

The Impact of Basel Accords on the Lender's Profitability under Different Pricing Decisions

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Abstract

In response to the deficiencies in financial regulation revealed by the global financial crisis, the Basel Committee on Banking Supervision (BCBS) is proposing to introduce a new capital regulatory standard to improve the banks' ability to absorb shocks arising from financial and economic stress. The regulatory capital requirements in the third of Basel Accords is conceptually similar to the mixture of Basel I (risk-invariant requirements) and Basel II (risk-based requirements), it introduce a non-risk based measure to supplement the risk-based minimum capital requirements and measures.

We look at how the interest rate charged to maximise a lender's profitability is affected by the different versions of the Basel Accord that have been implemented in the last 20 years. We investigate three types of pricing models on a portfolio of consumer loans, which are the fixed price model, a two price model and a variable risk based pricing model. We investigate the result under two different scenarios, Firstly where there is an agreed fixed price the lender has to pay to acquire capital in the market and secondly when the lender decides in advance how much if its equity capital can be used to cover the requirements of a particular loan portfolio. We develop an iterative algorithm for solving

these latter cases based on the solution approaches to the former. We also look at the sensitivity of the lending policy not only to the different Basel Accords but also to the riskiness of the portfolio and the costs of capital and loss given default values.

1 Introduction

Between 1988 and 2006, Basel regulations required that banks set aside a fixed percentage of equity capital to cover all their risks in lending (Basel 1). It required banks would set at a minimum regulatory capital equivalent to a uniform 8% of risk weighted assets. Kirstein (2001) pointed out this might result in adverse incentive influences. As the cost of regulatory capital is the same for high and low riskiness of borrowers, the bank is tempted to charge higher interest rates for more risky loans. The bank might like to replace low risk customers with high risk customers in order to maximize profit. Thus the Basel committee had approved a reform, known as The Basel 2 capital requirement, it provides for a greater sensitivity of capital requirements to the credit risk inherent in bank loan portfolio. The Basel 2's proposal requires the bank to set different regulatory capital ratios for borrowers with different default risks. Beginning of the year 2010, a new global regulatory standard on bank capital regulation was introduced by the Basel Committee. The third of the Basel Accords (Basel 3) was developed in a response to the deficiencies in financial regulation revealed by the global financial crisis. The regulatory capital required by Basel 3 which is same to that for the Basel 2 is also the risk-based requirement, but it sets out higher and better quality capital, better risk coverage. Basel 3 requires banks to hold 4.5% of common equity and 6% of Tier I capital of risk-weighted assets. The total capital requirement is 8.0% that is the same to requirement of Basel 2 on Tier 1 Capital, and the difference between the total capital requirement and the Tier 1 requirement can be met

with Tier 2 capital. It also introduces a mandatory capital conservation buffer of 2.5%. The purpose of the conservation buffer is to ensure that banks maintain a buffer of capital that can be used to withstand future periods of financial and economic stress. Moreover, Banks will be required a discretionary countercyclical buffer, which allows regulators to impose up to another 2.5% of capital during periods of high credit growth. Thus Basel 3 seems like to be an “enhanced version” of Basel 2 to some extent. The following Table shows major different between Basel 2 and 3.

Basel 2	Basel3
a. Tier 1 Capital	
Tier 1 capital ratio = 4% Core Tier 1 capital ratio = 2%	Tier 1 Capital Ratio = 6% Core Tier 1 Capital Ratio (Common Equity after deductions) = 4.5%
The total capital requirement is 8.0%.	The total capital requirement is 8.0%.
b. Capital Conservation Buffer	
There no capital conservation buffer is required.	Banks will be required to hold a capital conservation buffer of 2.5% to absorb losses during periods of financial and economic stress.
c. Countercyclical Capital Buffer	
There no Countercyclical Capital Buffer is required.	A countercyclical buffer within a range of 0% – 2.5% of common equity or other fully loss absorbing capital will be implemented.

Putting these together means that Total Regulatory Capital Ratio = [The total capital requirement ratio] + [Capital Conservation Buffer] + [Countercyclical Capital Buffer], hence Banks could be setting aside 13% of Risk Weighted Assets.

The Basal Accords require the banks to set aside regulatory capital to cover unexpected losses on a loan. If the chance of the loan defaulting l_D is Loss Given Default (LGD), the fraction of the defaulted amount that is actually lost. The minimum capital requirement (MCR) per unit of loan with a probability of being good is defined as $l_D \cdot K(p)$. This is given under four different MCRs:

. Basel 0: Describes the situation pre-1998 when there were no regulatory capital requirements so

$$K(p) = K_0 = 0$$

. Basel 1: Describes the MCR under the first Basel Accord where

$$K(p) = K_1 = \frac{0.08}{l_D}$$

. Basel 2: Describes the MCR under the second Basel Accord where

$$l_D \cdot K_2(p) = l_D \cdot N \cdot \left[\left(\frac{1}{1-R} \right)^{1/2} \cdot N^{-1} \cdot (1-p) + \left(\frac{R}{1-R} \right)^{1/2} \cdot N^{-1} \cdot \right.$$

$0.999 - l_D \cdot 1 - p$, where $R=0.04$ (credit cards)

where $N(\cdot)$ is the cumulative normal distribution and $N^{-1}(\cdot)$ is the inverse cumulative normal distribution.

.Basel 3: the MCR for the third of Basel Accord can be written as:

$$K_3(p) =$$

$$\left\{ \frac{[(\text{The total capital requirement}) + (\text{Capital Conservation Buffer}) + (\text{Countercyclical Capital Buffer})]}{\text{The total capital requirement for Basel2}} \right\} \cdot K_2(p)$$

$$\begin{aligned}
&= \left(\frac{8\% + 2.5\% + 2.5\%}{8\%} \right) \cdot K_2(p) \\
&= \left(\frac{13}{8} \right) \cdot K_2(p) \\
&= 1.625 \cdot K_2(p)
\end{aligned}$$

Kashyap and Stein(2003) pointed out that there are many potential benefits to risk-based capital requirements, as compared to the “one-size-fits-all” approach embodied in the Basel 1 regulation. The objective of this paper is to understand the influence of different Basel regulatory capital requirements (Basel 1, Basel 2 and Basel 3) on the lender’s profitability under different pricing decisions (such as fixed (one) price model, two prices model, and variable pricing model) in portfolio level, which are expressed in terms of the tradeoffs between optimal interest rate, default risk, maximizing expected profits.

Fixed-rate has been the dominate form of pricing of loans until the early 1990s. More recently the development of the internet and the telephone as new channels for loan applications has made the offer process more private to each individual (Thomas 2009), and developments in credit scoring and response modelling have assisted the banks in marketing their products more efficiently, and in increasing the size of their portfolios of borrowers (Chakravoriti and To 2006). The banks are able to “price” their loan products at different interest rates by adopting methods such as channel pricing, group pricing, regional pricing, and product versioning. Variable pricing, therefore, can improve the profitability of the lender by individual bargaining and negotiation. Since the lender can segment the applicants depending on their default risk and offer different loan terms to each segments, so the simplest variable pricing model is actually two prices model where we assume there are only two types of borrowers which differ in their ability to repay a loan (default risk).

Allen DeLong and Saunders (2004) outline the Basel accord and credit scoring,

and they observe how corporate credit models are modified to deal with small business lending. Ruthenberg and Landskroner (2008) analyze and estimate the possible effects of Basel 2 regulation on the pricing of bank loans related to the two approaches for capital requirements (internal and standardized). They indicate that big banks might attract good quality firms since the reduction in interest rates produced by adopting the IRB approach. On the other hand lower quality firms will be benefit by borrowing from small banks, which are more likely to adopt the standardized approach. Perli and Najda (2004) suggest an alternative approach to the Basel capital allocation. They offer a model for the profitability of individual borrower revolving loan they use it to imply that the regulatory capital should be some percentile of the profitability distribution of the loan, but there is no reference to the effect on operating decision. Olive and Thomas (2005) analyze the changes in the operating decision of which borrowers for loans to accept and which to reject because of the effects of different Basel regulations imposed on the retail bank in portfolio level with predetermined capital. Based on the achievements from the paper of Olive and Thomas, our paper will analyze in depth how the impact of different Basel regulations has on different pricing decisions in portfolio level with or without the condition where we assume the bank decides in advance how much of its equity capital can be set aside to cover the requirements of a loan portfolio.

This paper is organised as follows. Section 2 looks at the profitability model of different pricing decisions (fixed price, two prices and variable pricing). Section 3 use several numerical examples to discuss what the impact of Basel Accords will have on different pricing models associated with a portfolio of such loans that wants to maximise expected profit and optimal interest rate. Section 4 extends the models by assuming the bank decides in advance how much of its equity capital can be set aside to cover the required regulatory capital of a loan portfolio. Section 5 draws some conclusions from this work.

2 Pricing Models in portfolio level

Before we consider the pricing models in portfolio level it may be useful to review the model that applies to an individual loan. Consider a loan of one unit offered by a lender to a borrower with a probability of being good of p . If the rate charged on the loan is r and if the loan defaults, so the loss given default on any loan is l_D . Let r_Q be the required return on equity capital, the risk free rate at which the lender can borrow the money is r_F . Let B denotes as sum of cost of regulatory capital $r_Q l_D K(p)$ and risk free rate r_F . Under the situation where the interest rate is r and the take probability is $q(r, p)$. Hence the expected profit from an individual loan is

$$E(\text{Profit}) = q(r, p) \cdot [(r - B) \cdot p - (l_D + B) \cdot (1 - p)] \quad \text{Eq.1}$$

where $B = r_Q l_D K(p) + r_F$.

