

# RBS Risk Management

How wrong is your model?

– Conservatism, use and uncertainty.

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# How wrong is your model?

## Conservatism

*Essentially, all models are wrong, but some are useful.*

George Box and Norman Draper

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Any graphs or tables shown are based on mock data and are for illustrative purposes only.

# How wrong is your model?



## Overview

- Conservative adjustment – a working definition and when to apply it
- Standard error – the basic solution
- Sample size and the impact of correlation
- Effective Sample Size and Conservatism

## A working definition

### Conservatism

- The adjustment of a model's assumptions, build or output, required to ensure that the model remains fit for purpose in the face of recognised, specific weaknesses in its development or implementation.
- This presentation concentrates on conservative adjustment of output.

## Conservative Adjustment and Basel II

- *BIPRU 4.3.88: A firm must add to its estimates a margin of conservatism that is related to the expected range of estimation errors. Where methods and data are less satisfactory and the expected range of errors is larger, the margin of conservatism must be larger (see also BIPRU 4.3.48,.74, BCD Annex VII Part 4 point 49, BIS 411, 416).*
- *BIPRU 4.3.95 (Low Default Portfolios): A firm must use a statistical technique to derive the distribution of defaults implied by the firm's experience, estimating PDs (the "statistical PD") from the upper bound of a confidence interval set by the firm in order to produce conservative estimates of PDs in accordance with BIPRU 4.3.88 R.*
- Conservative adjustment is a Basel II requirement
- The adjustment required must respond to information
- Confidence intervals can be used to quantify the adjustment

## When to apply Conservatism for IRB models

- ✓ The model's parameters and estimates have a range of possible values consistent with the development data (confidence interval / standard error).
- ✗ The model's output is a distribution whose observed values have a natural variation (prediction error).
  
- ✓ The model uses 3 years data but should use 5 years.
- ✓ To build the model, the data has been over-sampled (the dataset has multiple consecutive months for each obligor, but each record is considered independent).
- ✓ A recognised factor hasn't been included in the model.
- ? Future shifts in population could affect model's forward-looking view.

## Basic Conservatism Principle

- To compensate for simple model weakness, add a proportion of Model Standard Error (SE) to the model output.

$$\text{Conservative output} = \text{Model output} + (\text{SE} * \text{Confidence Factor})$$

- The Confidence Factor depends on
  - ▶ distribution of standard error
    - Normal distribution reasonable for large samples
  - ▶ appetite for conservatism, expressed in terms of statistical confidence level.
    - In some regulatory contexts (Low Default Portfolios) a 50% (2-sided) or 75% (1-sided) confidence level is acceptable

$$\text{Confidence Factor} = 0.674$$

## Model weakness – over-sampling example

- A simple average EAD model has been built on an over-sampled dataset
- 12 months of data for 1000 customers, assuming incorrectly that the 12000 records are independent.
- A quoted standard error of 0.32% (for example) should be replaced with a larger standard error to compensate for this duplication of information.

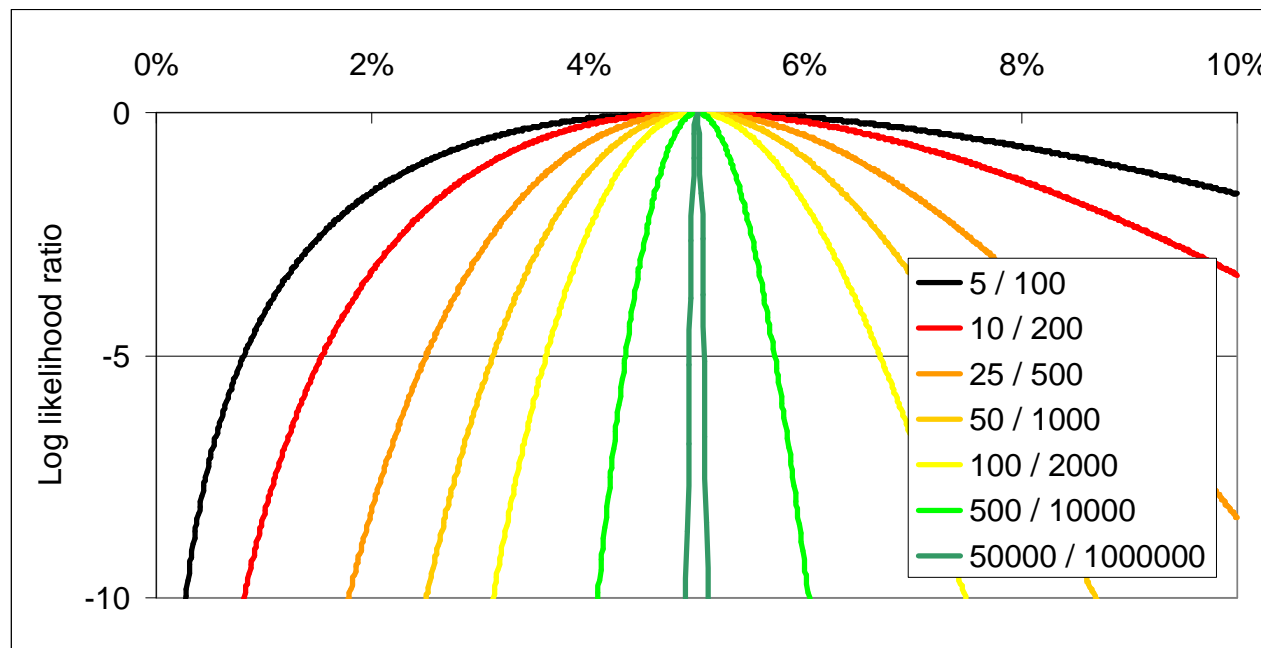
$$\text{Compensated Standard Error} = 0.32 * \text{SQRT}( 12000 / 1000 ) = 1.11\%$$

