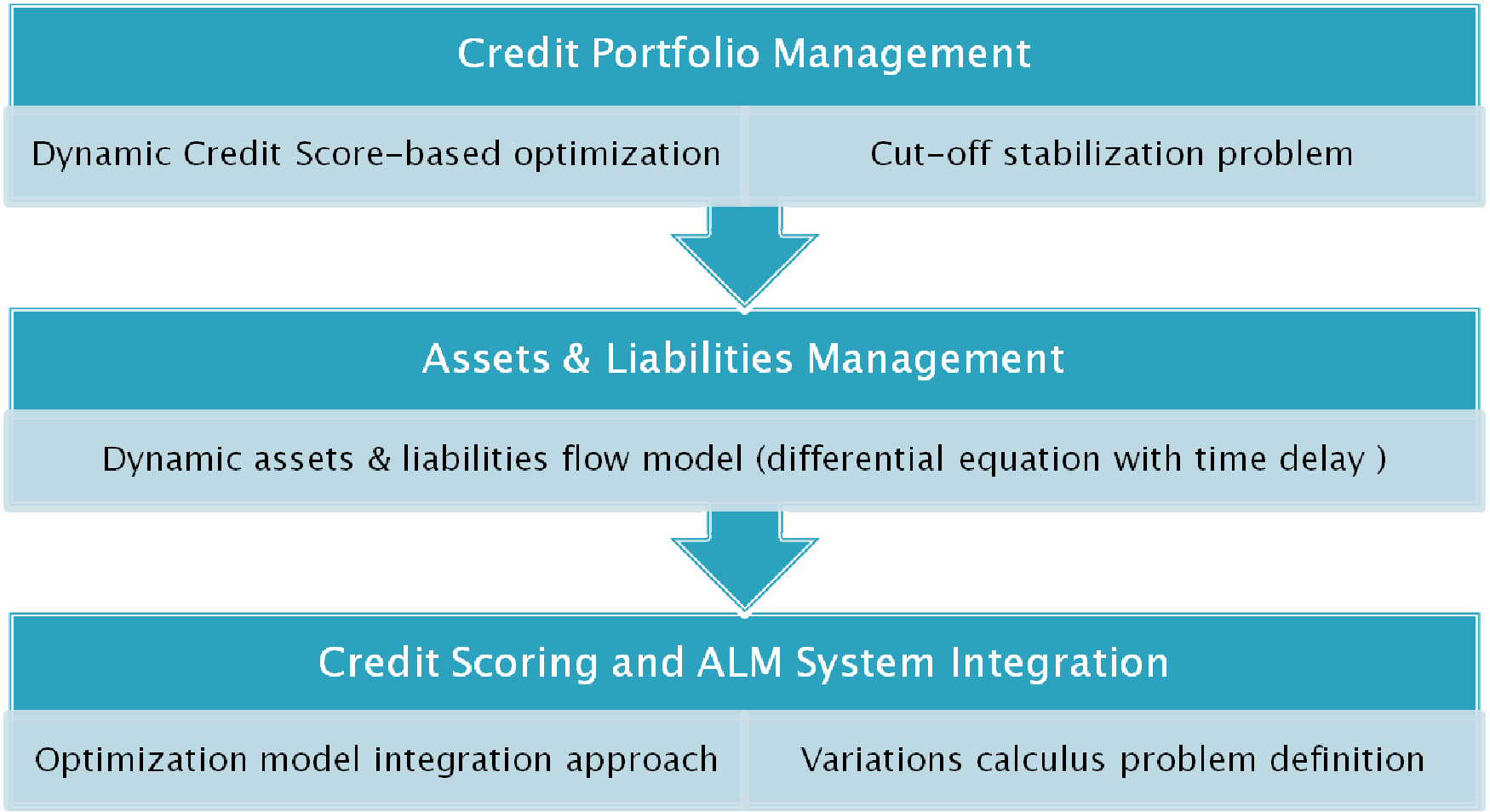
Modelling Optimal Dynamic Control of Consumer Loans Portfolio: Credit Scoring and ALM Systems integration

Osipenko Denis,
Retail Risk Management,
'Raiffeisen Bank Aval' JSC, Kiev, Ukraine