2.1 Fixed (one) price model in portfolio level

For fixed price model, we assume the lender only offer one interest rate r to all potential borrowers. We suppose response rate function is the linear function which will be defined as:

$$q(r, p) = \min\{\max[0, 1 - b \cdot (r - r_L) + c \cdot (1 - p), 1]\}, \text{ for } r \geq r_L > 0.$$

Eq. 2

This means the borrower's response rate is dependent of the probability of the default risk. We assume $r_L = 0.04$, $b = 2.5$, and $c = 2$. This implies that an 1% increase on interest rate drops the take probability by 0.025 while if the default probability of the borrower goes up by 0.01 (Good drops by this amount) the take probability goes up by 0.02. For borrower with a default rate of 0.01, thus 100% of them would take a loan of rate 6%, while only 50% of them would

take a loan with interest rate 25%. The lender are tempted to accept the loans they have positive expected profit, but reject all the loans that is to be unprofitable since profit is main objective concerned by the lender. This gives us the definition of a cut-off probability (or cut off point p_c) which is the probability if the borrowers being good at which the expected profit is zero. Thus the lender should accept all the customers with probability of being good above the cut off point $p_c = \frac{l_D+B}{r+l_D}$, and reject all the applications with probability of being good p_c or lower. Meanwhile, we assume the probability of the borrowers being good has a uniform distribution. The probability density function is $f(p) = \frac{1}{1-a}, a \leq p \leq 1$, so it means no one with probability of being good less than a is in the potential borrowers' population. Therefore, the cut off point for a portfolio actually is $\max(a, p_c)$.

If we define $EP(r, p)$ to be the expected profit from a portfolio with cut off point $\max(a, p_c)$, we find

$$EP(r, p) = \text{Max}_{r, p_c} \int_{\max(a, p_c)}^1 q(r, p) \cdot [(r - B) \cdot p - (l_D + B) \cdot (1 - p)] \cdot f(p) \cdot dp$$

Eq. 3

where $B = r_Q l_D K(p) + r_F$.

2.2 Two Prices Model in portfolio level

We assume the lender has two different rates that can be offered to potential borrowers. We also consider the situation where the probability of the borrowers being good has a uniform distribution. Suppose the rates are r_1 and r_2 , $r_1 < r_2$, the lender's strategy is given by two values (segment point p^* and cut off point $\max(a, p_c)$). Rate r_1 is made to the borrowers whose probability p of being good is $p \geq p^*$; rate r_2 is made to those who probability of being good is p ,

$$p^* > p \geq \max(a, p_c).$$

There a difference in two prices model is the optimal cut off point related to inferior rate r_2 , so $p_c = \frac{l_D+B}{r_2+l_D}$. In this case $\max(a, p_c)$ is ‘first round of screening’ to determine the range of potential borrowers’ population.

We believe the expected profit of the lender with inferior rate r_2 is always equal to or less than the expected profit of the lender with superior rate r_1 . So the probability of being good p^* , at which the lender start to offer r_1 rather than r_2 , can be achieved from inequality follows as

$$q(r_1) \cdot [(r_1 - B) \cdot p - (l_D + B) \cdot (1 - p)] \geq q(r_2) \cdot [(r_2 - B) \cdot p - (l_D + B) \cdot (1 - p)]$$

Eq. 4

which results in $p^* =$

$$\frac{\left[1 + \frac{1}{c} - \frac{b}{c} \cdot l_D - \frac{b}{c} \cdot r_1 - \frac{b}{c} \cdot r_2 + \frac{b}{c} \cdot r_L\right] + \sqrt{\left(1 + \frac{1}{c} - \frac{b}{c} \cdot l_D - \frac{b}{c} \cdot r_1 - \frac{b}{c} \cdot r_2 + \frac{b}{c} \cdot r_L\right)^2 + 4 \cdot \frac{b}{c} \cdot (l_D + B)}}{2}$$

Eq. 5

where p^* is denoted as the segmentation point setting to divide those who passed the lender’s ‘first round of screening’ into two groups which will be offered with different interest rates.

The choice between lender and borrowers is a typical interactive decision-making. The lender, therefore, has to consider whether the rate offered is really attractive to the borrowers, this lead to two constraints follow as :

$$q(r_1) = 1 - b \cdot (r_1 - r_L) + c \cdot (1 - p) > 0, \text{ which results into } p_1 = 1 + \frac{1}{c} - \frac{b}{c} \cdot (r_1 - r_L)$$

and

$$q(r_2) = 1 - b \cdot (r_2 - r_L) + c \cdot (1 - p) > 0, \text{ which leads to } p_2 = 1 + \frac{1}{c} - \frac{b}{c} \cdot (r_2 - r_L)$$

So if one offers r_1 only the borrowers who probability of being good is below p_1 will accept it, and only those whose probability of being good is below p_2 will accept r_2 .

Hence the expected value of lender’s total profit is showed by following equation,

$$EP(r, p) = \underset{r_1, r_2, p^*, p_c}{Max} \left\{ \int_{\max(a, p_c)}^{\min(p^*, p_2)} q(r_2, p) \cdot [(r_2 - B) \cdot p - (l_D + B)] \cdot f(p) \cdot dp \right. \\ \left. + \int_{p^*}^{\min(1, p_1)} q(r_1, p) \cdot [(r_1 - B) \cdot p - (l_D + B)] \cdot f(p) \cdot dp \right\}$$

Eq.6

where $B = r_Q l_D K(p) + r_F$.

2.3 Variable Pricing Model in portfolio level

Variable pricing (risk based pricing) means that the interest rate charged on a loan to a potential borrower depends on the lender's view of the borrower's default risk.

If the lender believes the borrower has a probability p of being Good, then the lender believes the expected profit if a rate $r(p)$ is charged to be

$$EP(r, p) = \underset{r, p}{Max} \int_a^1 q(r, p) \cdot [(r - B) \cdot p - (l_D + B) \cdot (1 - p)] \cdot f(p) \cdot dp$$

Eq.7

where $B = r_Q l_D K(p) + r_F$.

In order to find the optimal interest rate for a certain probability p of being Good, we differentiate this equation with respect to r and set the derivate to zero, to find when the profit is optimised. This gives a risk based interest rate of

$$r^*(p) = r_F + (l_D + B) \frac{1-p}{p} - \frac{q(r, p)}{\frac{\partial q(r, p)}{\partial r}} \quad \text{Eq.8}$$

Much of calculations of risk based interest rate can be found in the book by Thomas (2009).

3 What the impact of Basel Accords will have on different pricing models in portfolio level

We use some numerical examples under different Basel Accords by comparing various business measures such as the expected profit of lender, optimal interest rate and optimal cut off to disclose how Basel Accords effect on different pricing models.

3.1 Example for one price model

Consider the situation where $r_Q = 0.05$, $r_F = 0.05$, and $l_D = 0.5$. We can achieve the ratio of regulatory capital for each Basel rule respectively.

Basel 0: since $K(p) = K_0 = 0$, we have $= r_Q l_D K(p) + r_F = 0.05$.

Basel 1: $K(p) = K_1 = \frac{0.08}{l_D} = 0.16$, hence we have $= r_Q l_D K(p) + r_F = 0.054$.

Basel 2: since $K_2(p) = N \cdot \left[\left(\frac{1}{1-R} \right)^{1/2} \cdot N^{-1} \cdot (1-p) + \left(\frac{R}{1-R} \right)^{1/2} \cdot N^{-1} \cdot 0.999 - 1 - p \right]$, where $R=0.04$ (credit cards), then we have $B=r_Q l_D K_2 p + 0.05$.

Basel 3: due to we believe $K_3(p) = 1.625 \cdot K_2(p)$, we get $B = 1.625 \cdot r_Q l_D K_2(p) + 0.05$

The optimal interest rates and the expected profits achieved under different Basel Accords are shown in Table 1

a	r_{B0}	E(P) B0	r_{B1}	E(P) B1	r_{B2}	K(2)	E(P) B2	r_{B3}	K(3)	E(P) B3
0.6	0.376 8	0.0618	0.3778	0.0599	0.3782	0.2447	0.0589	0.3933	0.3983	0.0571
0.7	0.35	0.0778	0.3523	0.0757	0.3535	0.2380	0.0747	0.3556	0.3868	0.0728
0.8	0.314 0	0.0886	0.3162	0.0865	0.3169	0.2097	0.0859	0.3188	0.3408	0.0842
0.9	0.279 1	0.0942	0.2812	0.0922	0.2810	0.1491	0.0923	0.2823	0.2423	0.0911

We can see from Table 1 that optimal interest rates for Basel 1, Basel 2 and Basel 3 are always bigger than optimal interest rates achieved under Basel 0, this is because there are the costs of regulatory capitals needed to be covered. The optimal interest rates to charge drop but the expected portfolio profits rise up as the borrowers become less risky. The expected portfolio profits for all Basel regulations are less than that of made with no regulatory capital requirements case. Comparing column 4 and column 7, we found that the expected portfolio profits under Basel 2 is smaller than that achieved under Basel 1 until the probability of the borrowers being good is greater than 0.9. This is because Basel 2 regulation requires the lender to set flexible regulatory capital ratios for different risk types, the equity capital request decreases with

increasing of the quality of the borrowers. We can see that in column 10 which shows the regulatory capital ratios of Basel 2 are larger than regulatory capital ratio of Basel 1 (which is fixed at 0.16) except for the minimum good rate a reaches 0.9. The expected portfolio profits for Basel 3 is always smaller than that for Basel 1 and 2 capital requirements since Basel 3 regulation set much tighter capital restriction on equity capital than Basel 1 and 2. This can be seen from regulatory capital ratios of Basel 3 showed in column 9 where the ratios of regulation are extremely high by comparing with that for Basel 1 and 2.

3.2 Example for two prices model

In this example we take the same borrowing rates and the LGD as in Example 2.1, namely, $r_Q = 0.05$, $r_F = 0.05$, and $l_D = 0.5$. Meanwhile, assume the response rate function is linear with $r_L = 0.04$, $b = 2.5$, and $c = 2$.

Basel 0: no requirement for regulatory capital

a	r_2	$\max(a, P_c)$	r_1	p^*	Expected Profit
0.6	0.485568	0.6	0.317348	0.790765	0.07506936
0.7	0.415754	0.7	0.297977	0.845737	0.08508152
0.8	0.353758	0.8	0.279661	0.898441	0.09168503
0.9	0.297323	0.9	0.262075	0.94968	0.09493876

When the value of minimum good rate a increase that means the borrowers become less risky, optimal cut off point and segment point increase and the expected portfolio profits increase as well, but the optimal interest rates (r_1, r_2) to charge decrease.

$$\text{Basel 1: } K_1(p) = \frac{0.08}{l_D} = 0.16$$

a	r_2	$\max(a, P_c)$	r_1	p^*	Expected Profit
0.6	0.488475	0.6	0.319604	0.790703	0.07305781
0.7	0.418356	0.7	0.300156	0.845704	0.08304529
0.8	0.356118	0.8	0.281772	0.898428	0.0896575
0.9	0.299488	0.9	0.264128	0.949676	0.09294516

There is a small increase in the optimal interest rates being charged by comparing with the solutions for Basel 0 shown in Table 2, this is because there are not the costs of regulatory capitals needed to be considered in no regulatory requirement case. In two prices model the segment point p^* for Basel 1 is smaller than for Basel 0. We can see from Table 3 the expected portfolio profits for Basel 1 are smaller than that for Base 0.

