CREDIT SUISSE

M2N: Optimal collateral to credit allocation

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Agenda

- Collateral allocation: What do we need it for ?
- Reduce the complexity → cluster identification
- Desired property of an optimal allocation
- M2N – an optimization problem
- Remark, benefit & outlook

Collateral allocation: What do we need it for ?

Motivation

- For various applications it is neccessary to know what the coverage ratio of an individual credit line is: e.g. to detect collateral value shortfalls or for the calculation of a net exposure or EAD per credit line

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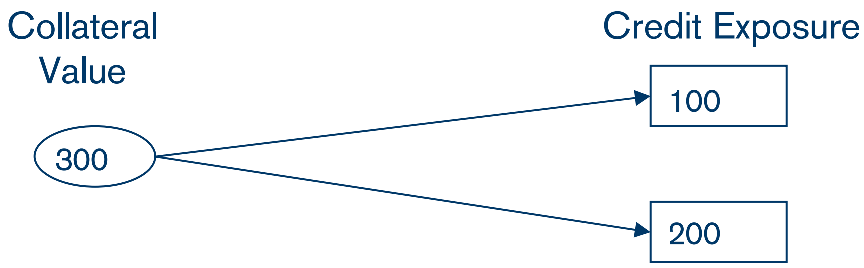

Collateral allocation: What do we need it for ?

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Intuitive: Exposure proportional allocation

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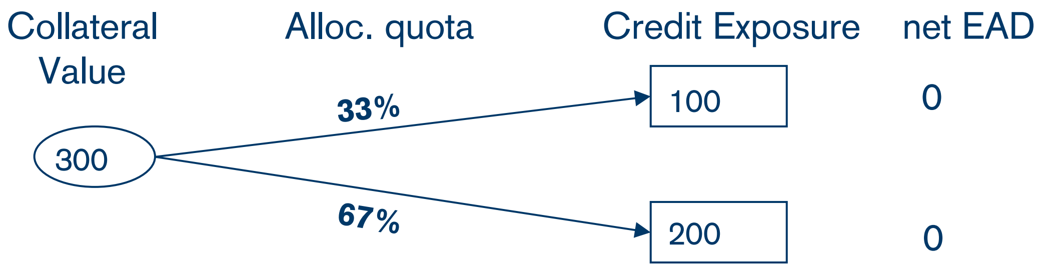

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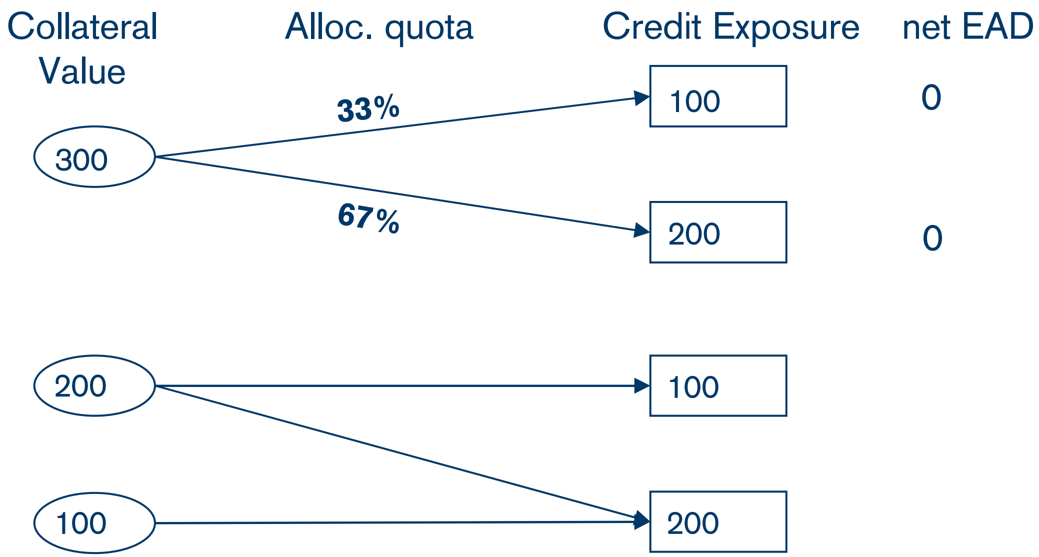