$$\text{Conservative add-on} = 0.674 * 1.11\% = 0.75\%$$

- Sample size drives standard error

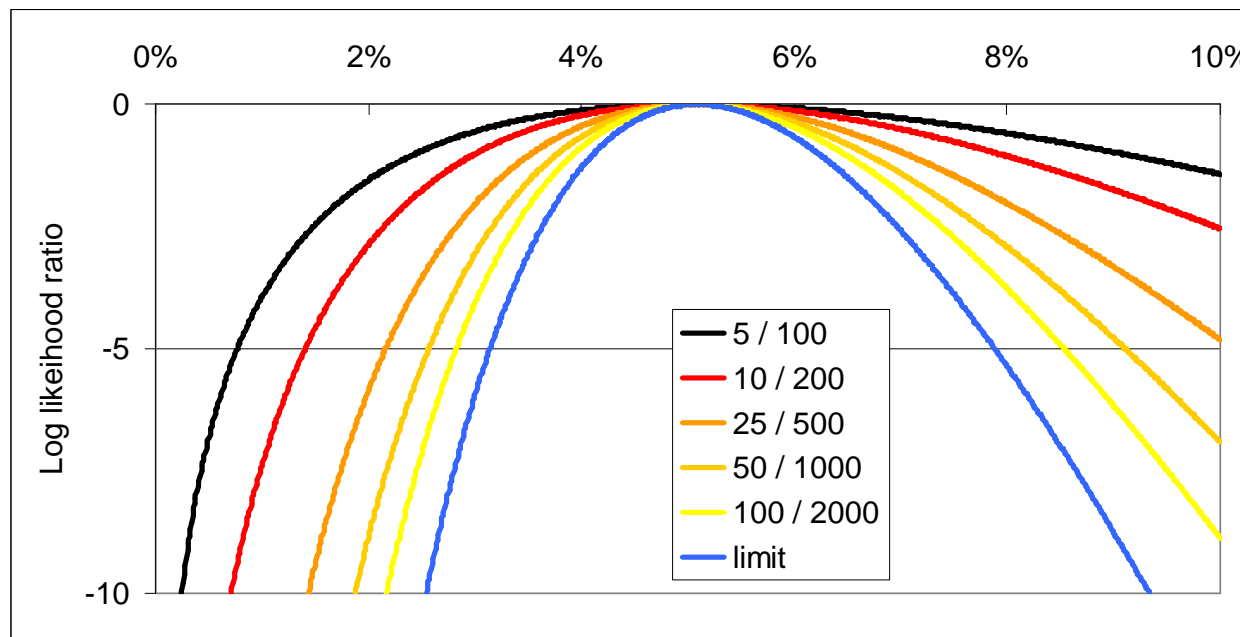
## How big is your sample?

- Assuming independence, the standard error of model output decreases to zero as sample size increases to infinity.
- Example PD=5%, with various sample sizes.
- Standard Error scales as the width of the log likelihood ratio curves.