Basel 2:

The Basel Accord 2's regulatory capital ratio is flexible, so we have to set two regulatory capital ratios. However, it is difficult to integrate the equation of $l_D \cdot K_2(p) = l_D \cdot N \cdot \left[\left(\frac{1}{1-R} \right)^{1/2} \cdot N^{-1} \cdot (1-p) + \left(\frac{R}{1-R} \right)^{1/2} \cdot N^{-1} \cdot (0.999) \right] - l_D \cdot (1-p)$, we will conservatively set the probability of the borrowers being good p in equation of $l_D \cdot K_2(p)$ to equal with p_c and p^* . The regulatory capital ratio for risky borrowers $K(p_c)$ is always larger than the ratio requested for good borrowers $K(p^*)$. We also built the model as assuming the probability of the borrowers being good in two regulatory capital ratios' equations respectively to be average value between $\text{Max}(p_c, a)$ and p^* (for risky group), and between p^* and p_1 (for higher quality group). We found there is no significant difference by comparing with the results produced from the case if we conservatively use p_c and p^* in equation of $l_D \cdot K_2(p)$.

a	r_2	$\max(a, P_c)$	r_1	p^*	Expected Profit
0.6	0.488913	0.6	0.31957	0.791461	0.072208
0.7	0.418609	0.7	0.299716	0.846327	0.082389
0.8	0.355927	0.8	0.280833	0.899	0.089405
0.9	0.298533	0.9	0.262549	0.950295	0.093397

It can be seen from Table 4 interest rate r_1 being charged on good quality borrowers under Basel 2 is always smaller than that for Basel 1, but inferior rate r_2 being charged on risky borrowers is higher than that for Basel 1 once the least good probability a the borrowers have reaches 0.8. So in this situation where the value of a is no less than 0.8 the inferior rate r_2 obtained under Basel 2 is between than for Basel 0 and Basel 1. It is Basel 1 that restricts the regulatory capital most with less risky borrowers. In this case the Basel 2 segment point is higher than the Basel 1 segment point. This is different with our expectation which the Basel 2 segment probability is higher than Basel 0 but below the Basel 1. It might be caused by setting cut off point for a portfolio equal to $\max(a, p_c)$, so the cut off point is actually fixed at a through using the Solver function in Excel. The expected portfolio profits for Basel 2 are smaller than that for Base 0. Moreover, the expected portfolio profits for Basel 2 are greater than that achieved under Basel 1 once the probability of the borrowers being good reaches 0.9. This is because Basel 2 regulatory capital requirements are flexible, the costs of equity capital request decreases with increasing of the quality of the borrowers.

Basel 3: $K_3(p) = 1.625 \cdot K_2(p)$

a	r_2	$\max(a, P_c)$	r_1	p^*	Expected Profit
0.6	0.491035	0.6	0.320974	0.79186	0.070447

0.7	0.420426	0.7	0.30082	0.846656	0.080727
0.8	0.357323	0.8	0.281591	0.899298	0.087995
0.9	0.299371	0.9	0.262907	0.950565	0.092441

The optimal interest rates and expected portfolio profits for Basel 3 is given in Table 5 where we can see the expected portfolio profits for Basel 3 is always smaller than that for Basel 1 and 2 as a result of Basel 3 regulation setting much tighter capital restriction than Basel 1 and 2. The segment probability is higher than that for Basel 2 and Basel 1. Both of optimal superior and inferior rates (r_1 and r_2) are always higher than that for Basel 2. Moreover, the inferior rate r_2 charged for Basel 3 is higher than that for Basel 1 when the value of a is no less than 0.9, and the superior rate obtained under Basel 3 is also smaller than that for Basel 1 once the value of a reaches 0.8.

3.3 Example for variable pricing model

We assume there is not any impact of adverse selection on this variable pricing model. Using the cost structure of the pervious example with $r_Q = 0.05$, $r_F = 0.05$ and $l_D = 0.5$, assume the parameters for the linear response rate function are $r_L = 0.04$, $b = 2.5$, and $c = 2$.

A. The optimal interest rates offered by lender under Basel Accords are shown in Table 6

p	$r(p)$ B0	$r(p)$ B1	$r(p)$ B2	$r(p)$ B3
0.35	1.015714	1.021428	1.022753	1.027153
0.4	0.8975	0.9025	0.904159	0.908321
0.5	0.72	0.724	0.725898	0.729584
0.6	0.588333	0.591667	0.59344	0.596632
0.7	0.482857	0.485714	0.487108	0.489765

0.8	0.39375	0.39625	0.397027	0.399076
0.9	0.315556	0.317777	0.317626	0.318922
0.95	0.279474	0.281579	0.280755	0.281555
0.96	0.272458	0.274542	0.27355	0.274233
0.97	0.265506	0.267567	0.266391	0.266945
0.98	0.258612	0.260654	0.259268	0.259678
0.99	0.251778	0.253798	0.252164	0.252406

The optimal interest rates to be charged decrease as the probability of the borrowers being good increase under all Basel regulation rules. Since there is no requirement for regulatory capital for Basel 0, the optimal interest rates for B0 are always less than those that for Basel 1, 2 and 3 regulations. We can see from Table 6 that the interest rates for Basel 2 are larger than that of made for Basel 1 until the probability of the borrowers being good is over 0.9, since Basel 1 requires the lender set a fixed equity ratio for loans to all risk types. The optimal interest rates for Basel 3 has similar trend to that of made for Basel 2, in which the optimal interest rates eventually become smaller than that for Basel 1 as the probability of the borrowers being good is good enough. However, there is the tighter capital restriction in Basel 3, so the optimal interest rates become smaller than that for Basel 1 only if the probability of the borrower being good rises above 0.95.

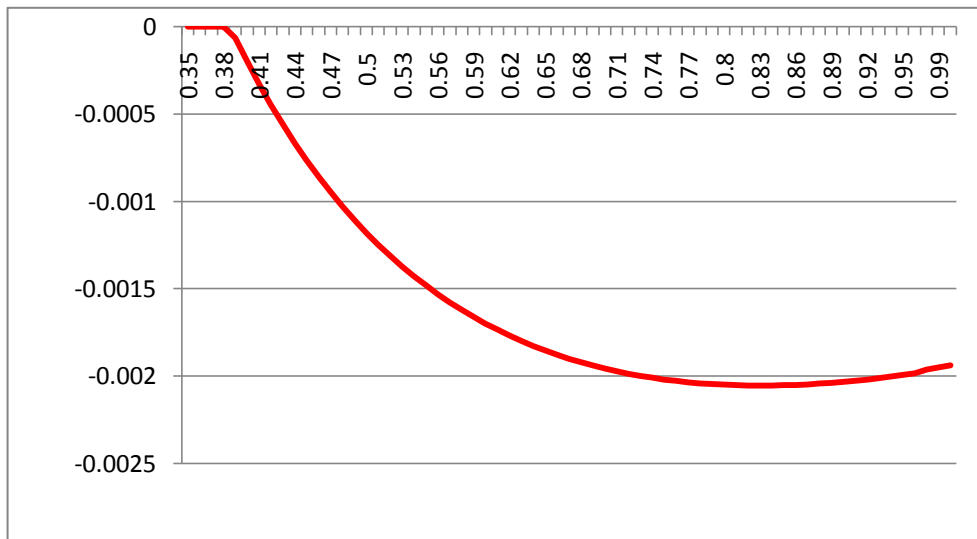
B. Expect portfolio profits for Basel Accords is given Table 7

p	$E(P)$ B0	$E(P)$ B1	$E(P)$ B2	$E(P)$ B3
0.35	0	0	0	0
0.4	0.000506	0.000306	0.00025093	0.000136
0.5	0.018	0.01682	0.016274144	0.01524
0.6	0.044204	0.042504	0.04161319	0.040034
0.7	0.068014	0.066057	0.06511253	0.063331
0.8	0.085078	0.083028	0.082395783	0.080741
0.9	0.094044	0.092011	0.092148375	0.090973

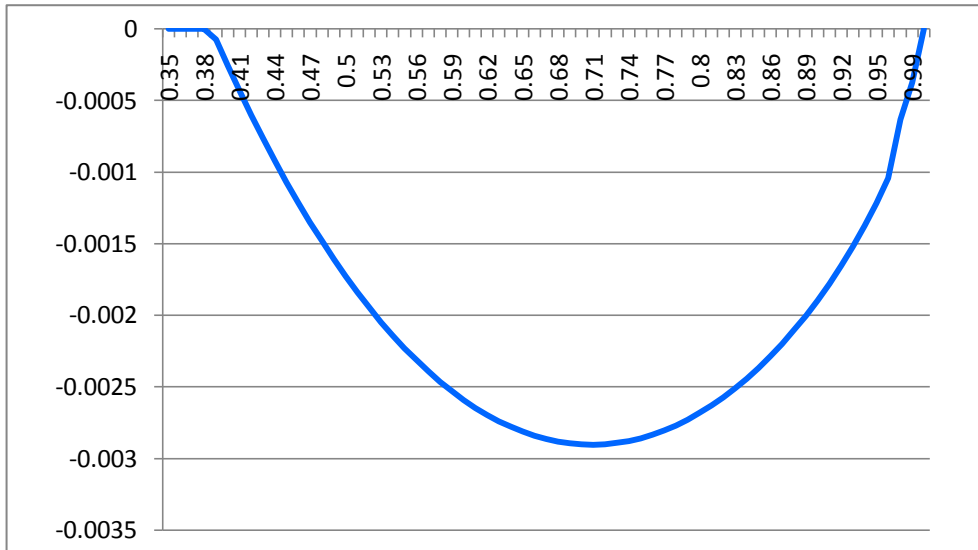
0.95	0.095501	0.093506	0.094284804	0.093529
0.96	0.095561	0.093576	0.094517747	0.093869
0.97	0.095545	0.093571	0.094694659	0.094165
0.98	0.095457	0.093493	0.094823436	0.094429
0.99	0.095295	0.093343	0.094920163	0.094686

From the Table 7 one sees that the expected profits for all Basel Accords increase as the borrowers become less risky. Though the regulatory capital rules in Basel 2 and Basel 3 depend on probability of the borrowers being good, the expected portfolio profits for both of two regulations will become to be larger than that for Basel 1 when probability of the borrowers being good is very high. This is more obvious with comparing the difference of profitability between each Basel regulations and Basel 0.

