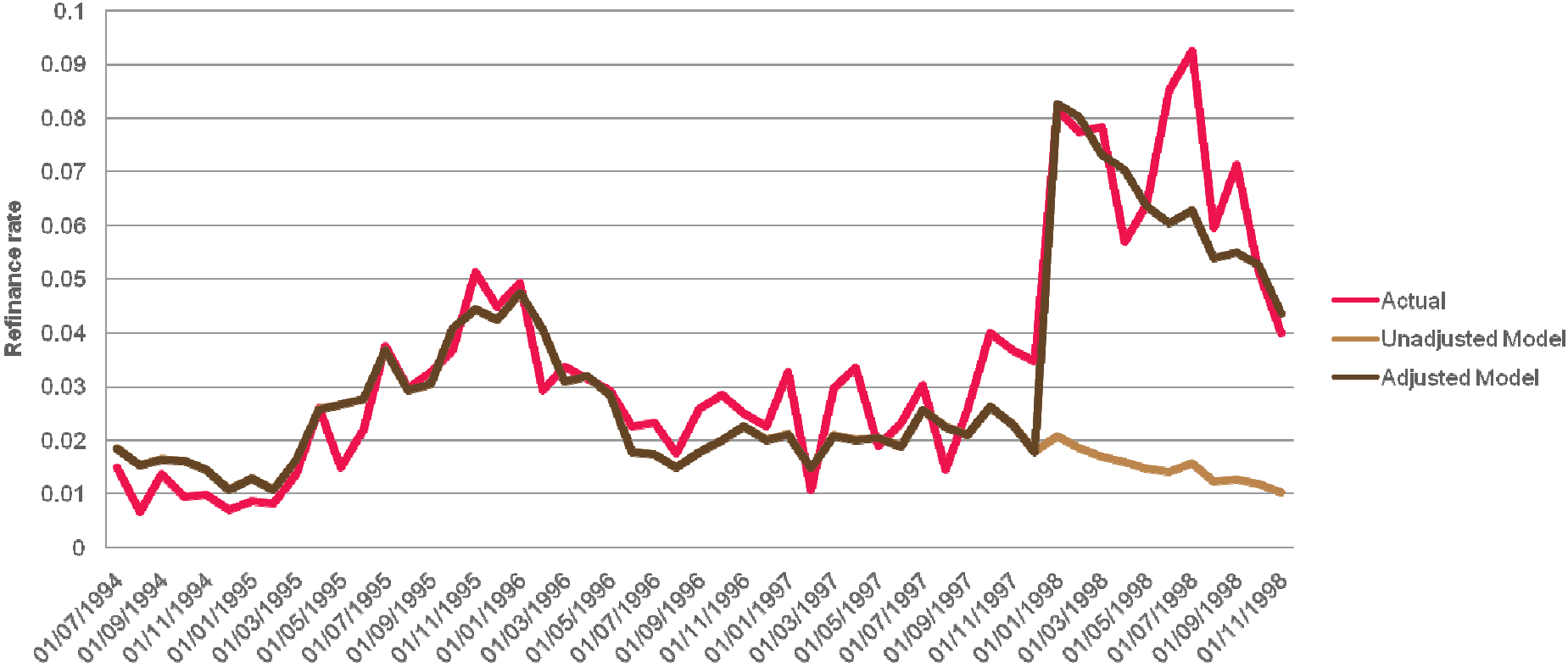
- Build population segments for development sample using tree segmentation technique (CHAID).
- Evaluate coefficients of value function for each population segment
- Build in-sample model
- Apply the model for out-of-period data and evaluate predicted response rate for each segment
- Use value function coefficients to evaluate out-of-period probability of prepayment for each population segment
- Apply scaling coefficients to make model output consistent with prospect theory prediction

# Prospect theory based out-of-period adjustment: results

	Training subset		Out-of-sample validation		Out-of-period validation		
	Actual	Model	Actual	Model	Actual	Model	Adjusted
Sample size	43,147		18,463		8,364		
Refinance rate	0.0242	0.0241	0.0247	0.0234	<b>0.0635</b>	<b>0.0142</b>	<b>0.0607</b>
Gini (%)		68.1		67.2		<b>54.1</b>	<b>53.6</b>
KS (%)		57.0		56.3		<b>44.8</b>	<b>43.4</b>

# Unadjusted vs. Adjusted Model

## Monthly Actual vs Modeled Refinance Rate



# Rational prepayment versus actual prepayment

- Does actual prepayment differs from what can be considered as rational?
- To determine if it is rational to refinance is a complex mathematical problem which is equivalent to pricing call option.
- The value of option is based on assumptions of underlying interest rate and mortgage rate stochastic process and risk adjustment coefficients. A standard way to solve the problem is first using lemma Ito to convert to a set of partial differential equations which can be solved using appropriate boundary conditions. The process is computationally intensive. Although the results are different depending on underlying process and risk adjustment parameters, still it is interesting to compare at least some of them with actual behavior.
- We used some recent results which allow for some simple underlying to express the rational prepayment condition in closed form.