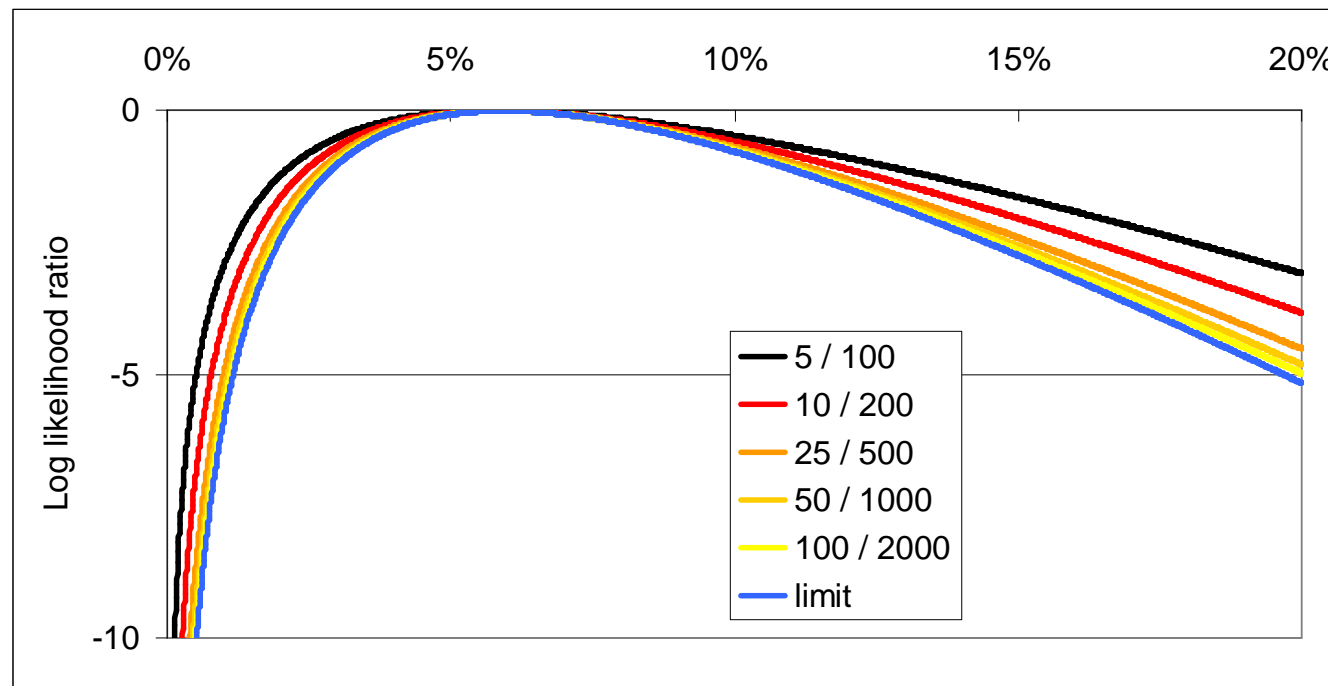
## How big is your sample?

- For portfolios with inter-obligor correlation, as sample size goes “to infinity” the marginal likelihood gets hung up on a limiting curve.
- Example PD=5% and asset correlation = 0.01.
- The limit looks like 36 defaults out of 700 independent obligors



## How big is your sample?

- Example PD=5% and asset correlation = 0.10
- The limit looks like 10 (?) independent obligors



## Correlation and Effective Sample Size

- Correlation reduces the free information that data can provide to the model.
- This can be expressed in terms of Effective Sample Size
  - ▶ the size of independent sample that would do the same job (i.e. produce similar inferences or shape of likelihood curve).
- A Design Effect approach is one way of quantifying this
  - ▶ Suppose, for each record  $a$ , the modelled quantity is  $X_a$ , and the correlations,  $corr(X_a, X_b)$ , are known or assumed.

