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# Modelling LGD using Bayesian methods

Katarzyna Bijak
Lyn C. Thomas

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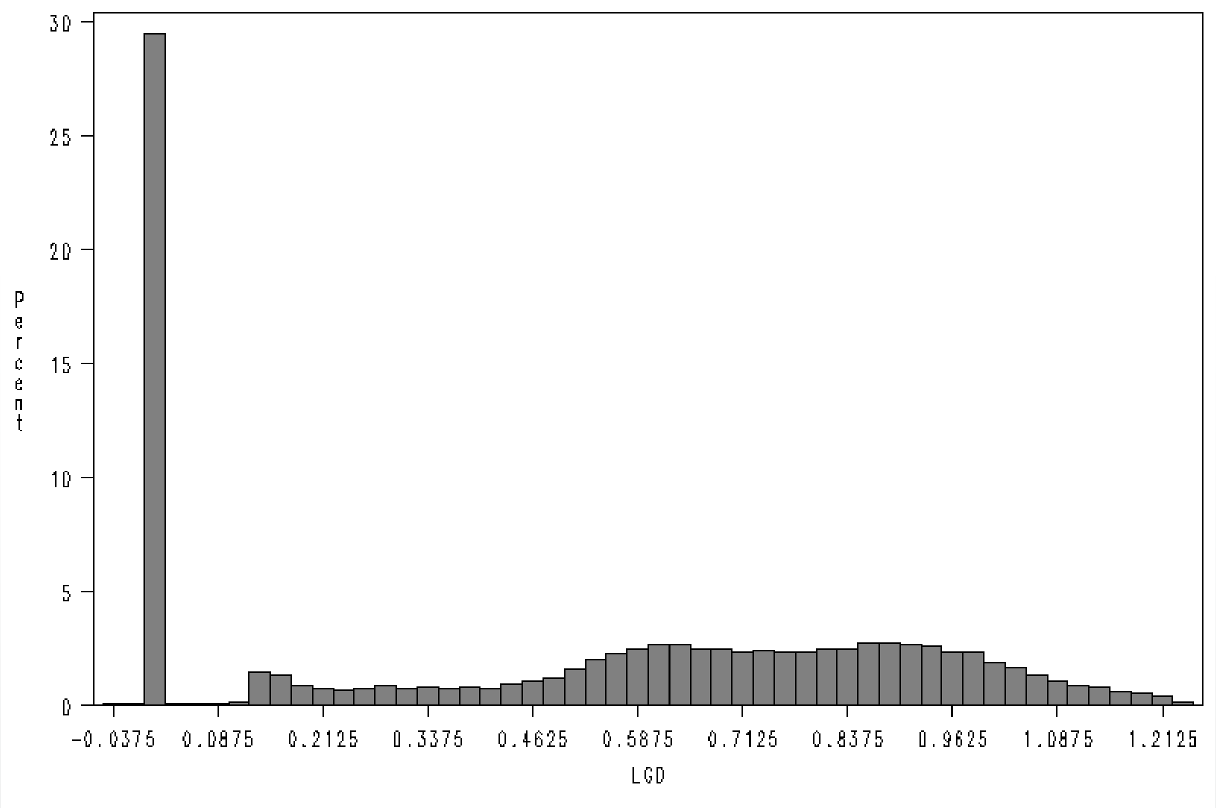
# Outline

- Introduction
- Modelling LGD: traditional approach
- Modelling LGD: Bayesian approach
- Empirical results
- Conclusions

# Loss Given Default (LGD)

- Loss borne by a bank when a customer defaults on a loan
- Often used in the form of LGD rate = 1 – Recovery Rate (RR)
- LGD estimates:
  - Expected LGD that helps calculate the expected loss
  - Downturn LGD, i.e. LGD in an economic downturn, under the new Basel Accord
- In this research: unsecured loans (no collateral)
- LGD is often found difficult to model, especially using the one-step approach, e.g. Bellotti and Crook (2008)

# LGD distribution example




- LGD distribution usually has a high peak at zero: many customers default but finally pay in full

# Modelling LGD: traditional approach

- Two-step approach
  - Matuszyk, Mues and Thomas (2010)
  - Loterman, Brown, Martens, Mues and Baesens (2009)
- There are two separate models estimated independently:
  - The first model (logistic regression) separates positive values from zeroes (and negative values, if any)
  - The second model (e.g. linear regression) allows for the estimation of the positive values
- The independent estimation can be considered problematic from the methodological point of view

