Is the Price Right?

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Introduction

- The personal financial product is a saturated market
- Many financial institutions are able to offer similar products
- Customer has more choices hence the acceptance rate decreases for these financial institutions

Introduction
Acceptance Probability

- A calculated probability of a customer accepting a particular variant of a product
- To help with deciding which offer to extend to the next customer with a set of particular characteristics

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The Issue of Pricing:
Credit Card Interest Rates

- Matching the interest rates on the credit card offered
- Depends on what type of customer
- Payment behavior defines the type of customer

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Which Interest Rates to Whom

- Payment behavior:

(1) Pays on time the whole amount

(2) Pays only the minimum amount required to still have active usage of the credit card

Which Interest Rates to Whom

- Let there be 2 interest rates charged, r_1 and r_2 where $r_2 > r_1$:

(1) Pays on time the whole amount (r_2)

(2) Pays only the minimum amount required to still have active usage of the credit card (r_1)

The Customer

- 2 kinds of customers: Transactors and (Non Transactors) Revolvers
- Definition of a Transactor: a credit card customer who pays all debt on time

The Customer

- Definition of a Non Transactor: (Revolver) A credit card customer who does not pay off the whole debt every month

The Model

- How would having a Transactor score affect profitability decisions?

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The Model

- Definitions:

  p = Probability of borrower not defaulting in the next 12 months

$$s = \log \frac{p}{1-p}$$

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The Model

- Definitions:

  t = Probability borrower will be a Transactor

$$v = \log \frac{t}{1-t}$$

The Model

Assume in each month:

a = Average value of credit card transactions

m = Merchant service charge rate

T = number of months a revolver takes to pay off current purchases

The Model

r_F = Risk-free rate at which the lender borrows money

r= Rate at which lender charges on credit card balance

l_D= loss given default (% of default finally lost)

The Model

$q(r,p)/q(r,t,p)$=
Probability borrower with chosen p being good (and t probability of being a Transactor) accepting card with rate r

$F_p(t)$ = Probability borrower with score s
(p probability of good) defaulting in T periods

The Model

$f_p(i)$= is the probability defaulting at period i given not defaulted before

The Model

Consider the case where the r is fixed:

Expected profit per month

$$E(P(r,p,t)) = q(r,p,t)(ma + a(r - r_F)(1-t)((1 - f_p(1)) + ((1 - f_p(1))(1 - f_p(2)) + \ldots \prod_{i=1}^{T}(1 - f_p(i)) - a(r_F + l_D)F_p(T)))$$

Decision to accept if the equation ≥0

The Model

$$(1-x)^{12} = p$$

$$f_p(i) = x \Rightarrow 1 - f_p(i) = 1 - x = p^{\frac{1}{12}}$$

$$F_p(T) = 1 - (1-x)^{12} = 1 - p^{\frac{T}{12}}$$

$$\begin{aligned}
&(1 - f_p(1)) + (1 - f_p(1))(1 - f_p(2)) + \cdots + \prod_{i=1}^{T} (1 - f_p(i)) \\
&= (1-x) + (1-x)^2 + \cdots + (1-x)^T \\
&= \frac{(1-x) - (1-x)^T}{1 - (1-x)} \\
&= \frac{p^{\frac{1}{12}}\left(1 - p^{\frac{T}{12}}\right)}{\left(1 - p^{\frac{T}{12}}\right)}
\end{aligned}$$

The Model

$$m + (1-t)(r-r_F)\frac{p^{\frac{1}{12}}\left(1-p^{\frac{T}{12}}\right)}{1-p^{\frac{1}{12}}} - (r_F + l_D)\left(1-p^{\frac{T}{12}}\right) > 0$$

$$\frac{m}{\left(1-p^{\frac{T}{12}}\right)} + \frac{p^{\frac{1}{12}}(1-t)(r-r_F)}{1-p^{\frac{1}{12}}} - (r_F + l_D) > 0$$

$$\frac{m}{\left(1-p^{\frac{T}{12}}\right)} + \frac{p^{\frac{1}{12}}(1-t)(r-r_F)}{1-p^{\frac{1}{12}}} > (r_F + l_D)$$

