Intensity models and transition probabilities for credit card loan delinquencies

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Agenda

Introduction

- Data & Definitions
- Intensity Models
- Research Objectives

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- Parameter Estimates
- Predictions

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Discussion

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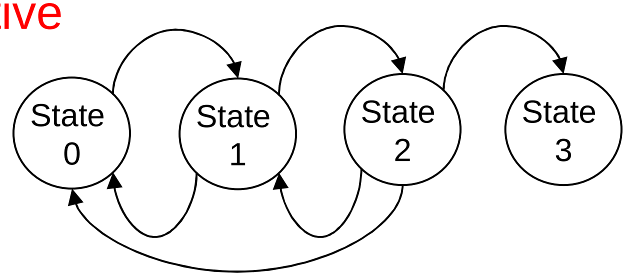
Introduction: Data & Definitions

Data

- Credit card loans, tracked monthly
- Application variables
- Behavioural variables, e.g. spending amounts, repayment amounts

States

- Four states defined: 0, 1, 2, 3, according to number of months in arrears
- Movements between states depend on (minimum) repayment amounts
- Months in arrears not necessarily consecutive



Definition of default

Introduction: Research Objectives

Intensity models for retail loans

- Based on survival models, can predict probabilities of different events occurring over time
- Inclusion of time-dependent covariates
- Predict for different states of delinquency and default
- Not previously applied on retail loans

Insights into factors that affect movements towards (recovery from) delinquency, and default (or not)

Prediction of events over time

Methodology

Transition Intensity

- Rate of change of number of accounts between any two states (*h* & *j*) at any time *t*

$$\alpha_{hji}(t) = Y_{hi}(t)\alpha_{hj0}(t)\exp\{\beta_{hj}{}^{T} Z_i(t)\}$$

- Indicator for whether individual *i* was in state *h* at time *t*
- Baseline transition intensity for state *h* to state *j* at time *t*
- Vector of unknown regression coefficients
- Vector of covariates for individual *i*

- Regression coefficients estimated via maximisation of partial likelihood
- Used to estimate baseline transition intensities $\alpha_{hj0}(t)$

Methodology

Generator Matrix

$$\mathbf{A}_i(t) = \begin{pmatrix} A_{00i} & A_{01i} & A_{02i} & A_{03i} \\ A_{10i} & A_{11i} & A_{12i} & A_{13i} \\ A_{20i} & A_{21i} & A_{22i} & A_{23i} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Matrix of sum of transition intensities from time 0 to *t*
- Estimated via transition intensities $\alpha_{hji}(t)$
  - Non-diagonals

$$A_{hji}\left(t;\hat{\beta}_{hj},\mathbf{Z}_i(t)\right) \cong \sum_0^t Y_{hi}(t)\alpha_{hj0}(t)\exp\left(\hat{\beta}_{hj}\mathbf{Z}_i(t)\right)$$

  - Diagonals

$$A_{hhi}\left(t;\hat{\beta}_{hj},\mathbf{Z}_i(t)\right) \cong -\sum_{h\neq j}\hat{A}_{hji}(t)$$

Methodology

Transition Matrix

$$\mathbf{P}_i(s,t)=\begin{pmatrix} p_{00} & p_{01} & p_{02} & p_{03} \\ p_{10} & p_{11} & p_{12} & p_{13} \\ p_{20} & p_{21} & p_{22} & p_{23} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Matrix of probabilities of transitions between each pair of states over specified time period *s* to *t*

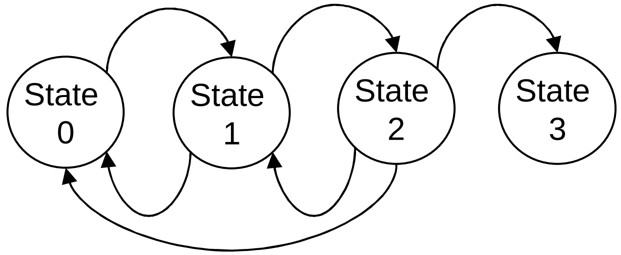
- Estimated via generator matrix $\mathbf{A}_i(t)$

$$\hat{\mathbf{P}}_i(s,t)\cong\prod_{(s,t]}\left(\mathbf{I}+\hat{\mathbf{A}}_i(u;\mathbf{Z}_i(u))-\hat{\mathbf{A}}_i(u-1;\mathbf{Z}_i(u-1))\right)$$

$$\hat{\mathbf{P}}_i(6,8)\cong\left(\mathbf{I}+\hat{\mathbf{A}}_i(7)-\hat{\mathbf{A}}_i(6)\right)\times\left(\mathbf{I}+\hat{\mathbf{A}}_i(8)-\hat{\mathbf{A}}_i(7)\right)$$