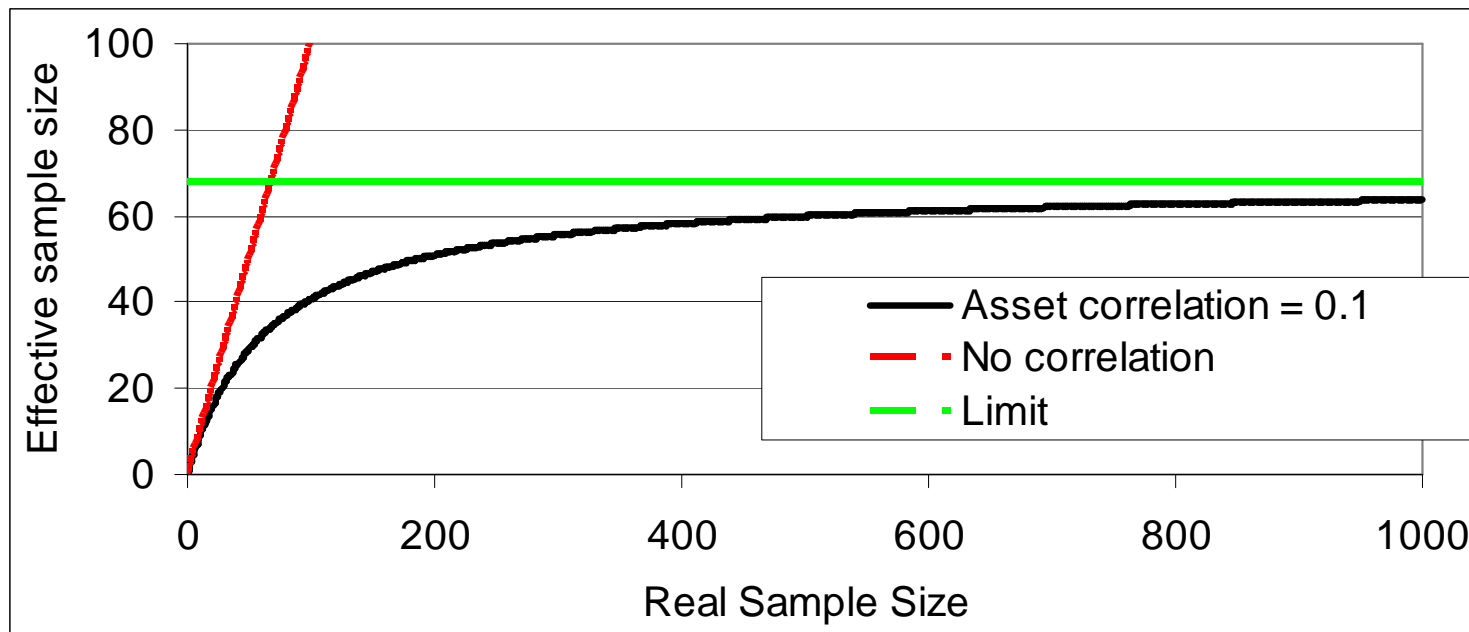
Effective Sample Size

$$= (\text{Number of records})^2 / (\text{Sum of } corr(X_a, X_b) \text{ over all pairs } a,b)$$

## Effective Sample Size – inter-obligor correlation

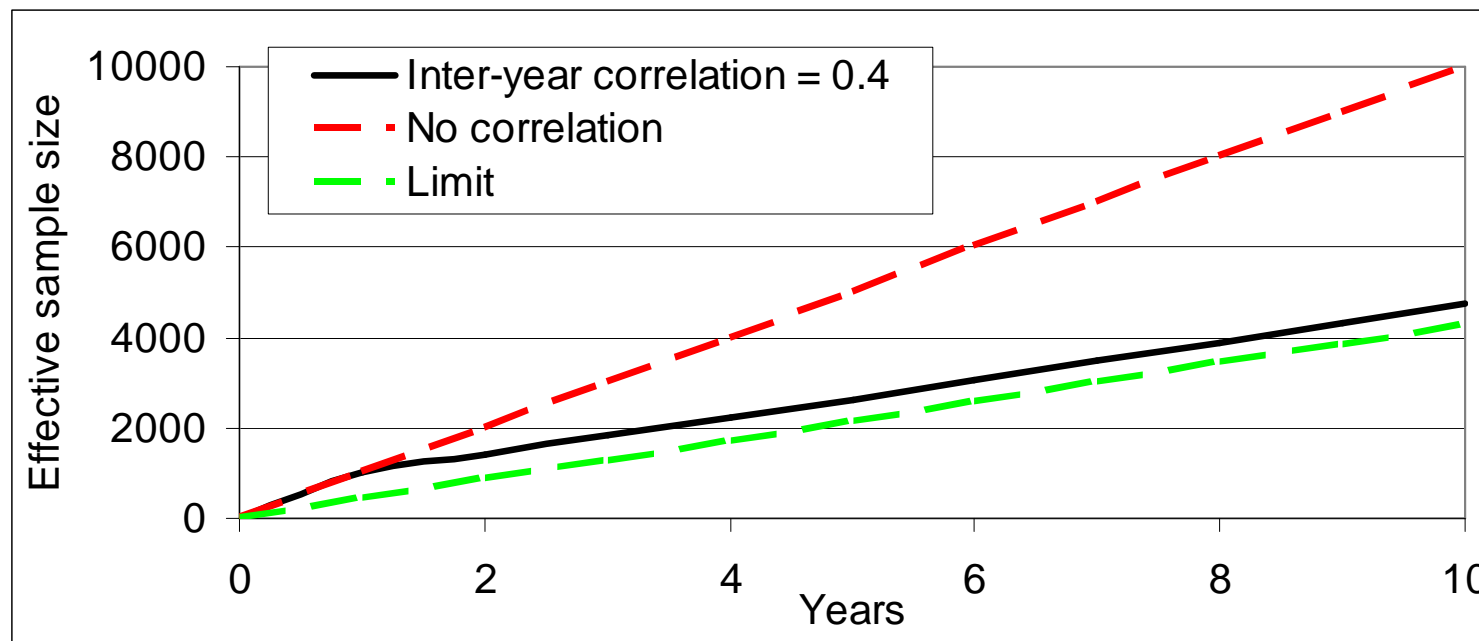
- Effective Sample Size can be estimated in practice.
- In a Vasicek Model of portfolio Default Rate, inter-obligor correlation caps the Effective Sample Size.