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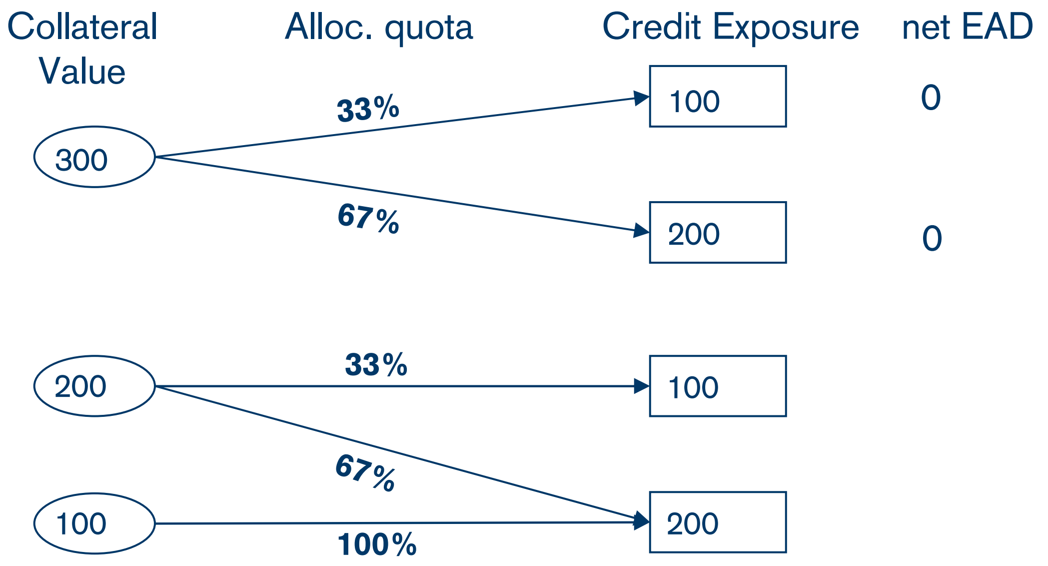

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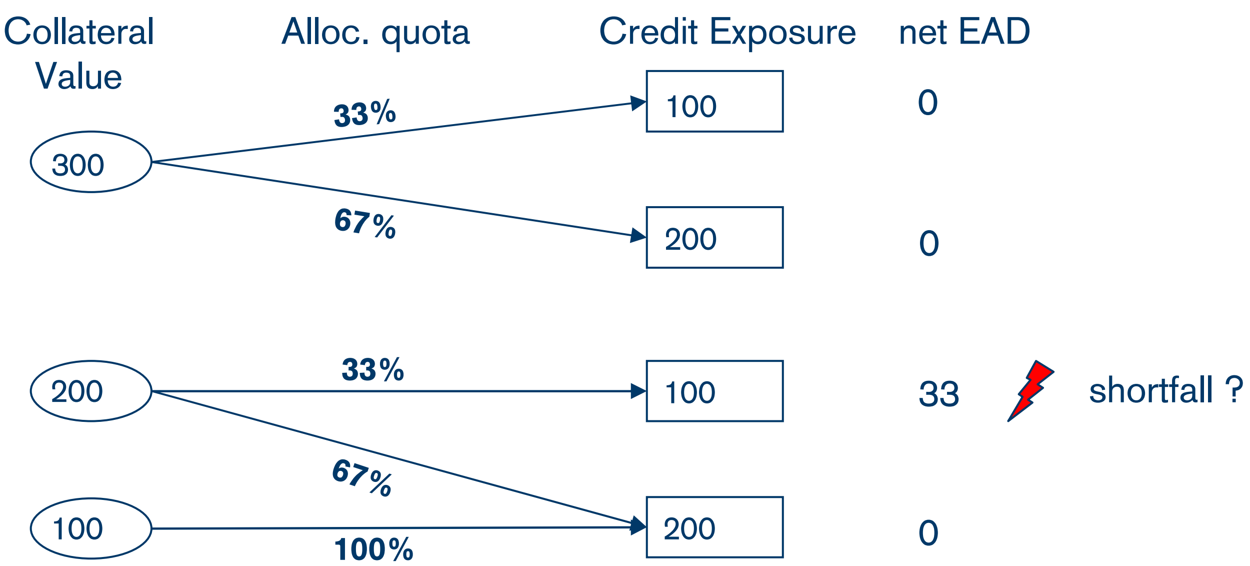
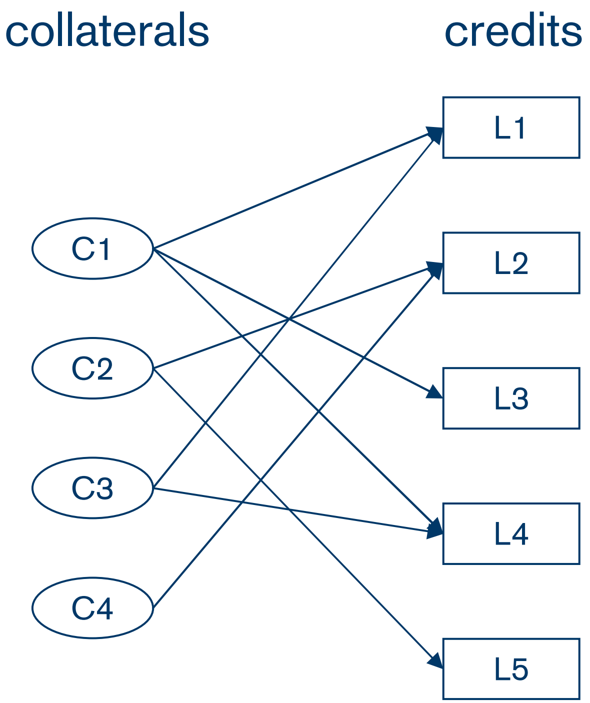