The Model

There exists p^*_T so for fixed interest rate of r, accept customers where:

$$p > p^*_T$$

The Model

Calculation of p:

$$E(P(r,p,t)) = q(r,p,t)\left(ma + \frac{a(r-r_F)(1-t)\left(1-p^{\frac{T+1}{12}}\right)}{1-p^{\frac{1}{12}}} - a(r_F+l_D)\left(1-p^{\frac{T}{12}}\right)\right)$$

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The Model

$$ma + (r_F - r)aTp(1-t) + (r_F + l_D)a(1-p) > 0$$

$$p((r - r_F)T(1-t) + (r_F + l_D)) > (r_F + l_D) - m$$

$$p > \frac{r_F + l_D - m}{(r - r_F)T(1-t) + (r_F + l_D)}$$

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The Model

If r is variable, $r(p)$: $\frac{\partial}{\partial r} = 0$

Expected profit per month:

$$q(r,p)[aTp(1-v)] + \frac{\partial q}{\partial r}[ma + (r - r_F)aTp(1-t)] - (r_F + l_D)a = 0$$

$$\frac{q(r,p)}{\frac{\partial q}{\partial r}} + \frac{m}{Tp(1-t)} + (r - r_F) - \frac{(r_F + l_D)}{Tp(1-t)} = 0$$

$$r(p,t) = r_F + \frac{(r_F + l_D)(1-p)}{Tp(1-t)} - \frac{q(r,p)}{\frac{\partial(r,p)}{\partial r}} - \frac{m}{Tp(1-t)}$$

The Model

Using the model, consider no Transactors, *t=0*

$$\frac{m}{\left(1 - p^{\frac{T}{12}}\right)} + \frac{p^{\frac{1}{T/12}}}{\left(1 - p^{\frac{1}{12}}\right)} > r_F + l_D$$

This will result in a p_{NT}^{*} where $p_{NT}^{*} < p_{T}^{*}$

The Model

Let $q = a - br - cp$, so $\frac{\partial q}{\partial r} = -b$

So $r = r_F - \frac{m}{(1-t)L_T(p)} + \left(\frac{(r_F + l_D)F^T(p)}{(1-t)L_T(p)}\right) - \frac{q}{\frac{\partial q}{\partial r}}$ becomes

$$r = r_F - \frac{m}{(1-t)L_T(p)} + \left(\frac{(r_F + l_D)F^T(p)}{(1-t)L_T(p)}\right) - \frac{(a - br - cp)}{(-b)}$$

$$2r = r_F - \frac{m}{(1-t)L_T(p)} + \left(\frac{(r_F + l_D)F^T(p)}{(1-t)L_T(p)}\right) + \frac{a}{b} - \frac{cp}{b}$$

The Model

If there are no Transactors,

$$2r = r_F + \left(\frac{-m + (r_F + l_D)F^T(p)}{(1-t)L_T(p)} \right) + \frac{a}{b} - \frac{cp}{b}$$

So if $p\uparrow$ then $r\downarrow$

So if $p\downarrow$ then $r\uparrow$

The Model

With variable pricing: response rate will be different for Transactors and Non Transactors.

$$q = t(a_1 - b_1 r - c_1 p) + (1 - t)(a_2 - b_2 r - c_2 p)$$

$$a_1 < a_2, b_1 < b_2, c_1 = c_2$$

The Model

$$r = r_F - \frac{m}{(1-t)L_T(p)} + \left(\frac{(r_F + l_D)F^T(p)}{(1-t)L_T(p)}\right) - \frac{t(a_1 - b_1 r - c_1 p) + (1-t)(a_2 - b_2 r - c_2 p)}{tb_1 - (1-t)b_2}$$

$$2r = r_F - \frac{m}{(1-t)L_T(p)} + \left(\frac{(r_F + l_D)F^T(p)}{(1-t)L_T(p)}\right) + \frac{ta_1 + (1-t)a_2}{tb_1 - (1-t)b_2} - \frac{tc_1 + (1-t)c_2}{tb_1 - (1-t)b_2}$$

So, if there are no Transactor ($t=0$),

$$r = r_F - \frac{m}{L_T(p)} + \left(\frac{(r_F + l_D)F^T(p)}{L_T(p)}\right) - \frac{t(a_1 - b_1 r - c_1 p) + (1-t)(a_2 - b_2 r - c_2 p)}{b_2}$$

$$2r = r_F - \frac{m}{L_T(p)} + \left(\frac{(r_F + l_D)F^T(p)}{L_T(p)}\right) + \frac{ta_1 + (1-t)a_2}{b_2} - \frac{tc_1 + (1-t)c_2}{b_2}$$