# Traditional approach: suggested solutions

- In order to apply this approach, one needs:
  - either to set a cut-off for the first model and take zero if Prob(LGD > 0) < cut-off and the estimated value otherwise **(cut-off approach)**
  - or to calculate a product of Prob(LGD > 0) and the estimated value **(probability times value approach)**
- In particular, one can draw a cut-off from U(0, 1) or draw a number from a Bernoulli distribution with Prob(LGD > 0) **(random cut-off approach)**
- The result is a point estimate of LGD for each customer

# Modelling LGD: Bayesian approach

- Bayesian graphical models
  - Single, hierarchical model instead of two separate ones
- <u>**Random cut-off approach**</u>
  - $p_i \sim \text{Logistic}(\mathbf{x}_i, \theta_1)$ – this is Prob(LGD > 0)
  - $b_i \sim \text{Bernoulli}(p_i)$ – this is equal to either 0 or 1
  - If $b_i = 0$ then $y_i \sim N(0, \sigma_1^2)$; else $y_i \sim N(\text{Linear}(\mathbf{z}_i, \theta_2), \sigma_2^2)$ – this is the estimated LGD value
- **Probability times value approach**

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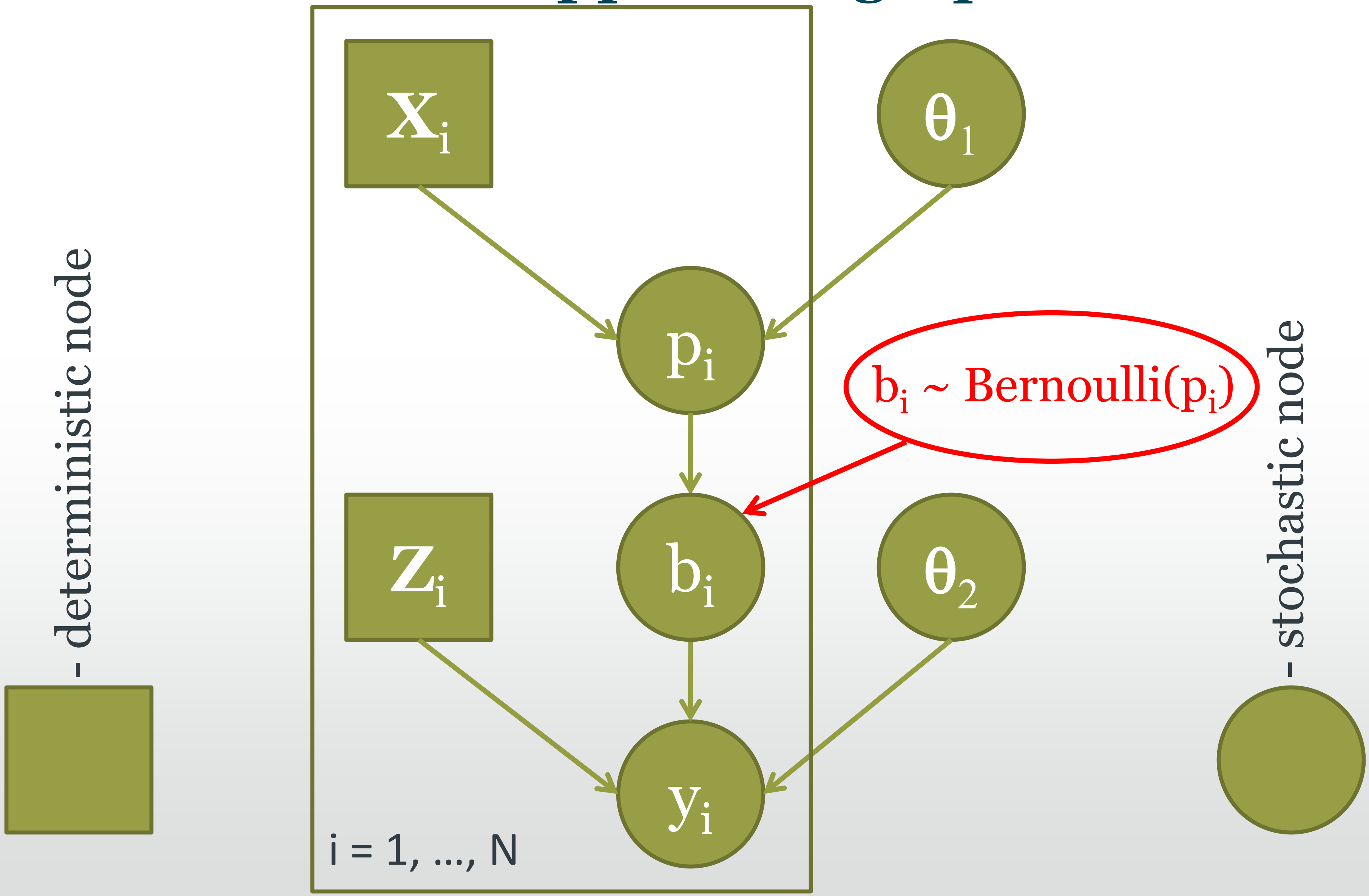
# Random cut-off approach: graph