Results: Parameter Estimates

4 states, 1 absorbing state (default), 6 possible transitions at any time point



Covariates

- Application type: number of cards, landline, time at address, time with bank, income, A, age group, employment status
- Behavioural type (lagged 3 months): credit limit, payment amount, proportion of credit drawn, rate of total jumps, improvement from state of 3 months previous

Covariates unchanged for different transition models – allow comparison of effects of covariates on different transitions

Code	Explanation	Towards delinquency			Towards recovery		
		0 to 1	1 to 2	2 to 3	1 to 0	2 to 1	2 to 0
NOCA	Number of cards at application	-	+*	+	-	-	+
LAND	Landline present	-	-	-	+	+	+*
TADD	Time at address, years	+	-	-	+	+	+*
TWBA	Time with Bank (months)	-	-	-	+	+	+
TWBM	Time with Bank, missing	-	-	-	+	+	+
INCL	Income, logged	-	+	+	-	-	+*
INCM	Income, missing	-	+	+	-	-	+
A1	Variable A Group 1	.	.	.	.	.	.
A2	Variable A Group 2	+	+	+	-	-	-
A3	Variable A Group 3	+	+	+	-	-	-
A4	Variable A Group 4	+	+	+	-	-	-
A5	Variable A Group 5	+	+	+	-	-	-
AGE1	Age (at application) group 1	.	.	.	.	.	.
AGE2	Age group 2	-	-	+	+*	-	-
AGE3	Age group3	-	-	+	+	-	-
AGE4	Age group 4	-	-	+	+	-*	-
AGE5	Age group 5	-	-	+	+	-	-
AGE6	Age group6	-	-	+	+	-	-
AGE7	Age group 7	-	-	+	+	-	-
AGE8	Age group 8	-	-	+	+	-	-
AGE9	Age group 9	-	-	+	+	-	-*
AGE10	Age group 10	-	-	+	+	-	+*
EEMP	Employed	.	.	.	.	.	.
ESEL	Self-employed	+	+	+*	-	+	-*
ENOT	Not employed	-	+*	+*	-*	-*	+
EUNE	Unemployed	+	+	-*	+	+	+*
CLI3	Credit limit, logged and lagged	-	-	-	+	+	+
PAY3	Payment amount, logged and lagged	+	+	-	+	+*	+*
PDR3	Proportion of credit drawn, lagged	+	+	+	-	+	-
RJT3	Rate of total jumps, lagged	+	+	-	-	+	-
RSD3	Indicator for improvement in state from 3 months previous	+	+	.	-	.	.

Results: Parameter Estimates

Observations

- Most variables behave intuitively
- Certain groups of people better at managing themselves in delinquency without going into default
  - Younger debtors are at higher risk of going into delinquency, but when in 2 months in arrears, they are also more likely to move out towards recovery
  - Self-employed debtors have higher risk of going into delinquency, but when in delinquency, employed debtors have higher risk of default
  - Debtors who frequently go into states of delinquency (and recover) are more likely to be delinquent again but less likely to default

Results: Baseline Intensities

Transition from State 0

- Transit to state 1
- Risk of transition highest at the beginning of the loan

Transition from State 1

- Transit to either state 0 (full recovery) or state 2
- Higher risk of transition to state 0 than state 2
- Higher risk of transition to state 2 at the beginning of the loan: debtors that struggle from the very beginning go from state 0 to 2 in a short period

Transition from State 2

- Transit to state 0 (full recovery), state 1 (recover) or state 3 (default)
- Highest risk of transition to state 3, then state 1 and state 0

Results: Predictions I

Typical account created for each employment type

Matrix of transition of probabilities computed for time 6 to 12 months

Debtors of different employment types behave differently when in different states

While in delinquency, self-employed or unemployed debtors seem more adept at keeping themselves in arrears but not default

- If account is non-delinquent at time 6, likely to remain non-delinquent at time 12
- Small chance of going into the different states of delinquency
- How delinquent debtor is will affect chance of recovery, further delinquency or default

Results: Predictions

Apply intensity models onto test set to predict for transitions from time 6 to 12

Use training set transition proportions to get cut-off values

How to handle accounts that are censored between 6 and 12 months?