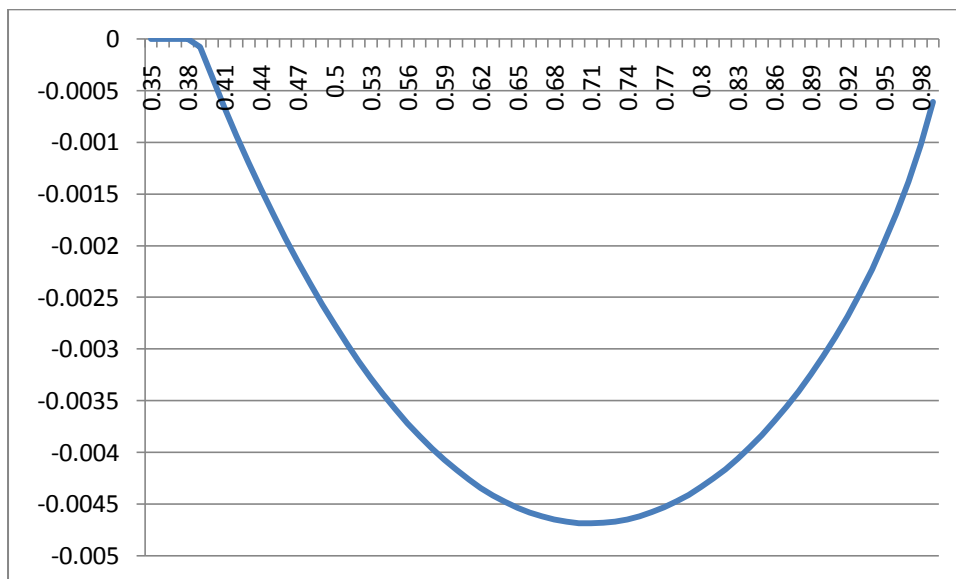
a. Figure 1: Difference in profitability between Basel 1 and Basel 0



b. Figure 2: Difference in profitability between Basel 2 and Basel 0



c. Figure 3: Difference in profitability between Basel 3 and Basel 0



We have above three figures by using expected profits for Basel 1, Basel 2 and Basel 3 minus profit achieved with Basel 0. We can see that profit made with Basel 0 is always not less than profit made with Basel 1, 2 and 3 regulations. And the difference between Basel 2 and Basel 0 will reach the largest value when portability of the applicant being good equals to 0.71, then the value of the difference will start to drop down until the difference of profitability between Basel 2 and Basel 0 becomes to zero. It means there is no any regulatory capital required when the probability of the borrowers being good

equals to 1 due to Basel 2 requires flexible amounts of equity set aside for loan to various risk types of borrowers. By comparing between Figure 2 and Figure 3, the same trend is evident in the difference of profitability between Basel 3 and Basel 0. Comparing with there is a dramatic change in Figure 2 and Figure 3, for Basel 1 the curve becomes flatter along with the probability of the applicant being good increase. This is because Basel 1 requires a fixed amount of equity set aside (0.08) to all risk types applicants (even the probability of being good equals to 1).

4 How Basel Accords affect different pricing models in portfolio level with predetermined equity capital

The previous model looked at what impact the Basel Accord requirements for regulatory capital have on the performances of lender's different pricing decisions assuming a known required rate of ROE at the portfolio level. A more realistic model is to consider the problem in portfolio level model with predetermined equity capital which means there is no known acceptable ROE. Instead we impose that the lender decides in advance how much of its equity capital can be used to cover the minimum capital requirements of a particular loan portfolio.

Initially we assume that the funds for portfolio that is only raised by borrowing from the market at the risk free rate r_F and loss given default on any loan is l_D irrespective of the rate charged. With ignoring the regulatory capital and the cost of funding some of the borrowing from equity, the profit from a loan is modified from Eq.1 to

$$MaxE(Profit) = q(r, p) \cdot [(r - r_F) \cdot p - (l_D + r_F) \cdot (1 - p)] \quad Eq. 9$$

where linear take probability $q(r, p)$ is

$$\min\{\max[0, 1 - b \cdot (r - r_L) + c \cdot (1 - p), 1]\}.$$

We assume the probability of these portfolio loans being good also has a uniform distribution. If we define $EP(r, p)$ to be the expected profit from a portfolio with cut-off probability p_c , we find

$$EP(r, p) = \text{Max}_{r, p_c} \int_{\max(a, p_c)}^1 q(r, p) \cdot [(r - r_F) \cdot p - (l_D + r_F) \cdot (1 - p)] \cdot f(p) \cdot dp \quad \text{Eq.10}$$

The regulatory capital set aside to cover such a portfolio, in predetermined equity capital case, cannot exceed the equity capital Q set aside for this portfolio lending.

Thus, given the limit on the equity capital provided, we need to solve the following constrained optimization problem in order to maximize the expected profit from the portfolio.

$$EP(r, p) = \text{Max}_{r, p_c} \int_{\max(a, p_c)}^1 q(r, p) \cdot [(r - r_F) \cdot p - (l_D + r_F) \cdot (1 - p)] \cdot f(p) \cdot dp \quad \text{Eq. 11}$$

subject to

$$\int_{\max(a, p_c)}^1 l_D \cdot k(p) \cdot f(p) \cdot q(r, p) \cdot dp \leq Q \quad \text{Eq. 12}$$

This is equivalent (Thomas 2009) to solving the cut-off point while the equity capital Q set aside is given a certain predetermined amount, and then we can obtain the maximize expected profit by substituting cut off probability into Eq. 11 .

Consider the probability of these portfolio loans being good also has a uniform distribution, and response probability of the borrowers should always larger

than zero ($1 - b \cdot (r - r_L) + c \cdot (1 - p) \geq 0$). Thus Eq.12 can be rewritten into

$$\frac{l_D}{1-a} \int_{\max(p_c, a)}^{\min(1, \frac{1-b(r-r_L)}{c}+1)} K(p) \cdot q(r, p) \cdot dp \leq Q \quad \text{Eq. 13}$$

$$\text{let } I[r, \max(p, a)] = \int_{\max(p_c, a)}^{\min(1, \frac{1-b(r-r_L)}{c}+1)} K(p) \cdot q(r, p) \cdot dp$$

$$\text{thus we have } \frac{l_D}{1-a} \cdot I[r, \max(p, a)] \leq Q \quad \text{Eq. 14}$$

It is difficult to solve above integral function under Basel 2 and Basel 3 regulatory capital requirements since the minimum capital requirement $K(p)$ in both Basel rules include this following function $N \cdot \left[\left(\frac{1}{1-R} \right)^{1/2} \cdot N^{-1} \cdot (1 - p) + (R1-R)12 \cdot N - 1 \cdot 0.999 - l_D \cdot 1 - p$.

Thus, this was done by setting $I[r, \max(p, a)] = \sum_{i=p}^1 K(i) \cdot q(r, i)$,

For example, if an interest rate r being charged to borrowers whose probability of being good is not less than 0.7, so the equity capital set aside of Q is given by following calculations:

$$p = 0.99, I[r, 0.99] = K(0.99) \cdot q(r, 0.99) \cdot 0.01;$$

$$p = 0.98, I[r, 0.98] = K(0.98) \cdot q(r, 0.98) \cdot 0.01 + K(0.99) \cdot q(r, 0.99) \\ = K(0.98) \cdot q(r, 0.98) \cdot 0.01 + I[r, 0.99];$$

$$p = 0.97, I[r, 0.97] = K(0.97) \cdot q(r, 0.97) \cdot 0.01 + I[r, 0.98];$$

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$$p = 0.7, I[r, 0.7] = K(0.7) \cdot q(r, 0.7) \cdot 0.01 + I[r, 0.71].$$

Unlike the earlier part of this paper there is no market price of equity. In order to finding the cut-off point we need to estimate the shadow prices x for equity

capital set aside. We apply different shadow prices x in following equation:

$$EP(r, p) = \text{Max}_{r, p_c} \int_{\max(a, p_c)}^1 q(r, p) \cdot [(r - B) \cdot p - (l_D + B) \cdot (1 - p)] \cdot f(p) \cdot dp$$

Where $B = x \cdot l_D \cdot k(p) + r_F$

to obtain optimal interest rates for different cut-off probability p_c by using the Solver function in Excel and use these results in Eq. 14 to estimate value of Q , then to find which one is the closest value of Q estimated to meet regulatory requirements predetermined. Finally, the maximize expected profit from the portfolio with predetermined equity capital is found by applying the optimal interest rate and cut off probability which can produce the closet estimated value of Q into Eq. 11.

4.1 Numerical Examples

We use some numerical examples to explore how the impact of Basel Accord requirements have on different pricing models in portfolio level with predetermined equity capital through comparing optimal interest rate, optimal cut off and expected portfolio profits yielded under the different Basel regulations.

4.1.1 Example for one price model

Using the cost structure of the examples from earlier part of this paper with $r_F = 0.05$ and $l_D = 0.5$, assume the parameters for the linear response rate function are $r_L = 0.04$, $b = 2.5$, and $c = 2$. We also assume the loan portfolio is of credit cards and so under the Basel 1 requirements the regulatory capital is $K(p) = K_1 = 0.08/l_D$ while the correlation in the Basel 2 and Basel 3 capital requirement is $R = 0.04$. We can now apply the values in Eq. 11 and Eq. 13

under the three regulatory regimes (Basel 1, Basel 2, Basel3).