# MCMC methods

- Markov chain Monte Carlo (MCMC) methods
  - Stochastic simulation methods that are used in the Bayesian inference to generate samples from posterior distributions
  - Based on the construction of a Markov chain that converges to the posterior distribution
  - Under some assumptions, as $t \rightarrow \infty$, $\theta^{(t)}$ converges to its equilibrium distribution (posterior distribution) that is independent from its initial state $\theta^{(0)}$
  - Results: **distributions** of model parameters and outcomes as well as model performance measures

# MCMC algorithm

1) Selection of the initial values $\theta^{(0)}$
2) Generating $T$ values until the equilibrium is reached
3) Convergence monitoring
4) Discarding the first $B$ values (burnin period)
5) Treating $\{\theta^{(B+1)},\ldots,\theta^{(T)}\}$ as the sample (MCMC output)
6) Analysis of the posterior distributions: calculating posterior summary statistics, plotting densities etc.

Based on: Ntzoufras, I. (2009) Bayesian Modeling Using WinBUGS. Hoboken, NJ: Wiley.

# Bayesian framework

- Software: OpenBUGS 3.0.3
- Method: Markov chain Monte Carlo (MCMC)
- Burnin period: 10K iterations
- MCMC output: 100K iterations
- Sampling lag (thinning interval) $L = 5$
  - MCMC output is not independent: there are correlations between $\theta^{(t)}$ and $\theta^{(t+k)}$, autocorrelations of lag $k = 1, 2, \ldots$
  - Keeping the first value from each batch of $L$ iterations → independent sample

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# Empirical research

- Research design
  - Traditional approach
    - Random cut-off
    - Probability times value
  - Bayesian approach
    - Random cut-off
    - Probability times value
- Data on ca 48K personal loans granted by a large UK bank
- Random training and validation samples (10K records each)