PD = 5%, asset correlation parameter = 0.10.



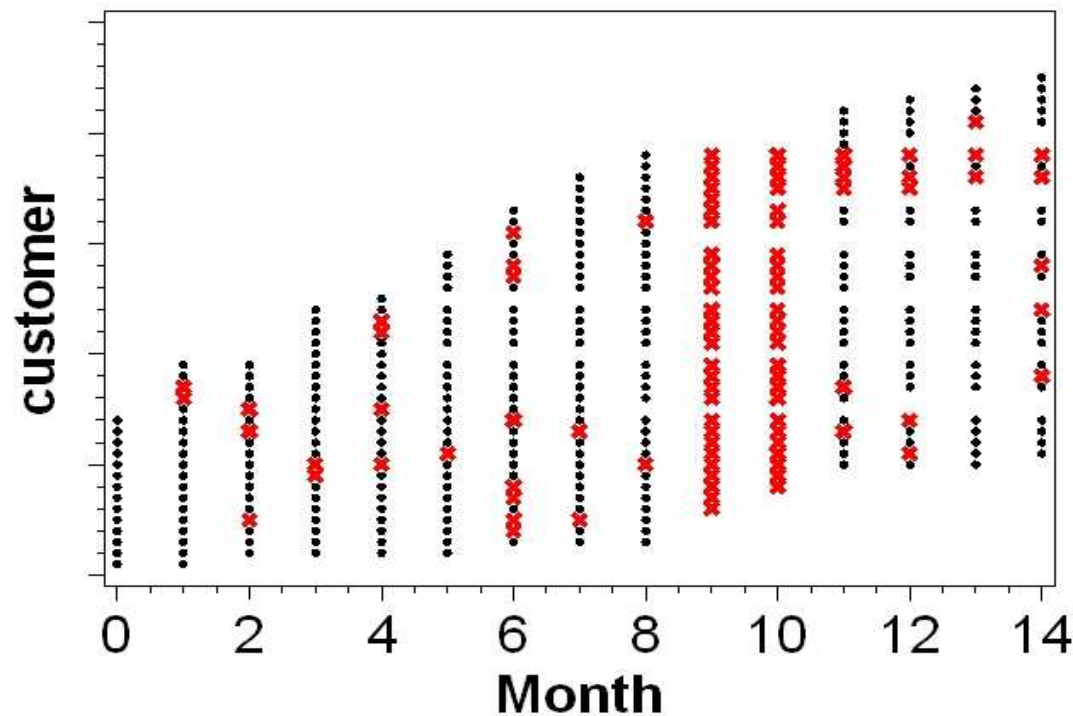
## Effective Sample Size – inter-year correlation

- Where records span a series of years, Effective Sample Size grows almost linearly, but correlation reduces the rate of growth.
- For example, in a portfolio with 1000 obligors over several years  
PD = 5% , inter-year correlation = 0.40



## Effective Sample Size in practice

- Example EAD model: 45 obligors, 14 months of data with some gaps X



Sample Size	275
<i>EAD</i>	101.22%
<i>SE</i>	0.37%

Correlations  
 Inter-obligor 0.02  
 Inter-month 0.90

Effective Sample Size	30.34
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## Effective Sample Size in practice

- Conservative adjustment assuming independence.

	●	● X
Raw Sample Size	275	371
Standard Error	0.37%	0.32%
Conservative add-on	+0.25%	+0.22%

- Use Effective Sample Size to compute the conservative adjustment under correlation assumptions.

	●	● X
Effective Sample Size	30.34	30.73
Standard Error	1.11%	1.10%
Conservative add-on	+0.75%	+0.74%

# How wrong is your model?

## Conclusion

- Conservatism is required in IRB modelling.
- Conservatism is challenging, scientific, useful, and can be built up from basic statistical principles
  - Start simply – use standard error.
  - Adjust for correlation – use Effective Sample Size.
- Conservatism needs a thorough understanding of model assumptions, data correlations and error structures.