Some results
Fixed *r*

m	*T*	*rF*	*ld*	*r*	*t*	*p*
0.02	6	0.005	0.5	0.01	0.5	0.868409
0.02	6	0.005	0.5	0.015	0.5	0.81785
0.02	6	0.005	0.5	0.01	0	0.81785
0.02	6	0.005	0.5	0.01	1	0.922361

Some results

Calculating the *r*

$m=0.02,\ T=6,\ r_F=0.005,\ a_1= 3.1, b_1=2.5,\ c_1 =c_2= 2$

$a_2 =4, b_2 =3.5$

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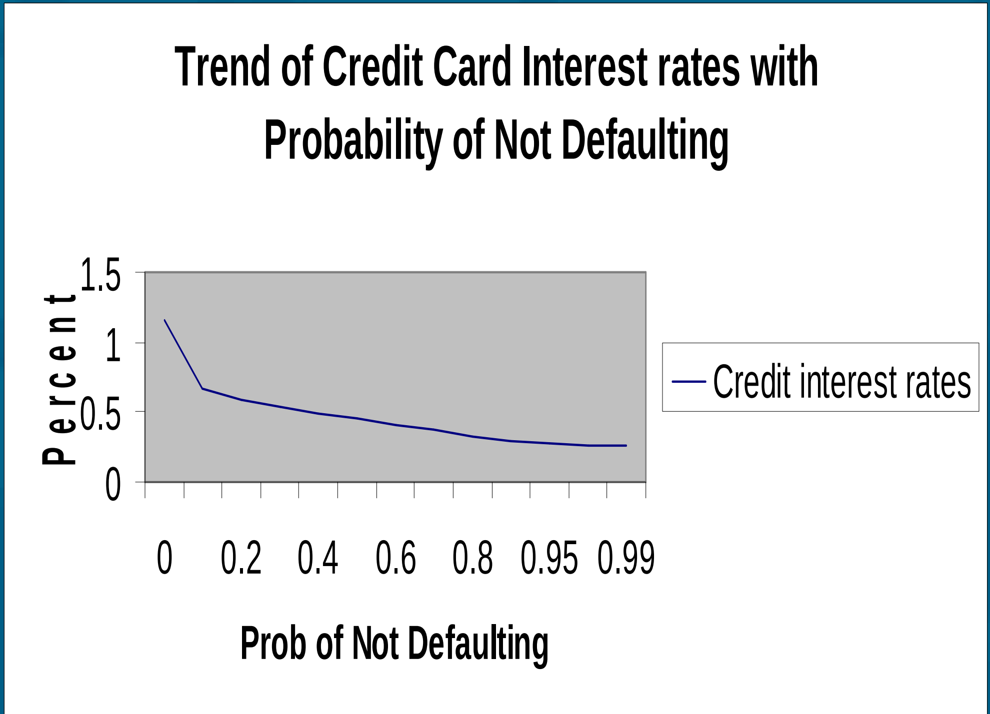
Some results

Calculating the r $(t=0.5)$



p	r
0.0001	1.153801
0.1	0.661468
0.2	0.59479
0.3	0.542796
0.4	0.496587
0.5	0.453468
0.6	0.412269
0.7	0.372375
0.8	0.333423
0.9	0.295183
0.95	0.27628
0.98	0.264998
0.99	0.261247

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$t=0.5$

p	r
0.0001	1.153801
0.1	0.661468
0.2	0.59479
0.3	0.542796
0.4	0.496587
0.5	0.453468
0.6	0.412269
0.7	0.372375
0.8	0.333423
0.9	0.295183
0.95	0.27628
0.98	0.264998
0.99	0.261247

$t=0.99$

p	r
0.0001	28.60516
0.1	5.613701
0.2	3.906622
0.3	2.933761
0.4	2.25012
0.5	1.721043
0.6	1.287914
0.7	0.920006
0.8	0.599237
0.9	0.314081
0.95	0.182323
0.98	0.106279
0.99	0.081401

Conclusions

- Able to determine the credit risk rate that max expected profit and with high acceptance probability for Transactors and Non Transactors

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Thank you

Questions?

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