While on the whole structure there is obviously enough collateral, the simple allocation logic produces a collateral value shortfall on one credit.

Reduce the complexity → cluster identification

Cluster: Set of collaterals and credits that share a direct connection



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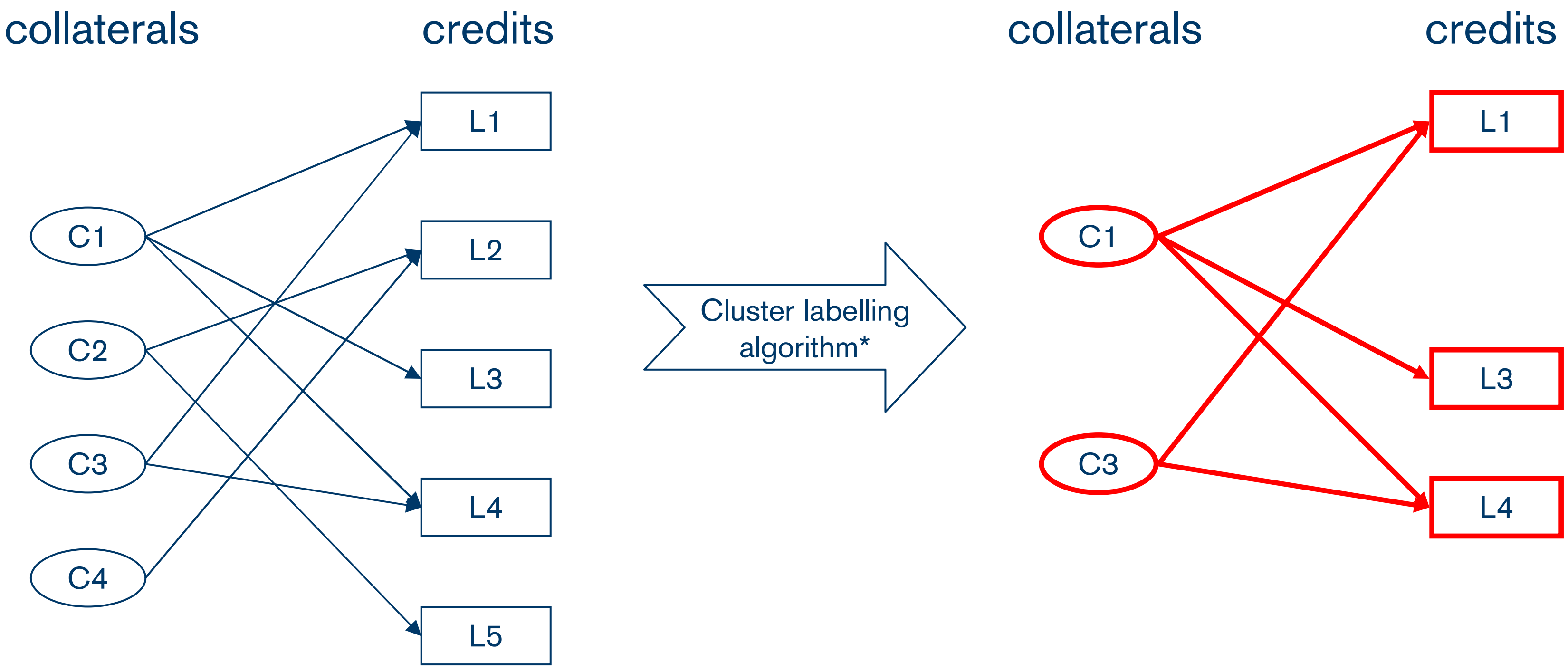