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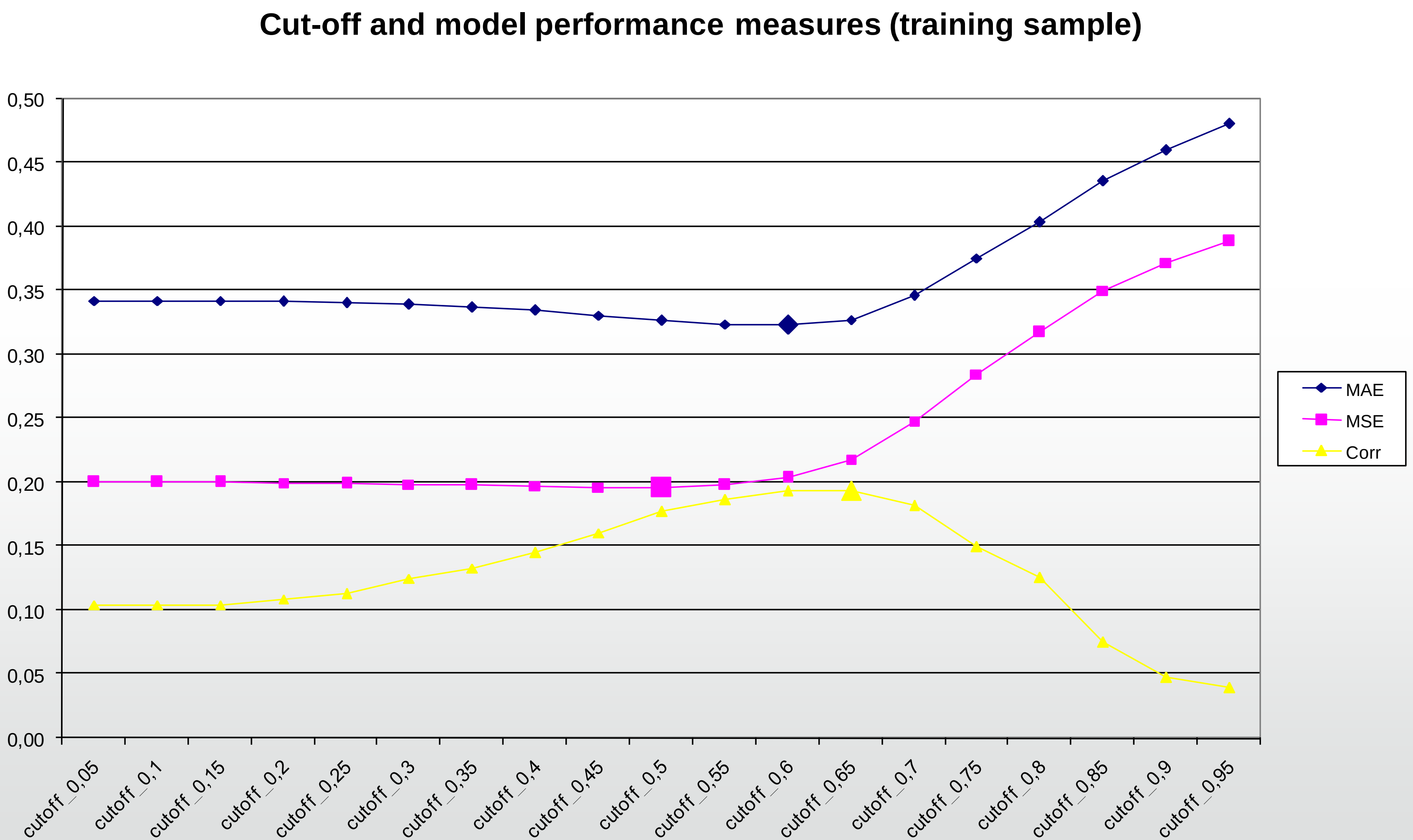
# Traditional approach: separate models

| Logistic regression | Training | Validation |
| --- | --- | --- |
| MAE | 0.367 | 0.370 |
| MSE | 0.183 | 0.186 |
| **Gini** | **0.420** | **0.421** |
| **KS** | **0.309** | **0.317** |

| Linear regression | Training | Validation |
| --- | --- | --- |
| MAE | 0.182 | 0.181 |
| MSE | 0.057 | 0.056 |
| Pearson correlation | 0.398 | 0.405 |
| **R-squared** | **0.159** | **0.164** |