- Leave them but exclude from performance measurements
- Let state at time 12 = state at time 6
- Let state at time 12 = last known state

- Cut-off values vary depending on which method used

Order of comparison of states: 3 2 1 0 vs 3 1 2 0

Results: Predictions

		Predicted at time 12											
		Unknowns as is (A)				Unknowns as initial (B)				Unknowns as latest (C)			
OBS time 6		State 0	State 1	State 2	State 3	State 0	State 1	State 2	State 3	State 0	State 1	State 2	State 3
	State 0	1.001	0.709	2.132	14.244	0.995	0.762	2.229	16.439	1.000	0.727	2.178	16.146
	State 1	0.897	0.801	1.577	16.429	0.979	0.612	1.690	22.071	0.902	0.711	1.432	21.643
	State 2	0.360	0.327	0.647	11.667	0.386	0.306	0.404	13.067	0.372	0.316	0.538	13.067
	State 3	0	0	0	0	0	0	0	0	0	0	0	0

– Cohort Accuracy Table: ratio of total number of observations predicted to be in each state at time 12, over actual numbers

– Cells representing recovery-transitions are less than one: number of accounts predicted to recover under-estimated

– Cells representing delinquency-transitions are much higher than one: number of accounts predicted to go into delinquency (especially default) over-estimated

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Results: Predictions

		Predicted at time 12											
		Unknowns as is (A)				Unknowns as initial (B)				Unknowns as latest (C)			
		State0	State1	State2	State 3	State 0	State 1	State 2	State 3	State 0	State 1	State 2	State 3
OBS T12	State 0	0.907	0.790	0.723	0.792	0.911	0.760	0.750	0.756	0.908	0.822	0.753	0.779
	State 1	0.077	0.132	0.185	0.164	0.075	0.156	0.167	0.188	0.076	0.114	0.162	0.168
	State 2	0.005	0.022	0.076	0.034	0.005	0.015	0.063	0.047	0.005	0.016	0.072	0.043
	State 3	0.001	0.005	0.053	0.155	0.001	0.004	0.049	0.155	0.001	0.004	0.046	0.167

- Account level accuracy: number of accounts in each cell divided by different sums, so not directly comparable
- Diagonal cells give level of accuracy
  - For accounts in state 0, model predicts at least 90% correctly
  - For accounts in states 1, 2 and 3, model doesn't predict as well
- Non-diagonal cells give idea of where the "wrong" predictions are
  - Upper (lower) triangle represent accounts that are predicted to be in a state worse (better) than they actually are
- Model is able to differentiate between delinquent and non-delinquent accounts, but not so successful at predicting *which* delinquent state

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Results: Predictions

Method	Accuracy	Over-cautious	Over-optimistic
Unknowns as is	83.17%	9.20%	7.61%
Unknowns as initial state	83.60%	8.97%	7.41%
Unknowns as last known state	83.27%	9.20%	7.51%

– Overall, accuracy is high

– When the model does get it wrong, it seems to err more on the conservative side, predicting a state that is worse than it should be

Conclusions

Set of semi-parametric multiplicative intensity models to predict for delinquency

- Based on survival models
- Dynamic framework with probabilities for when events might occur
- Previously not applied on retail loans

How covariates affect risk of delinquency

- Parameter estimate signs are intuitive
- Some groups of people seem to be better at keeping themselves in delinquency and not default: debtors with no steady income, debtors who frequently go into arrears, young debtors

Conclusions

Predictions for test set

- Number of transitions predicted fairly accurate
- Model seems to be able to differentiate delinquent and non-delinquent accounts but not the correct delinquent state
- Model tends to over-estimate the number of accounts that go into delinquency and default

Further work

- Inclusion of macroeconomic variables
- Alternative way for translating probabilities into predictions: different method, different cut-off values?

Discussion

Thank you

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Q&A