*e.g. an adapted Hoshen-Kopelman algorithm

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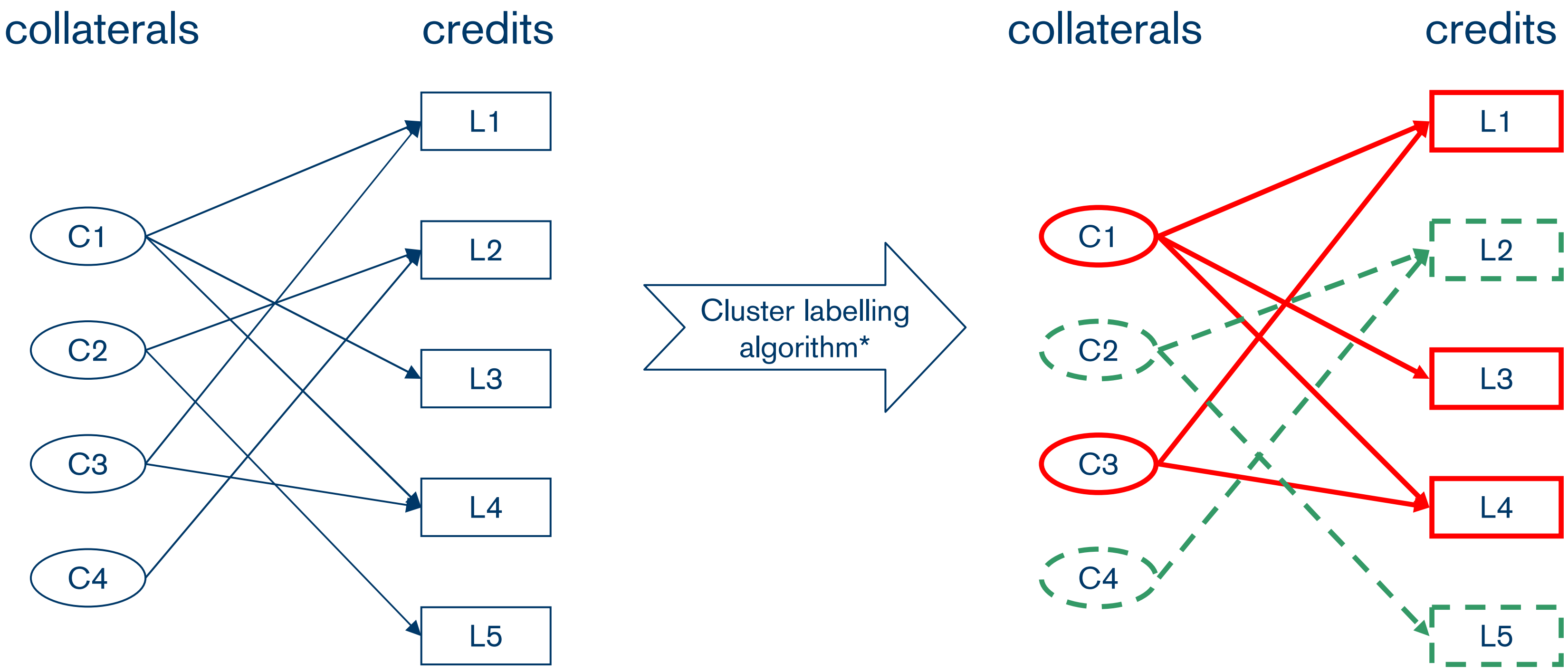