- a. Optimal interest rate, optimal cut off and expected portfolio profits for Basel 1 is shown in Table 8

a	0.6		
Q	r	$\max(a, P_c)$	$E(P)$
0.01	0.401949	0.816895	0.032002
0.02	0.386739	0.740423	0.050027
0.03	0.37903	0.679048	0.059286
0.04	0.376854	0.627242	0.061889
a	0.7		
Q	r	$\max(a, P_c)$	$E(P)$
0.01	0.407457	0.841031	0.034311
0.02	0.393195	0.775777	0.056396
0.03	0.384206	0.724084	0.070762
0.04	0.36	0.7	0.0776
a	0.8		
Q	r	$\max(a, P_c)$	$E(P)$
0.01	0.414136	0.869762	0.036977
0.02	0.401949	0.816897	0.064002
0.03	0.37	0.8	0.081575
0.04	0.32	0.8	0.088533
a	0.9		
Q	r	$\max(a, P_c)$	$E(P)$
0.01	0.43	0.9	0.040137
0.02	0.380001	0.9	0.070034
0.03	0.330001	0.9	0.088055
0.04	0.279998	0.9	0.094201

- b. Optimal interest rate, optimal cut off and expected portfolio profits for Basel 2 is shown in Table 9

a	0.6		
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Q	r	$\max(a, P_c)$	$E(P)$
0.01	0.374611	0.82688	0.034369
0.02	0.38006	0.759399	0.047545
0.03	0.37953	0.708077	0.055931
0.04	0.377708	0.664649	0.060537
0.05	0.377513	0.629911	0.061878
a	0.7		
Q	r	$\max(a, P_c)$	$E(P)$
0.01	0.369972	0.854468	0.038429
0.02	0.378746	0.787716	0.056217
0.03	0.380226	0.744864	0.066864
0.04	0.379595	0.70992	0.074234
0.05	0.351949	0.7	0.077804
a	0.8		
Q	r	$\max(a, P_c)$	$E(P)$
0.01	0.363238	0.884763	0.045531
0.02	0.374369	0.828519	0.068081
0.03	0.370705	0.8	0.081397
0.04	0.315316	0.8	0.088609
a	0.9		
Q	r	$\max(a, P_c)$	$E(P)$
0.01	0.353655	0.918846	0.064084
0.02	0.329441	0.9	0.088189

c. Optimal interest rate, optimal cut off and expected portfolio profits for Basel 3 is shown in Table 10

a	0.6		
Q	r	$\max(a, P_c)$	$E(P)$
0.01	0.366959	0.868989	0.025911
0.02	0.376899	0.808587	0.038035
0.03	0.379852	0.766591	0.046211
0.04	0.380205	0.736586	0.05156

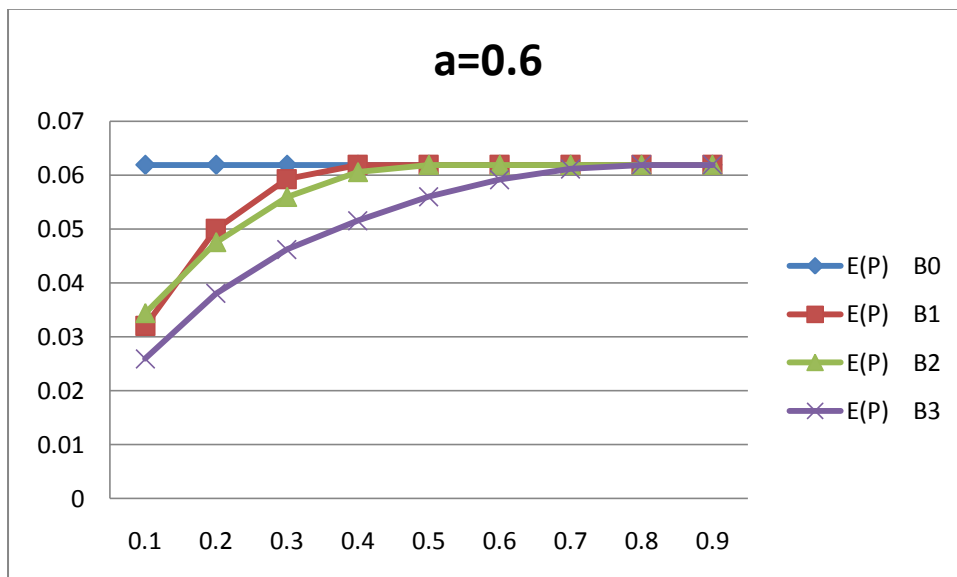
0.05	0.379504	0.707384	0.056026
0.06	0.378389	0.680736	0.059173
0.07	0.37736	0.655069	0.061134
0.08	0.37686	0.630074	0.061881
a	0.7		
Q	r	$\max(a, P_c)$	$E(P)$
0.01	0.362857	0.88628	0.029951
0.02	0.374217	0.829535	0.045115
0.03	0.378328	0.793269	0.054766
0.04	0.37979	0.768332	0.061179
0.05	0.380211	0.748692	0.065968
0.06	0.380007	0.724143	0.07143
0.07	0.379273	0.701342	0.07577
0.08	0.355765	0.7	0.077742
a	0.8		
Q	r	$\max(a, P_c)$	$E(P)$
0.01	0.354892	0.914746	0.033662
0.02	0.367484	0.866581	0.052787
0.03	0.373242	0.835957	0.065086
0.04	0.375172	0.822824	0.070368
0.05	0.36719	0.8	0.082265
0.06	0.332626	0.8	0.087838
a	0.9		
Q	r	$\max(a, P_c)$	$E(P)$
0.01	0.349461	0.931685	0.053989
0.02	0.356492	0.909443	0.071506
0.03	0.340678	0.9	0.085203
0.04	0.292207	0.9	0.093795

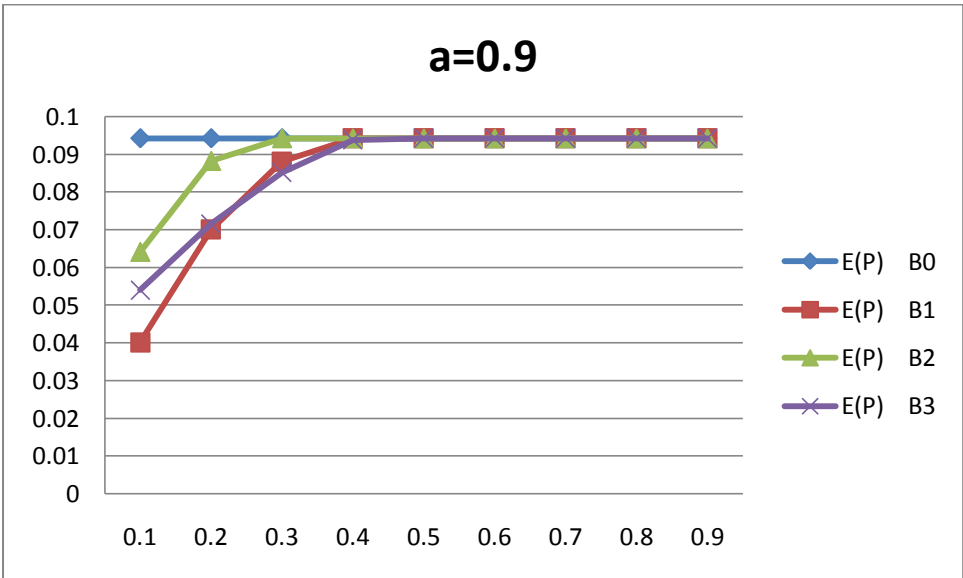
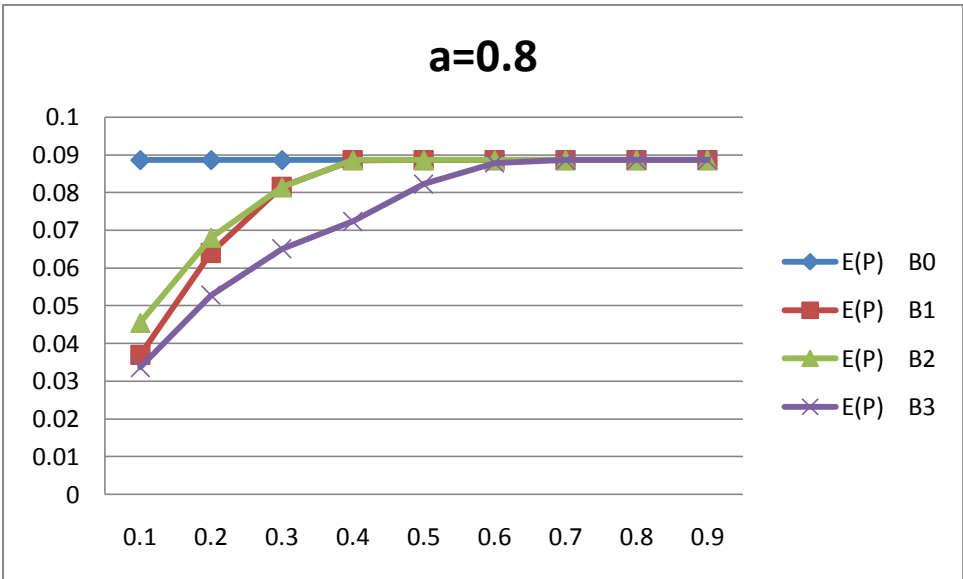
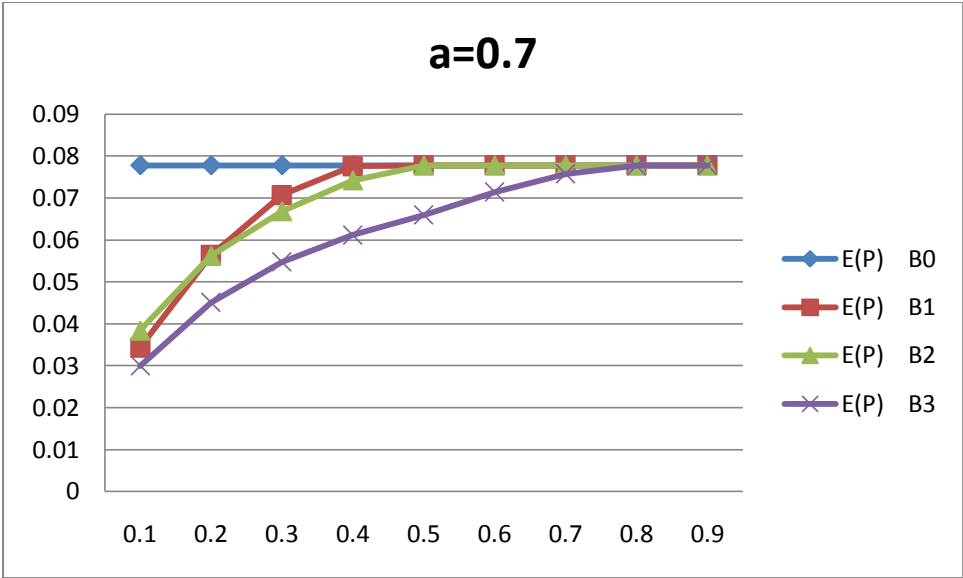
It is seen from these tables that the expected portfolio profits always increase as the borrowers become less risky. Meanwhile, the larger amount Q of equity set aside for a portfolio the lender have, the more profits one can achieve.