$$r - r_0 \leq x^* \equiv \frac{1}{\psi} (\phi + W(-\exp(-\phi)))$$

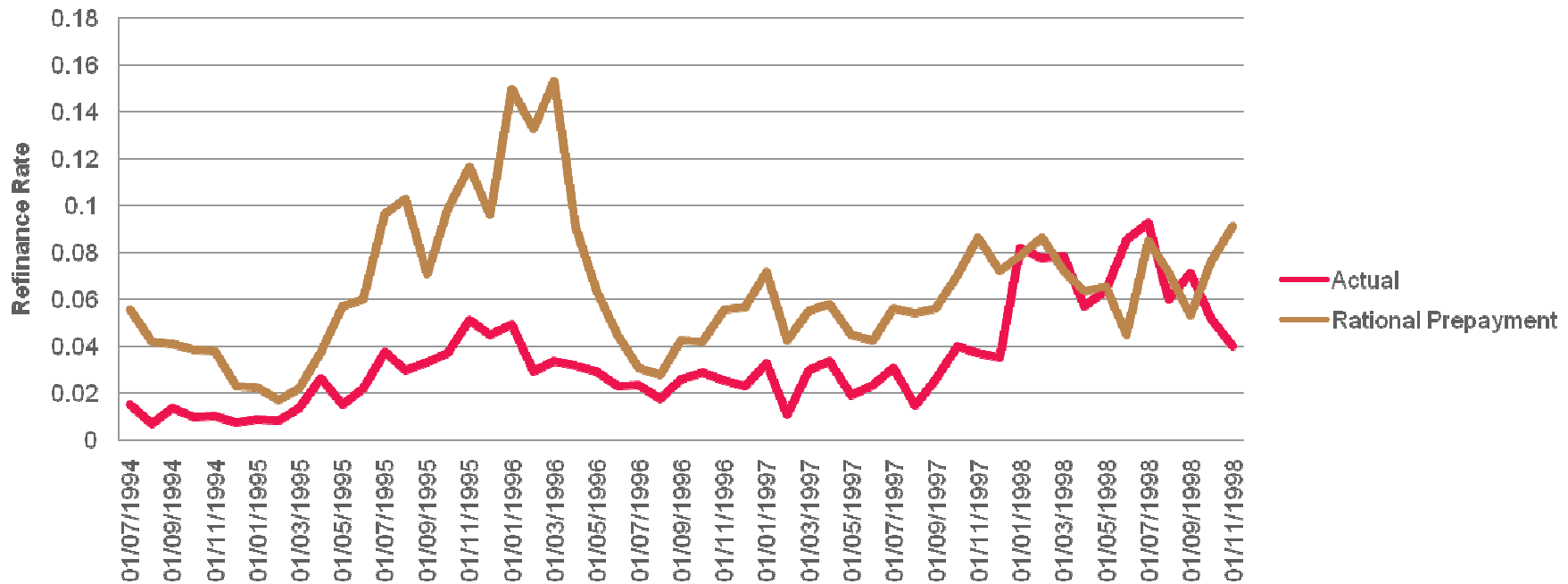
$$\text{where } \psi = \frac{\sqrt{2(i + \lambda)}}{\sigma} \quad \phi = 1 + \psi(i + \lambda) \frac{C}{B_t(1 - \tau)} \quad \lambda = p_{reloc} + \frac{r_0}{\exp(r_0 T) - 1} + i_{inf}$$

$W(\cdot)$  is the Lambert W-function

Our results show that actual prepayment differs significantly from rational decision.

# Rational prepayment versus actual prepayment

## Actual vs Rational Prepayment Rates



	# of records	Threshold (b.p.)	Predicted #	Prediction rate	Actual #	Actual rate
In-period	61,593	251	3,908	<b>0.063</b>	1,486	<b>0.024</b>
Out-of-period	8,352	195	584	<b>0.070</b>	519	<b>0.062</b>

# Conclusion and future research

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## Conclusion

- Econometric history is important to predict prepayment behavior
- Models often do not produce reliable prediction for out-of-period prepayment rate
- Prospect theory can be used as a common ground for out-of-sample and out-of-period prediction and greatly improve prediction quality
- Actual prepayment behavior pattern differs significantly from the result of applying a normative theory, which can be considered as rational behavior

## Future research

- As future steps of this research it would be interesting to apply this approach to recent data. Analyze “optional default” events which are common for recent housing market downturn
- Use new data to analyze disposition effect