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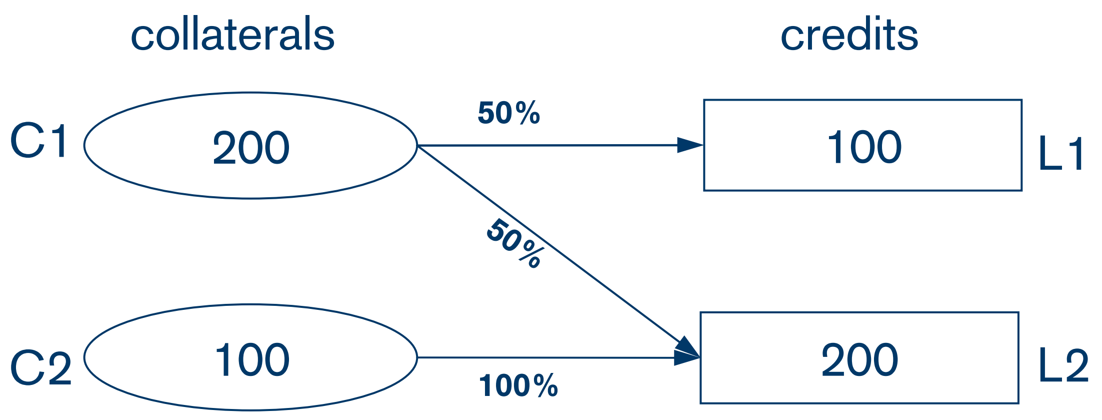
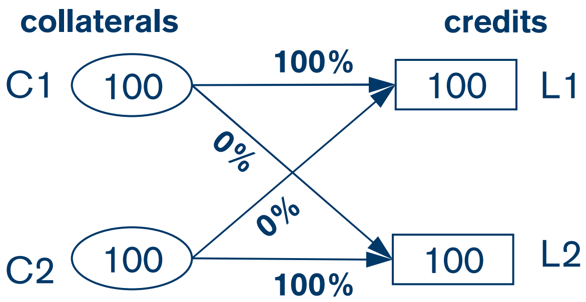
Desired property of an optimal allocation

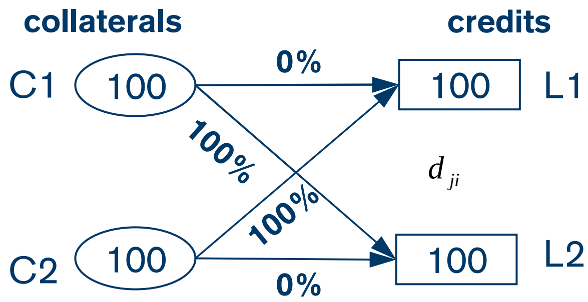
- Allocation of collateral value, such as the coverage ratio of every credit of a structure (connected cluster of credits and collaterals) is as close as possible to the coverage ratio of the whole structure
- I.e. for all L_i in a cluster $(\sum_j d_{ji} C_j / L_i - \alpha_0)^2$ is to be minimized, with
  - $\alpha_0 = \sum_j C_j / \sum_i L_i$ (average cluster coverage ratio)
  - L_i = exposure of credit line i=1,..,n
  - C_j = collateral value of collateral j=1,..,m
  - d_{ji} = quota of collateral j, that is allocated to credit i
- In the example: $\sum C_j / \sum L_i = 1$