Credit Scoring and Credit Control XII conference
August 24-26, 2011

Table of contents

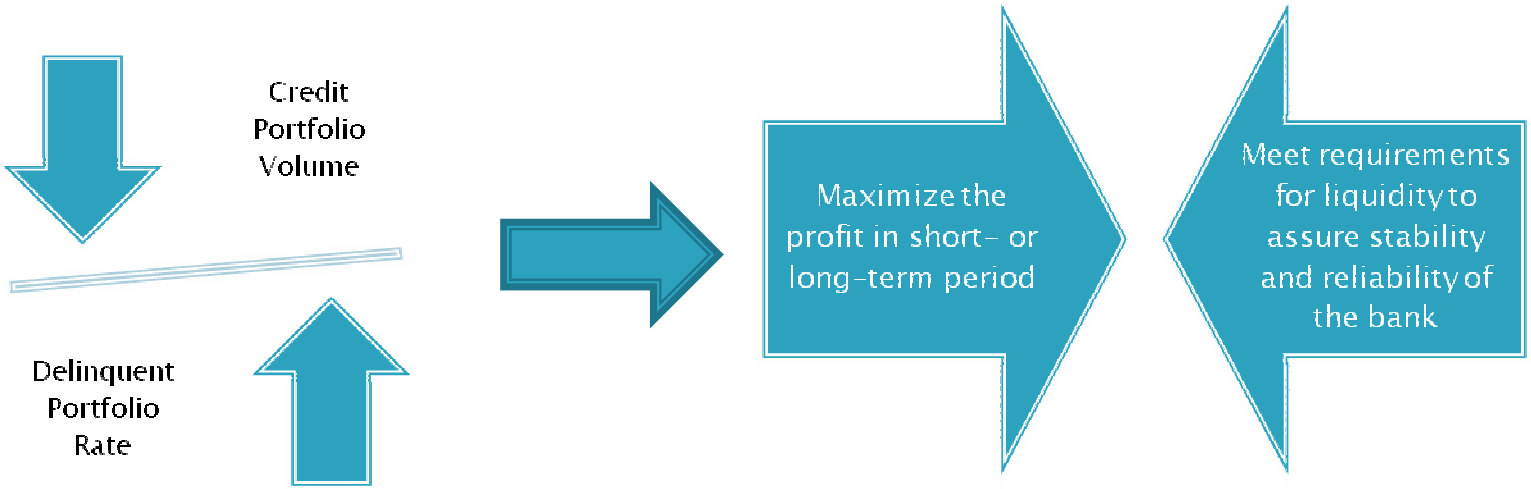


Background

- By the reason of risks interpenetration:

  Credit Risk => Losses => Balance Liquidity Risk

- Banking management should consider all aspects of banking activity in the decision making process.

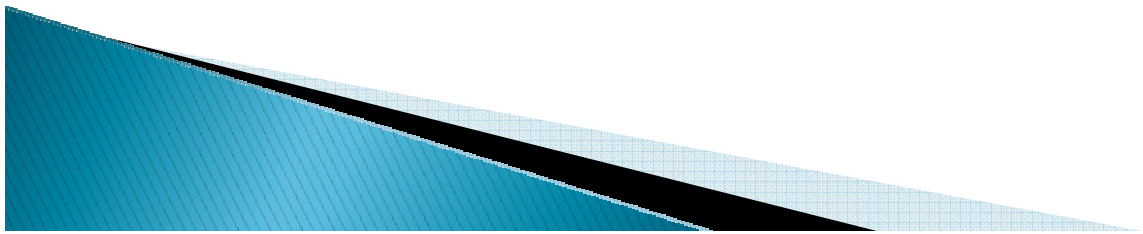


Background

- *Credit scoring systems* are the key tools of modern risk management, particularly, in retail banking. However, credit scoring in portfolio management should not be applied in isolation from the entire set of bank's business processes.

- One of the solutions to the problem of consistent approach to Risk Management in retail banking is integration of *Credit Scoring Models* and *Common Dynamic Flow Models* of the bank, that possesses the main features of *Assets & Liabilities Management System*.

- As a result, the problem of *optimal control for maximization of profitability* in a given time period can be solved.



Background

- Approaches to solving optimal control problem for credit portfolio:
  - optimal cut-offs in portfolio level model with predetermined equity capital (Lyn C. Thomas, 2009);
  - profit maximization decisions and default-based scores (Lyn C. Thomas, 2002);
  - profit-based cut-offs setting - profit modelling approach (Raymond Anderson, 2007).

- The fundamental aspects of Assets & Liabilities Management System:
  - risk management integration model and liabilities management are defined by Joseph F.Sinkey, Jr. (2007);
  - Sealy (1977) proposed the common approach to the modelling of banking activities as the financial firm activity;
  - This approach was reviewed and improved by N.Yegorova, A.Smulov (2002) to obtain simulation model of the commercial bank.