The optimal interest rate to charge drops as increasing of amount Q of equity set

aside for a portfolio under Basel 1 regulation. If the lender has a little equity in Basel 2 and 3 regulations, then one will take the best borrowers first. So in such situation the lender will be able to attract more good quality borrowers through dropping the interest rate down. Therefore, the optimal interest rates showed in Basel 2 and 3 regulations at first increase and then drop down along with increasing of the amount Q of equity set aside for a portfolio. It is clear to see that the optimal cut off decreases or is constant as amount of Q increases, which shows that the lender is likely to take more risky borrowers if one has enough equity but the optimal cut off probability cannot be below the value of a .

- d. Compare between expected profits for 4 regulation rules (with no regulatory requirements, Basel 1, Basel 2, and Basel 3) under one price model.





These Figures describe what happens to the expected profit as the equity capital set aside for a portfolio is increased from 0.01. As equity capital increases, the expected portfolio profits increase first. However, it is obvious to see the expected portfolio profits do not depend on size of regular capital. Once the expected portfolio profits reach the largest values which are the profits achieved under the Basel 0 regulation, the lender should stop to increase the equity capital but if the lender continues to do so and the expected portfolio profits will start to drop. Thus, we can see in these Figures that if the lender has enough equity they will end up in the Basel 0 case regardless which Basel requirement is used. Moreover, if the lender only has a little equity then the lender will take the best borrowers first and so the lender can make more profit with Basel 2 than Basel 1, but if the lender has more equity then they are taking more of the risky borrowers and so now Basel 1 gives more profit than Basel 2. For example, when $a = 0.7$ with $Q = 0.01$ the Basel 2 curve is above the Basel 1 curve. When the quality of the applicant rises up, this trend will be more obvious such as Basel 2 curve shown in Figure of $a = 0.9$ is all above the Basel 1 curve. Basel 3 regulation set much tighter capital restriction on equity capital than Basel 1 and 2, thus we can see from the Figures for value of a respectively equals to 0.6, 0.7 and 0.8 that the expected portfolio profit for Basel 3 is always lower than that for Basel 1 and 2 capital requirements before the profit reaches the Basel 0 solution. With higher quality of borrowers such as $a = 0.9$, the expected profit for Basel 3 lies between the Basel 1 and Basel 2 as the capital restriction of Q is small. This reflects a typical feature of Basel 3 regulation which is to be considered as a mixture of Basel 1 and Basel 2 capital requirements.

4.1.2 Example for two prices model

For two prices model, there a difference with one price model is the customers are actually segmented into two groups by the lender. Thus the expected profit from the portfolio given the limit on the equity capital provided is modified from Eqs 11 and 12 to ,

$$EP(r, p) = \left\{ \int_{\max(a, p_c)}^{p^*} q(r_2, p) \cdot [(r_2 - r_F) \cdot p - (l_D + r_F)] \cdot f(p) \cdot dp + \int_{p^*}^1 q(r_1, p) \cdot [(r_1 - r_F) \cdot p - (l_D + r_F)] \cdot f(p) \cdot dp \right\}$$

Eq. 15

subject to

$$\int_{\max(a, p_c)}^{p^*} l_D \cdot K(p) \cdot f(p) \cdot q(r_2, p) \cdot dp + \int_{p^*}^1 l_D \cdot K(p) \cdot f(p) \cdot q(r_1, p) \cdot dp \leq Q$$

Eq. 16

We take the same values used in Example of one price model so that the risk free rate is 0.05 and loss given default value is set as 0.5. The response rate function is also the same as in that example with $r_L = 0.04$, $b = 2.5$, and $c = 2$. In this model, the calculations applied in Eqs 15 and 16 give under the different regulatory regimes

- a. Expected portfolio profits for Basel 1 is shown in Table 11

a	0.6				
Q	$r(2)$	p_c	$r(1)$	p^*	$E(P)$
0.01	0.683867	0.6	0.5218	0.695166	0.028009
0.02	0.642	0.6	0.416654	0.747501	0.053499
0.03	0.549533	0.6	0.363899	0.785416	0.069658
0.04	0.462713	0.6	0.321212	0.822716	0.074444
a	0.7				
Q	$r(2)$	p_c	$r(1)$	p^*	$E(P)$
0.01	0.602429	0.7	0.4683	0.796964	0.033909
0.02	0.534286	0.7	0.394249	0.840751	0.061933
0.03	0.452909	0.7	0.350426	0.874397	0.078165
0.04	0.38143	0.7	0.311416	0.908174	0.082882
a	0.8				
Q	$r(2)$	p_c	$r(1)$	p^*	$E(P)$
0.01	0.524875	0.8	0.425687	0.889277	0.038543
0.02	0.44645	0.8	0.380003	0.920323	0.067117
0.03	0.379451	0.8	0.340577	0.950356	0.083614
0.04	0.321595	0.8	0.306222	0.97924	0.088974
a	0.9				
Q	$r(2)$	p_c	$r(1)$	p^*	$E(P)$
0.01	0.442378	0.9	0.409888	0.96187	0.041124
0.02	0.381178	0.9	0.371385	0.987832	0.070219
0.03	0.33	0.9	0.33	1	0.088054
0.04	0.28	0.9	0.28	1	0.0942

b. Expected portfolio profits for Basel 2 is shown in Table 12

a	0.6				
Q	$r(2)$	p_c	$r(1)$	p^*	$E(P)$
0.01	0.70086	0.6	0.565591	0.673925	0.019044
0.02	0.667522	0.6	0.47042	0.715598	0.040543
0.03	0.595003	0.6	0.370189	0.783517	0.063123
0.04	0.531574	0.6	0.345776	0.791545	0.072315

a	0.7				
Q	$r(2)$	p_c	$r(1)$	p^*	$E(P)$
0.01	0.622841	0.7	0.496214	0.771448	0.02711
0.02	0.582029	0.7	0.394252	0.816559	0.057225
0.03	0.499511	0.7	0.350488	0.84706	0.075035
0.04	0.427069	0.7	0.317271	0.877187	0.083153
a	0.8				
Q	$r(2)$	p_c	$r(1)$	p^*	$E(P)$
0.01	0.55037	0.8	0.422559	0.862038	0.042676
0.02	0.469157	0.8	0.350204	0.896181	0.072328
0.03	0.385125	0.8	0.315797	0.923432	0.087388
0.04	0.316628	0.8	0.305726	0.943876	0.089298
a	0.9				
Q	$r(2)$	p_c	$r(1)$	p^*	$E(P)$
0.01	0.444199	0.9	0.34515	0.94428	0.06456
0.02	0.330684	0.9	0.323985	0.961707	0.08876
0.03	0.24824	0.9	0.24824	0.998752	0.091937

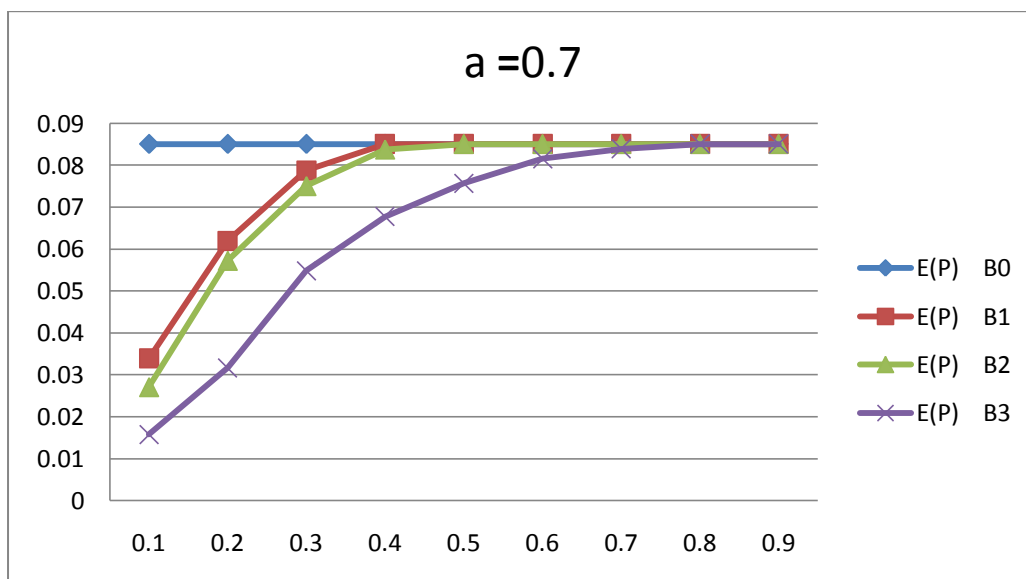
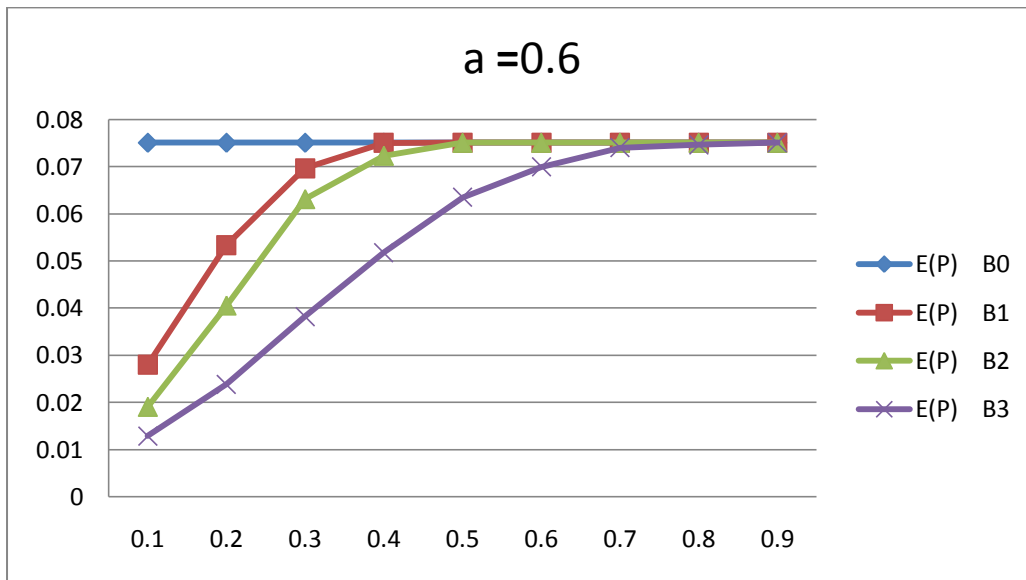
c. Expected portfolio profits for Basel 3 is shown in Table 13