# Traditional approach: entire model



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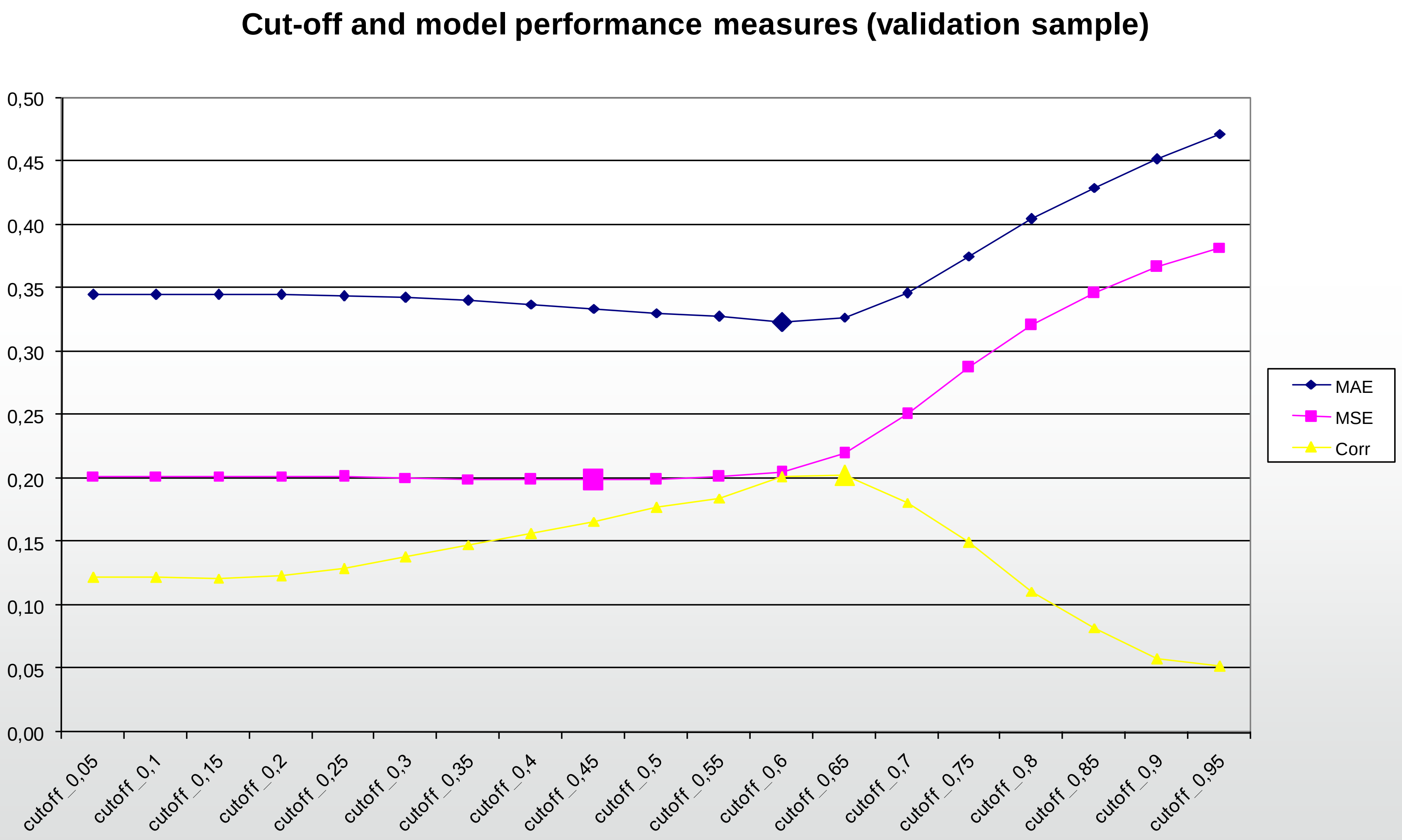
# Traditional approach: entire model



# Random cut-off approach: performance

| | MAE (Training) | MAE (Validation) |
|---|---|---|
| Traditional - mean | 0.364 | 0.365 |
| Bayesian - post. mean | 0.364 | 0.365 |

| | MSE (Training) | MSE (Validation) |
|---|---|---|
| Traditional - mean | 0.244 | 0.245 |
| Bayesian - post. mean | 0.243 | 0.245 |

| | Corr (Training) | Corr (Validation) |
|---|---|---|
| Traditional - mean | 0.081 | 0.085 |
| Bayesian - post. mean | 0.082 | 0.085 |

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# Random cut-off approach: performance

| | MAE (Training) | MAE (Validation) |
|---|---|---|
| Traditional - sd | 0.0029 | 0.0030 |
| Bayesian - post. sd | 0.0031 | 0.0031 |

| | MSE (Training) | MSE (Validation) |
|---|---|---|
| Traditional - sd | 0.0028 | 0.0028 |
| Bayesian - post. sd | 0.0030 | 0.0031 |

| | Corr (Training) | Corr (Validation) |
|---|---|---|
| Traditional - sd | 0.0101 | 0.0103 |
| Bayesian - post. sd | 0.0104 | 0.0105 |

# Random cut-off approach: estimates

| Variable (logistic regression) | Traditional | Bayesian (p. mean) |
|---|---|---|
| Intercept | 1.084 | 1.094 |
| Age of exposure (months) | -0.545 | -0.547 |
| Amount of loan at opening | 0.338 | 0.342 |
| Total number of advances/ arrears within the whole life of the loan | -1.478 | -1.510 |
| Number of months with arrears >0 within the life of the loan | 0.073 | 0.069 |
| Number of months with arrears >1 within the last 12 months | -0.529 | -0.539 |

# Random cut-off approach: estimates

| Variable (linear regression) | Traditional | Bayesian (p. mean) |
|---|---|---|
| Intercept | 0.719 | 0.718 |
| Joint applicant present | -0.012 | -0.012 |
| Total number of advances/ arrears within the whole life of the loan | -0.143 | -0.141 |
| Term of loan (months) | -0.037 | -0.037 |
| Worst arrears within the life of the loan | 0.178 | 0.177 |
| Number of months with arrears >2 within the last 12 months | -0.053 | -0.052 |