Assumptions and Limitations

- In the current context by the **Assets and Liabilities Management System** we understand mathematical model of banking activities that contains only such financial instruments as credits and deposits

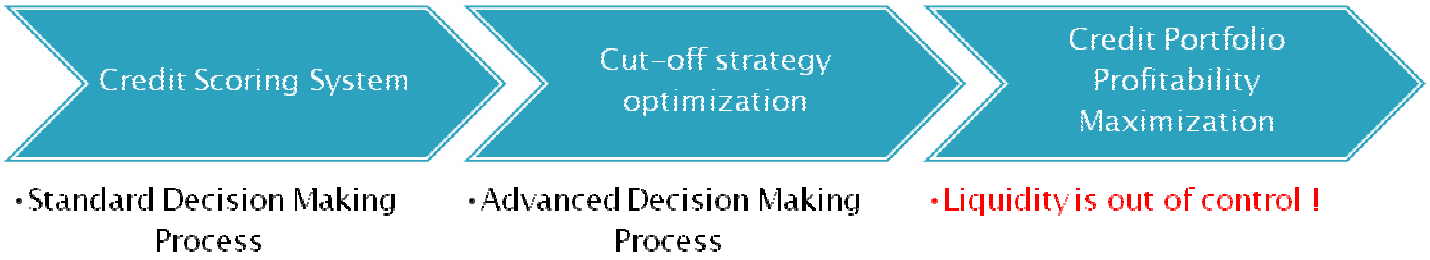
- In the current context by the **Credit Scoring System** we understand Application Scoring System only, although it's possible to extend the approach to the *full cycle credit scoring system*

- **Common Dynamic Flow Models** based on system of differential equations with delays. Because of the analytical solution for these types of equations is complicated and possible for limited set of tasks, we propose to shape the problem with mentioned approach, but to use the calculus of approximations

- Deposit supply is stable and has market average rate

Problem definition (1 / 2)

- **Profitability maximization problem at Credit Portfolio Level**



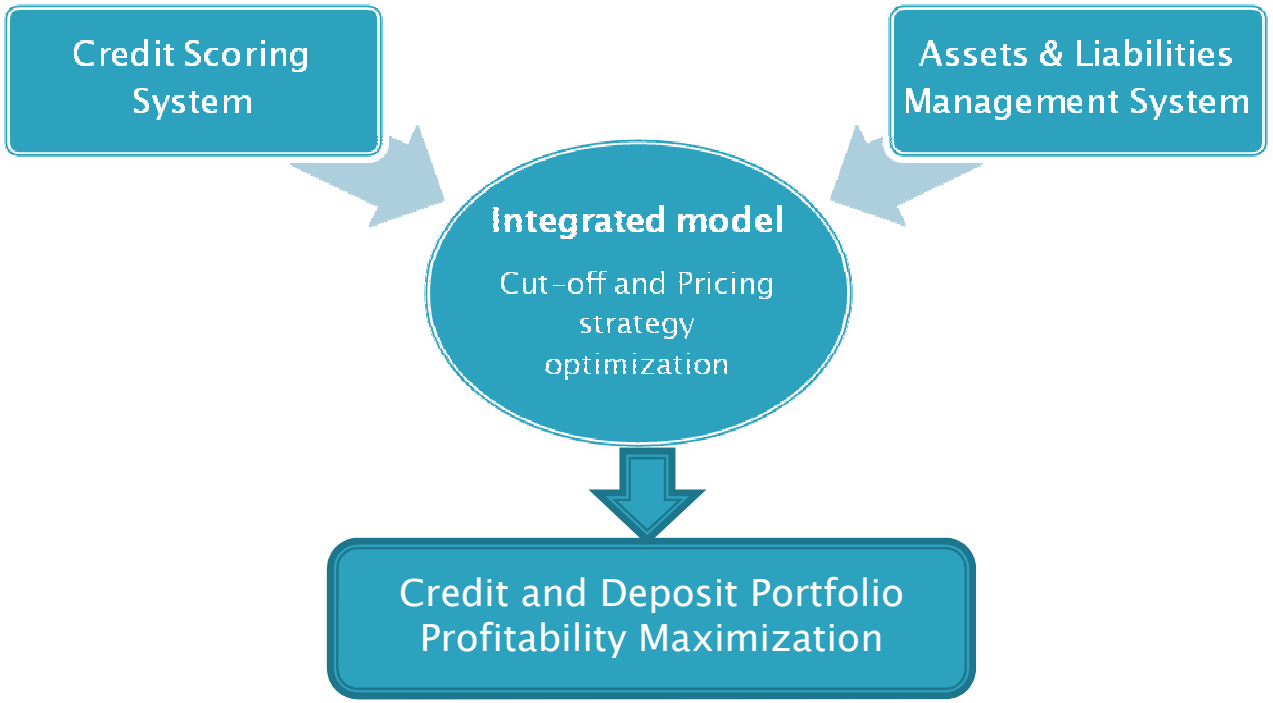
  - *Liquidity risk: funding gap can arise in future periods*

- **Profitability maximization problem at Credit and Deposit Portfolio Level**

  - *Credit risk: bad loans rate can increase in future periods*

Problem definition (2/2)