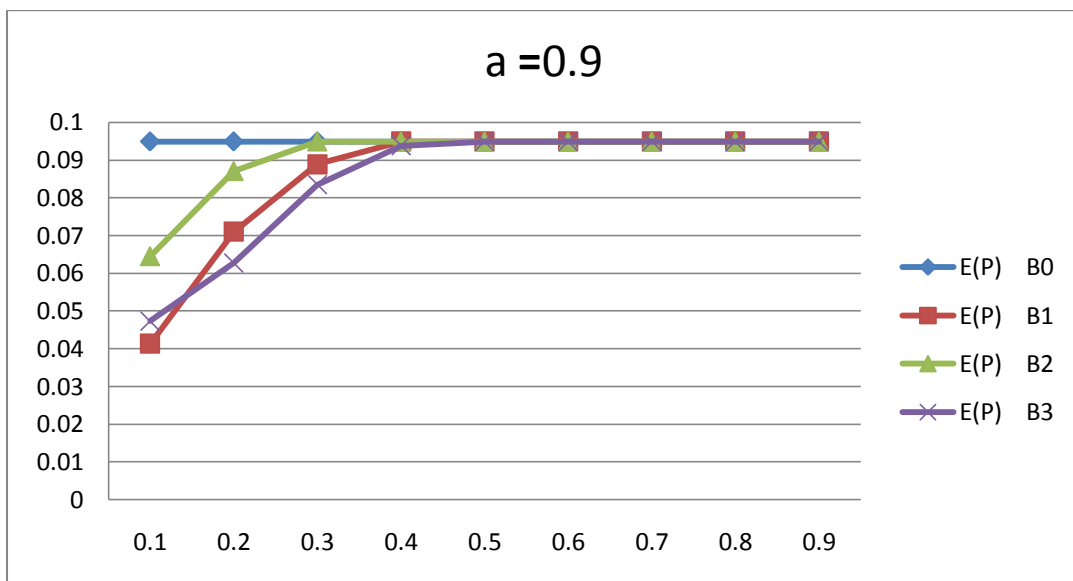
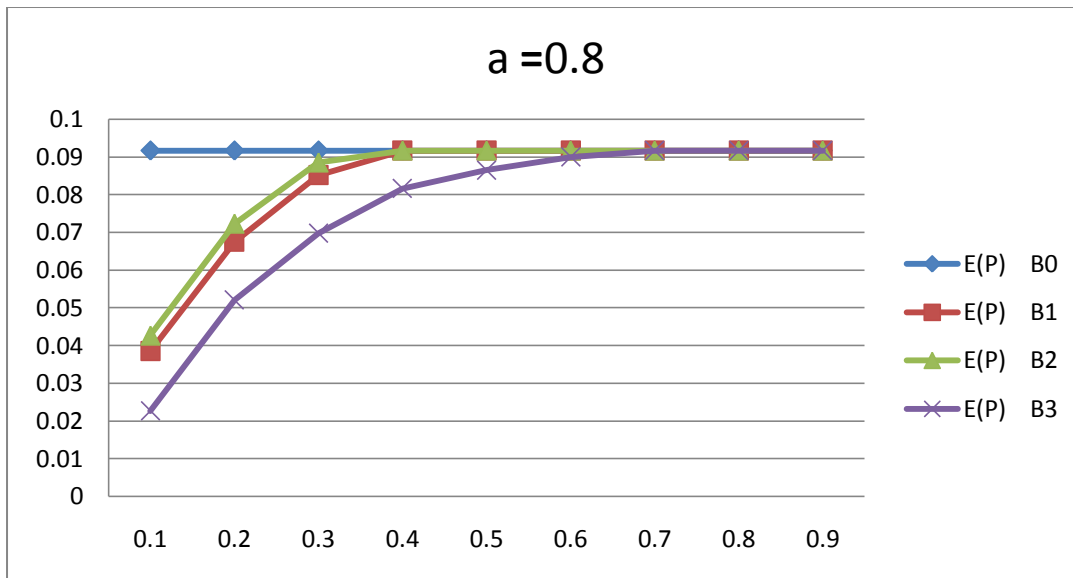
a	0.6				
Q	$r(2)$	p_c	$r(1)$	p^*	$E(P)$
0.01	0.712272	0.6	0.60043	0.65966	0.012894
0.02	0.69277	0.6	0.541607	0.684038	0.023875
0.03	0.670923	0.6	0.479696	0.711346	0.03821
0.04	0.65086	0.6	0.426343	0.736425	0.051786
0.05	0.604818	0.6	0.37967	0.763313	0.063485
0.06	0.55227	0.6	0.360161	0.783517	0.069943
0.07	0.503224	0.6	0.333337	0.803533	0.074032
a	0.7				
Q	$r(2)$	p_c	$r(1)$	p^*	$E(P)$
0.01	0.637763	0.7	0.541699	0.752796	0.015736
0.02	0.617494	0.7	0.48032	0.778133	0.031666

0.03	0.589681	0.7	0.401002	0.812899	0.054844
0.04	0.537868	0.7	0.370212	0.832402	0.067769
0.05	0.495313	0.7	0.348409	0.848716	0.075714
0.06	0.448958	0.7	0.326724	0.867696	0.08155
0.07	0.40661	0.7	0.309015	0.886368	0.083899
<i>a</i>	0.8				
<i>Q</i>	<i>r</i> (2)	<i>p_c</i>	<i>r</i> (1)	<i>p</i> *	<i>E</i> (<i>P</i>)
0.01	0.565519	0.8	0.475025	0.843101	0.022708
0.02	0.537932	0.8	0.392737	0.873352	0.05207
0.03	0.479349	0.8	0.355845	0.892778	0.069755
0.04	0.42524	0.8	0.329214	0.910719	0.081652
0.05	0.393073	0.8	0.318014	0.920958	0.086485
0.06	0.337366	0.8	0.307135	0.937818	0.089947
<i>a</i>	0.9				
<i>Q</i>	<i>r</i> (2)	<i>p_c</i>	<i>r</i> (1)	<i>p</i> *	<i>E</i> (<i>P</i>)
0.01	0.48946	0.9	0.383945	0.932331	0.047316
0.02	0.44974	0.9	0.348914	0.942966	0.062763
0.03	0.368954	0.9	0.321644	0.957436	0.083498
0.04	0.292207	0.9	0.292207	0.977169	0.093795

If the lender only has very limited equity capital of Q (such as $Q=0.01$) the expected portfolio profits for all Basel regulations in two prices model are lower than those in one price model, but the lender can make more profits than one price model along with increasing of capital resection of Q . In two prices model both of superior rate r_1 and inferior rate r_2 in three Basel regimes decrease as increasing of the capital restriction of Q , and the segment probability in three Basel regimes increase along with increasing of Q . Moreover we can see in three Basel regimes the differences between superior rate r_1 and inferior rate r_2 are likely to be small if the lender has more equity capitals, and two prices will eventually be the same in the situation where the value of a equals to 0.9 if the lender has enough equity capitals.

d. Compare between expected profits for 4 regulation rules (with no regulatory requirements, Basel 1, Basel 2, and Basel 3) under two prices model.





Form these figures, we can see again the expected portfolio profits for Basel 2 regulatory capital requirements are larger than that of Basel 1 until the value of a equals to 0.8, and the expected portfolio profits for Basel 3 is always smaller than and Basel 2. In the situation where the value of a equals to 0.9, the expected portfolio profits for Basel 3 is higher than that for Basel 1 but below that for Basel 2 as the amount of Q equity capital set aside equals to 0.01.

4.1.3 Example for variable pricing model

Variable pricing means that the interest rate charged on a loan to potential borrower depends on the default risk of individuals. Thus in that case the expected profit is

$$EP(r, p) = \text{Max}_{r, p} \int_p^1 q(r, p) \cdot [(r(p) - r_F) \cdot p - (l_D + r_F)] \cdot f(p) \cdot dp$$

Eq. 17

Subject to $\sum_p^1 l_D \cdot K(p) \cdot q(r, p) \leq Q$ Eq. 18

Using the cost structure of all the previous examples in this section, we have parameters $r_L = 0.04, b = 2.5, c = 2, r_F = 0.05$ and $l_D = 0.5$. As we known again unlike the earlier part of this paper there is no market price of equity, so the optimal price of equity is the marginal return on equity which is a ratio of the marginal expected profit to the marginal increase in equity. Thus we have the marginal return on equity function as follows:

$$\theta = \frac{EP(r, p)}{l_D \cdot K(p)} \quad \text{Eq. 19}$$

Much of the analysis about the marginal ROE can be found in the book by Thomas (2009). Though θ is the marginal profit increase for an extra unit of equity, it will be useful as the lender face to decide whether or not to expand the retail credit portfolio by comparing it with the real market price of equity. In predetermined equity capital case, we believe variable pricing is a knapsack problem. Therefore, the lender will take applicants with largest ratio first until use up all equity they have.