# Bayesian approach: outcomes

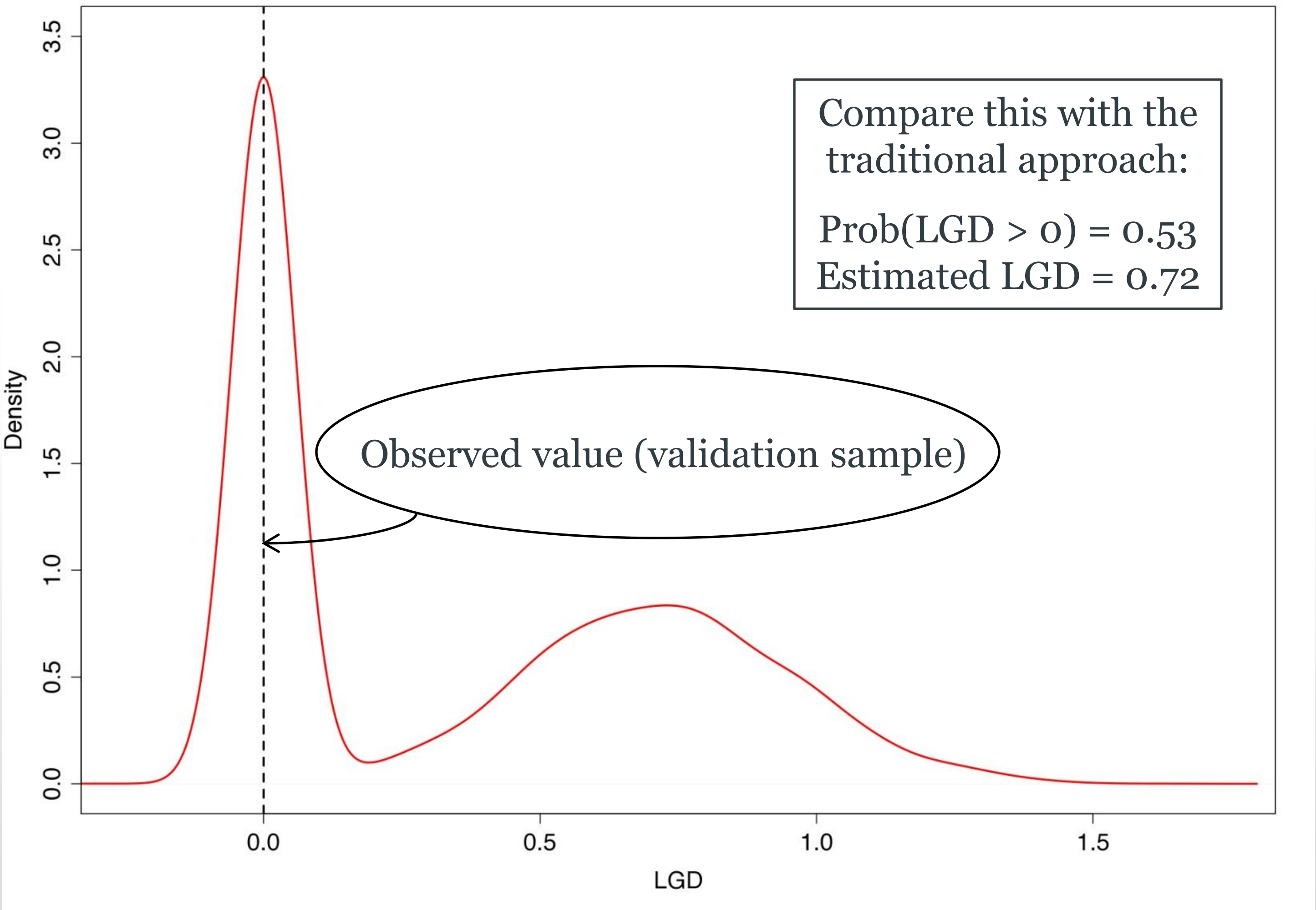
- In the traditional approach: **a point estimate** of LGD (a single number) for each customer
- In the Bayesian approach: **an individual distribution** of LGD for each customer → various characteristics, e.g.:
  - Mean
  - Median
  - Other quantiles