Caveat regarding unique solution

Solution not necessarily unique





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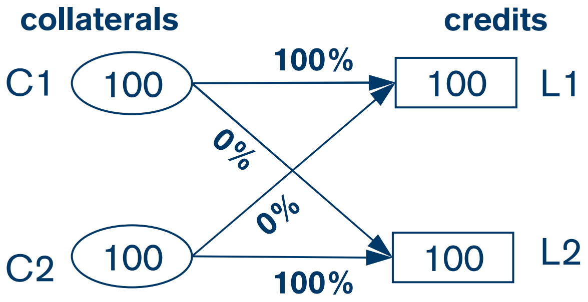
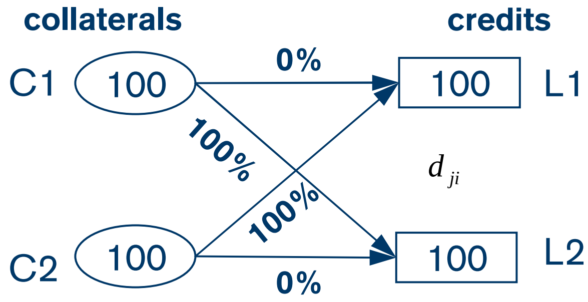
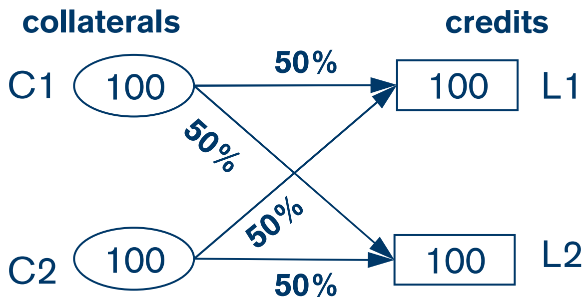




→ Introduce second optimality criterion: Allocation quotas of a collateral are to be close to 1/(number of connected credit), i.e. minimize $\sum_j \sum_j | d_{ji} - d_{j0} |$, with

d_{ji} = allocation quota of collateral j to credit i,

d_{j0} = 1/(number of credits connected to collateral j)



M2N – an optimization problem

target function:

$$F(d_{ji}) = \overbrace{\sum_{i=1}^{n} \left|\alpha_i - \alpha_0\right|}^{1} + \overbrace{\beta \cdot \sum_{j=1}^{m}\sum_{i=1}^{n} \left|d_{ji} - d_{j0}\right|}^{2}$$

$$= \sum_{i=1}^{n} \left| \sum_{j=1}^{m} d_{ji} \frac{C_j}{L_i} - \alpha_0 \right| + \beta \cdot \sum_{j=1}^{m}\sum_{i=1}^{n} \left|d_{ji} - d_{j0}\right|$$

with α_i = coverage ratio of credit i

α_0 = average cluster coverage ratio = $\sum_{j=1}^{m} C_j \Big/ \sum_{i=1}^{n} L_i$

d_{ji} = allocation quota of collateral j to credit i

d_{j0} = 1/(number credits connected to collateral j)

C_j = collateral j (j = 1, ..., m)

L_i = credit i (i = 1, ..., n)

β = target function weighting scalar

M2N – an optimization problem

constraints:

$$\sum_{i=1}^{n} d_{ji} = 1 \quad \forall\, j \in \{1,\ldots,m\}$$

$$d_{ji} \geq 0 \quad \forall\, i,j$$

$d_{ji} = 0 \quad \forall i,j$ if collateral j is not connected to credit i

-> next step: convert absolute values into linear form to be able to used efficient simplex solver implementations

M2N – a linear optimization problem

target function:

$$ZF(d_{ji}) = \sum_{i=1}^{n} \left|\alpha_i - \alpha_0\right| + \beta \cdot \sum_{j=1}^{m} \sum_{i=1}^{n} \left|d_{ji} - d_{j0}\right|$$

$$= \sum_{i=1}^{n} \gamma_i + \beta \cdot \sum_{j=1}^{m} \sum_{i=1}^{n} \rho_{ji}$$

under the constraints:

$$\alpha_i - \alpha_0 - \gamma_i \leq 0 \quad \forall i$$

$$\alpha_i - \alpha_0 + \gamma_i \geq 0 \quad \forall i$$

$$d_{ji} - d_{j0} - \rho_{ji} \leq 0 \quad \forall i, j$$

$$d_{ji} - d_{j0} + \rho_{ji} \geq 0 \quad \forall i, j$$

$$\sum_{i=1}^{n} d_{ji} = 1 \quad \forall\, j \in \{1, \ldots, m\}$$

$$d_{ji} \geq 0 \quad \forall\, i, j$$

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Matrices setup – getting ready for implementation