- a. Expected profits and Marginal ROE with Basel 1 is shown in Table 14

p	0.6	0.7	0.8	0.9	0.95	0.96	0.97	0.98	0.99
Q	E(P) B1	E(P) B1	E(P) B1	E(P) B1	E(P) B1	E(P) B1	E(P) B1	E(P) B1	E(P) B1
0.01	0	0.0216	0.044466	0.057945	0.061301	0.061717	0.062051	0.062304	0.062477
0.02	0.020767	0.047925	0.0675	0.07842	0.080698	0.080912	0.081048	0.081107	0.081091
0.03	0.0385	0.063125	0.0808	0.090242	0.091898	0.091995	0.092017	0.091964	0.091838
0.04	0.044204	0.068014	0.085078	0.094044	0.095501	0.095561	0.095545	0.095457	0.095295
Q	θ B1	θ B1	θ B1	θ B1	θ B1	θ B1	θ B1	θ B1	θ B1
0.01	0	0.270004	0.55582	0.724309	0.766258	0.771463	0.775633	0.778796	0.780964
0.02	0.259583	0.599065	0.843752	0.980245	1.008725	1.011403	1.013101	1.01384	1.013637
0.03	0.48125	0.789063	1.010001	1.128021	1.148725	1.149944	1.150214	1.149555	1.147979
0.04	0.552552	0.850179	1.063477	1.175556	1.193758	1.194506	1.194319	1.193209	1.191192

It is obvious to see that the expected profit goes up as the riskiness of the borrower decreases, and expected profit also increases if the capital restriction of Q increases. One sees that though the lender will take applicants with largest ratio of θ first in a knapsack problem, we found the probability of the borrowers being good with the largest marginal ratio in a given Q is also the one with the highest expected profits. Therefore we can conclude that the lender takes the most profitable borrowers rather than the borrowers with the least riskiness under Basel 1 regulation.

b. Expected profits and Marginal ROE with Basel 2 is given in Table 15

p	0.6	0.7	0.8	0.9	0.95	0.96	0.97	0.98	0.99
Q	E(P) B2	E(P) B2	E(P) B2	E(P) B2	E(P) B2	E(P) B2	E(P) B2	E(P) B2	E(P) B2
0.01	0	0	0.038629	0.073169	0.087079	0.089376	0.091431	0.093178	0.094495
0.02	0.005386	0.036631	0.063762	0.084465	0.091636	0.092722	0.093658	0.094411	0.094928
0.03	0.029328	0.055988	0.07691	0.090373	0.09402	0.094473	0.094822	0.095056	0.095155
0.04	0.041763	0.06604	0.083737	0.093442	0.095258	0.095382	0.095427	0.095391	0.095272
0.05	0.044204	0.068014	0.085078	0.094044	0.095501	0.095561	0.095545	0.095457	0.095295
Q	θ B2	θ B2	θ B2	θ B2	θ B2	θ B2	θ B2	θ B2	θ B2
0.01	0	0	0.368327	0.98119	1.789478	2.13202	2.660356	3.624304	6.17199
0.02	0.04394	0.307731	0.607981	1.132664	1.883122	2.211852	2.725131	3.67226	6.200261

0.03	0.23928	0.470342	0.73334	1.211898	1.932102	2.253607	2.759011	3.697346	6.215048
0.04	0.34073	0.554793	0.798445	1.253045	1.95754	2.275293	2.776605	3.710375	6.222727
0.05	0.360651	0.571376	0.811229	1.261126	1.962535	2.279551	2.780061	3.712933	6.224236

We found that the expected portfolio profits for Basel 2 are smaller than that for Basel 1 when the probability of the borrowers being default is high. Though Basel 2 regulatory capital requirements are flexible, the costs of equity capital request decreases with decreasing of the riskiness of the borrowers. Hence the expected portfolio profit for Basel 2 is greater than that achieved under Basel 1 once the probability of the borrowers being good reaches 0.9. In contrast to the situation of Basel 1 where the lender takes the most profitable borrowers, the probability of the borrowers being good with the largest marginal ratio in a given Q is always the borrower with the best quality. This means the lender in Basel 2 regulation like to take the borrower with the best quality first, even though they are not always the most profitable one.

c. Expected profits and Marginal ROE with Basel 3 is shown in Table 16

p	0.6	0.7	0.8	0.9	0.95	0.96	0.97	0.98	0.99
Q	E(P) B2	E(P) B2	E(P) B2	E(P) B2	E(P) B2	E(P) B2	E(P) B2	E(P) B2	E(P) B2
0.01	0	0	0.01756	0.063701	0.08326	0.086571	0.089565	0.092145	0.094133
0.02	0	0.012851	0.047611	0.077206	0.088708	0.090572	0.092227	0.093619	0.09465
0.03	0	0.031183	0.060062	0.082802	0.090965	0.09223	0.09333	0.09423	0.094864
0.04	0.017092	0.046096	0.070191	0.087354	0.092802	0.093578	0.094227	0.094726	0.095039
0.05	0.030063	0.056581	0.077313	0.090555	0.094093	0.094527	0.094858	0.095076	0.095162
0.06	0.039142	0.063922	0.082298	0.092795	0.094997	0.09519	0.095299	0.09532	0.095247
0.07	0.043558	0.067492	0.084724	0.093885	0.095436	0.095513	0.095514	0.095439	0.095289
0.08	0.044204	0.068014	0.085078	0.094044	0.095501	0.095561	0.095545	0.095457	0.095295
Q	θ B3	θ B3	θ B3	θ B3	θ B3	θ B3	θ B3	θ B3	θ B3
0.01	0	0	0.103037	0.525672	1.052912	1.270835	1.60373	2.2056	3.783562
0.02	0	0.066436	0.279369	0.637125	1.121813	1.329569	1.651389	2.240889	3.804364
0.03	0	0.161207	0.352429	0.683301	1.150358	1.353905	1.671134	2.255507	3.812983
0.04	0.085817	0.238302	0.411863	0.720865	1.173582	1.373702	1.687196	2.2674	3.819994

0.05	0.150937	0.292511	0.453652	0.747278	1.18991	1.387622	1.698492	2.275763	3.824924
0.06	0.196524	0.330459	0.482907	0.765769	1.201341	1.397367	1.706398	2.281618	3.828375
0.07	0.218697	0.348918	0.497137	0.774763	1.206901	1.402108	1.710244	2.284465	3.830054
0.08	0.221939	0.351616	0.499218	0.776077	1.207714	1.402801	1.710807	2.284882	3.830299

There is the same trend on the expected profits and the marginal ROE for Basel 3 regulation to that of made with Basel 2 case. Though the Basel 3 requires a much tighter capital restriction than Basel 2, the expected profits for Basel 3 are smaller than that for Basel 2.

5 Conclusion

Our analysis began with looking at the model of probability of a single loan, then we expand this to a particular portfolio loan related to different pricing decisions, which allows us to use several numerical examples to explore what the Basel Accords will effect various business measures including expected profit, optimal interest rate, and optimal cut off. Optimal interest rates for Basel 1, Basel 2 and Basel 3 are always bigger than optimal interest rates obtained under Basel 0 in one price model, two prices model and variable pricing model. In one price model, the expected portfolio profit for all Basel regulations are less than that of made with no regulatory capital requirements case. The expected portfolio profit for Basel 2 is smaller than that for Basel 1 until the value of a reaches 0.9. The expected portfolio profit for Basel 3 is always smaller than that for Basel 1 and 2 capital requirements since Basel 3 regulation set much tighter capital restriction. In two prices model, optimal segment probability in Basel 2 and 3 is always greater than the Basel 1 case. Superior rate r_1 under Basel 2 is always smaller than that for Basel 1, but inferior rate r_2 is higher than that for Basel 1 until the least good probability a the borrowers have reaches 0.8. The expected portfolio profits for Basel 2 are smaller than that

for Base 0, but greater than that achieved under Basel 1 once the probability of the borrowers being good reaches 0.9. Under Basel 3 regulation, the expected portfolio profits is always smaller than that for Basel 1 and 2. Both of r_1 and r_2 are always higher than that for Basel 2, but the inferior rate r_2 charged for Basel 3 is higher than that for Basel 1 when the value of a is no less than 0.9, and the superior rate obtained under Basel 3 is also smaller than that for Basel 1 once the value of a reaches 0.8. In variable pricing model, the expected portfolio profit made with Basel 0 is always no less than profit made with Basel 1, Basel 2 and Basel 3. The expected portfolio profits for Basel 2 and 3 regulations will become larger than that for Basel 1 when the quality of the borrowers is very good. Meanwhile, optimal interest rates for Basel 2 and 3 will be smaller than that for Basel 1 if the probability of the borrowers being good is good enough.

If we assume that the bank decides in advance how much of its equity capital can be used to cover the requirements of a particular loan portfolio. For one price model, optimal interest rate to charge drops as increasing on amount Q of equity set aside under Basel 1 regulation. If the lender has a little equity in Basel 2 and 3 regulations, the optimal interest rates increase at first and then drop down along with increasing of the amount Q of equity set aside. Optimal cut off decreases or is constant as amount of Q increases, which shows that the lender is likely to take more risky borrowers if one has enough equity but the optimal cut off probability cannot below the value of a . The expected portfolio profits increase as equity capital increases, but the lender should stop to increase the equity capital regardless which Basel requirement is used once the expected portfolio profits reach the largest values which are the profits achieved under the Basel 0 regulation. And if the lender only has a little equity then the lender will take the best borrowers first and so the lender can make more profit with Basel 2 than Basel 1, but if the lender has more equity then they are taking more of the risky borrowers and then Basel 1 gives more profit than Basel 2. With higher quality of borrowers such as $a = 0.9$, the expected profit for Basel 3 lies

between the Basel 1 and Basel 2 as the capital restriction of Q is small. This reflects a typical feature of Basel 3 regulation which can be considered as a mixture of Basel 1 and Basel 2 capital requirements. In two prices model both of superior rate r_1 and inferior rate r_2 in three Basel regimes decrease as there is an increase on the amount of Q , the differences between superior rate r_1 and inferior rate r_2 are likely to be small if the lender has more equity capitals in all three Basel regimes. The expected portfolio profit for Basel 2 regulatory capital requirements are larger than that of Basel 1 when the value of a is large, and the expected profit for Basel 3 is higher than that for Basel 1 but below the Basel 2 if the capital restriction of Q equals to 0.01 when the value of $a = 0.9$. In predetermined equity capital case, we believe variable pricing is a knapsack problem. We found the probability of the borrowers being good with the largest marginal ratio in a given Q is also the one with the highest expected profits under Basel 1 regulation. Therefore we conclude the lender takes the most profitable borrowers rather than the borrowers with the least riskiness under Basel 1 regulation. In contrast to the situation of Basel 1, the lender in Basel 2 and 3 regulations like to take the borrower with the best quality first, even though they are not always the most profitable one.

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