# Individual distributions: applications

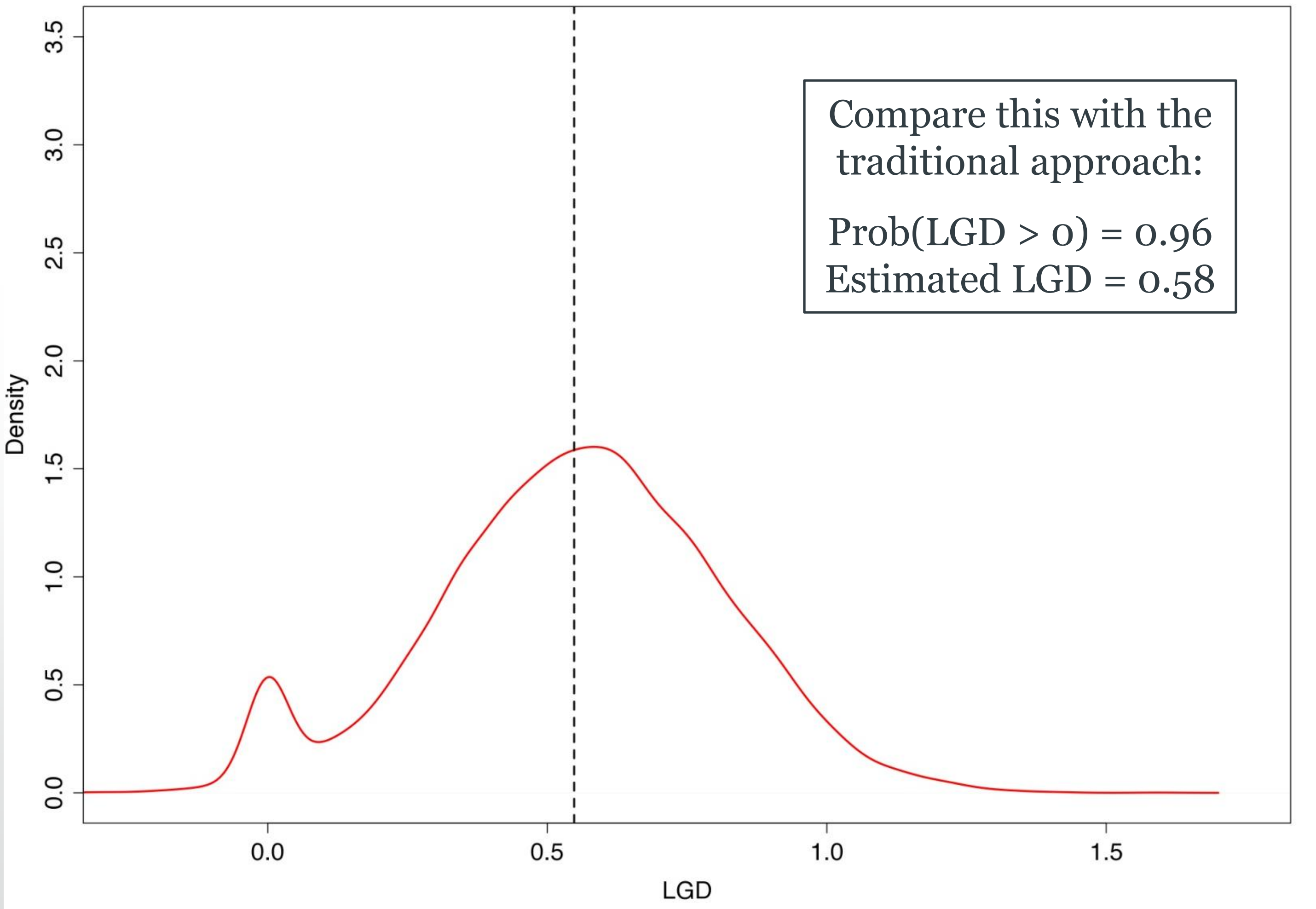
- The predictive mean can be treated as **the expected LGD** → expected loss EL = E(LGD) * E(PD) * E(EAD)

- The individual predictive distributions of LGD can be used to estimate **the downturn LGD**:
  - One could choose e.g. the 0.999th quantile
  - The choice of the quantile would depend on the user's perception of the severity of downturns (Kim, 2006)
  - This could be useful in case of lacking downturn data