General formulation:

$$\min_x \langle \vec{f}, \vec{x} \rangle \quad \text{s.d.} \quad \begin{cases} AD \cdot \vec{x} \le \vec{b} \\ A_{eq} D \cdot \vec{x} = \vec{b}_{eq} \end{cases} \qquad \vec{f} = \begin{pmatrix} \vec{1}_n \\ \beta \cdot \vec{1}_{m \cdot n} \\ \vec{0}_{m \cdot n} \end{pmatrix} \quad \vec{x} = \begin{pmatrix} \vec{\gamma}_n \\ \vec{\rho}_{m \cdot n} \\ \vec{d}_{m \cdot n} \end{pmatrix} \quad D = \begin{bmatrix} 1_{n+m \cdot n} & 0 \\ 0 & 1_{m \cdot n, j \leftrightarrow i} \end{bmatrix} \begin{cases} 1, if \;\; j \; and \; i \; are \; connected \\ 0, else \end{cases}$$

matrix and vector for inequalities constraints:

$$A = \begin{bmatrix} -1_{n,n} & 0_{n,m \cdot n} & A^1_{n,m \cdot n} \\ -1_{n,n} & 0_{n,m \cdot n} & -A^1_{n,m \cdot n} \\ 0_{m \cdot n,n} & -1_{m \cdot n,m \cdot n} & 1_{m \cdot n,m \cdot n} \\ 0_{m \cdot n,n} & -1_{m \cdot n,m \cdot n} & -1_{m \cdot n,m \cdot n} \\ 0_{m \cdot n,n} & 0_{m \cdot n,m \cdot n} & -1_{m \cdot n,m \cdot n} \end{bmatrix} \quad A_1 = \begin{bmatrix} C_1/L_1 & 0 & \cdots & 0 & & C_m/L_1 & 0 & \cdots & 0 \\ 0 & C_1/L_2 & \ddots & \vdots & \cdots & 0 & C_m/L_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 & \cdots & \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & C_1/L_n & & 0 & \cdots & 0 & C_m/L_n \end{bmatrix} \quad \vec{b} = \begin{pmatrix} \alpha_0 \cdot \vec{1}_n \\ -\alpha_0 \cdot \vec{1}_n \\ \vec{b}_1 \\ -\vec{b}_1 \\ \vec{0}_{m \cdot n} \end{pmatrix} \quad \vec{b}_1 = \begin{pmatrix} d_{10} \cdot \vec{1}_n \\ d_{20} \cdot \vec{1}_n \\ \vdots \\ d_{m0} \cdot \vec{1}_n \end{pmatrix}$$

matrix and vector for equalities constraints:

$$A_{eq} = \begin{bmatrix} 0_{m,n+m \cdot n} & A^{eq1}_{m,m \cdot n} \end{bmatrix} \qquad A^{eq1} = \begin{bmatrix} \overbrace{1 \;\; 1 \;\; \cdots \;\; 1}^{n} & \cdots & 0 & 0 & \cdots & 0 \\ 0 \;\; 0 \;\; \cdots \;\; 0 & \ddots & \vdots & \vdots & \ddots & \vdots \\ \vdots \;\; \vdots \;\; \ddots \;\; \vdots & \ddots & 0 & 0 & \cdots & 0 \\ 0 \;\; 0 \;\; \cdots \;\; 0 & \cdots & 1 & 1 & \cdots & 1 \end{bmatrix} \qquad b_{eq} = \vec{1}_m$$

Remark, benefit & outlook

- While the presented linear optimization problem could be formulated for a whole portfolio at once, only after slicing the portfolio in clusters the algorithm performed in reasonable time (without clustering several hours, with clustering couple of minutes)
- By applying the algorithm the average expected loss on the portfolio was reduced by ca. 10%, if applied in the context of RWA calculation a similar reduction is expected
- In some cases the presented target function does not yield an intuitively good solution, improvements could be made e.g. by using a convex (to still be able to use a simplex algorithm ideally a piecewise linear) function and/or non symmetric penalties for over- and under-collateralization of credit lines