# Individual distributions: example 1



# Individual distributions: example 2



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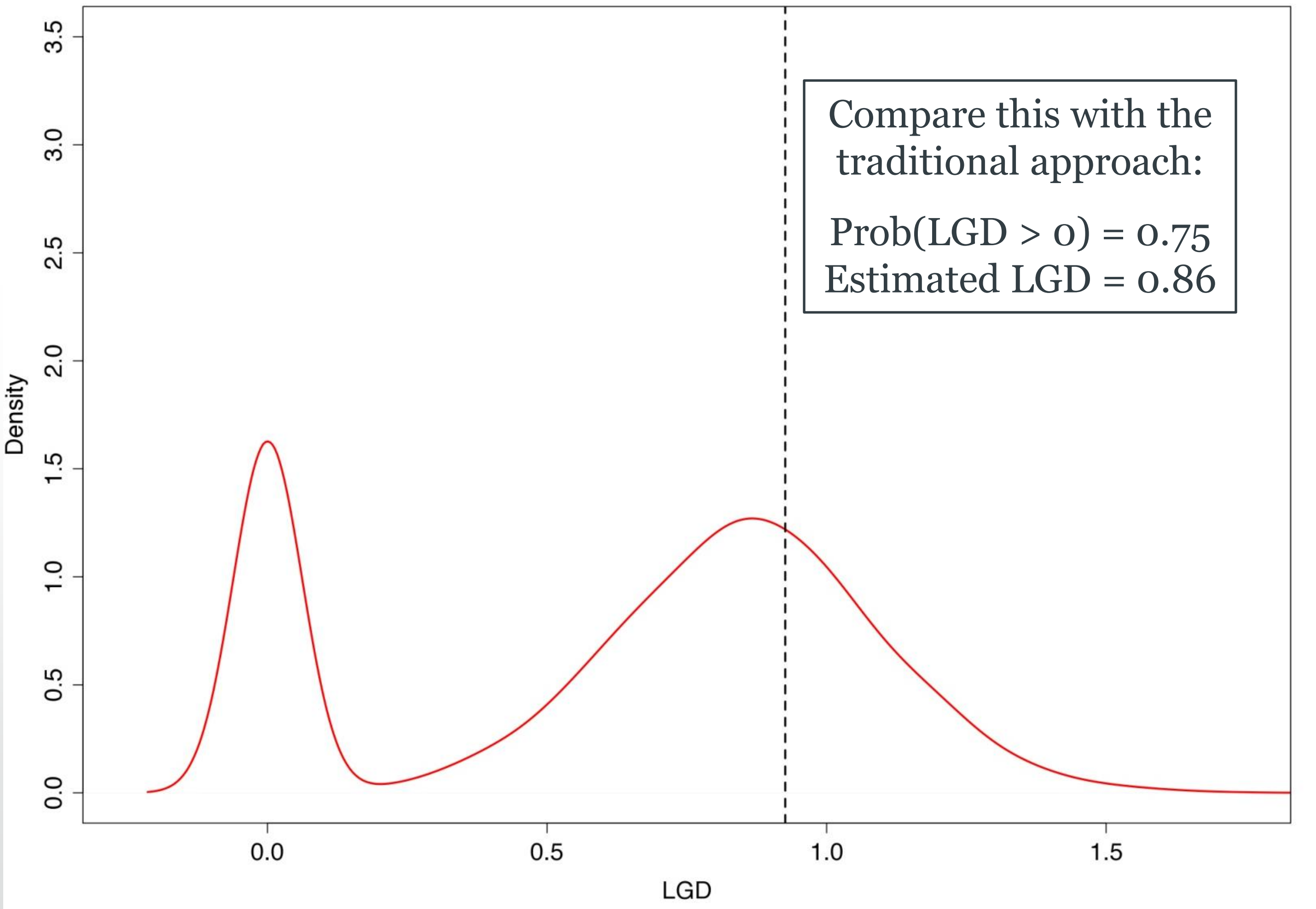
# Individual distributions: example 3



# Conclusions

- The (posterior) means of model performance measures and parameter estimates are very similar in both approaches: Bayesian and traditional (using a random cut-off)

- Advantages of the Bayesian approach:
  - More coherent approach: a single hierarchical model instead of two separate ones
  - Much better description of uncertainty (distributions!) than in the traditional approach

# Conclusions

- In the Bayesian approach, there is an individual distribution of LGD for each customer (rather than just a point estimate as in the traditional approach)
- The individual distributions of LGD can be used to estimate:
  - the expected LGD → expected loss
  - the downturn LGD
- What does it mean for the user?
  - More information
  - Need to make choices and decisions

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# References


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- Loterman, G., Brown, I., Martens, D., Mues, C., Baesens, B. (2009) Benchmarking State-Of-The-Art Regression Algorithms For Loss Given Default Modelling, CRC 2009.
- Matuszyk, A., Mues, C., Thomas, L.C. (2010) Modelling LGD for unsecured personal loans: decision tree approach, „Journal of the Operational Research Society", 61, 393-398.

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# Thank you!