- **Profitability maximization problem at Credit and Deposit Portfolio Level using Credit Scoring System**



- *The liquidity control and credit risk management problems are solved in complex in the framework of integrated model*

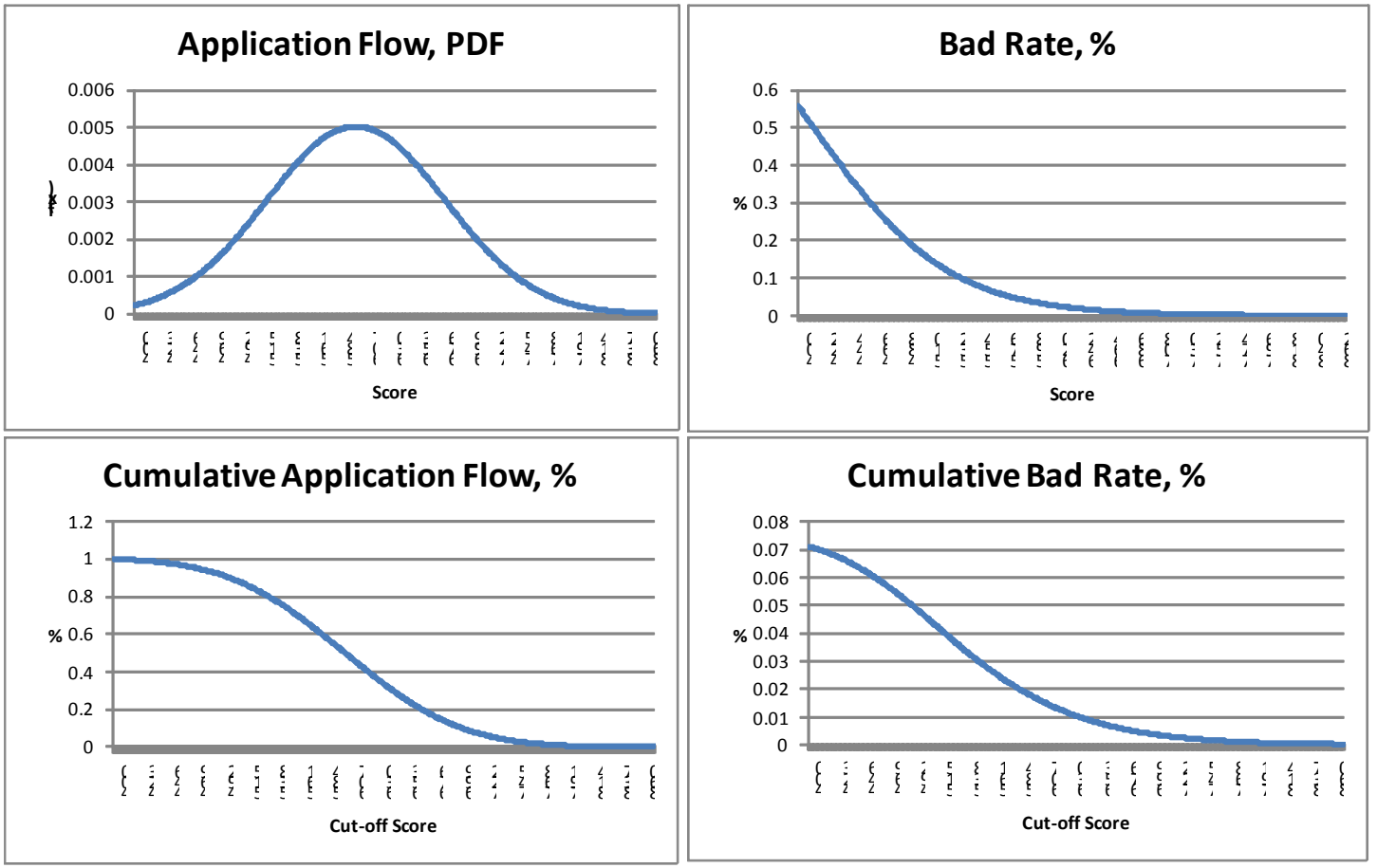
Credit Portfolio Profitability Model (1 / 4)

- The key elements which are used in the proposal for Credit Portfolio Automatic Control System are Acceptance Rate function, Bad Rate and Cumulative Bad Rate functions (Probability of Default), and Profitability functional.
- For example, *the cumulative Bad Rate distribution* function is derived from the population distribution function (PDF) and the Bad Rate distribution function for the application flow with cut-off score S:

$$BR(S) = \frac{\int_{S}^{S\max} ar(s) \cdot br(s) ds}{\int_{S}^{S\max} ar(s) ds}$$

- where
  - *ar(s)* is a population distribution function;
  - *br(s)* is a function of the bad loans share in the score *s;*
  - *S* is cut-off score.

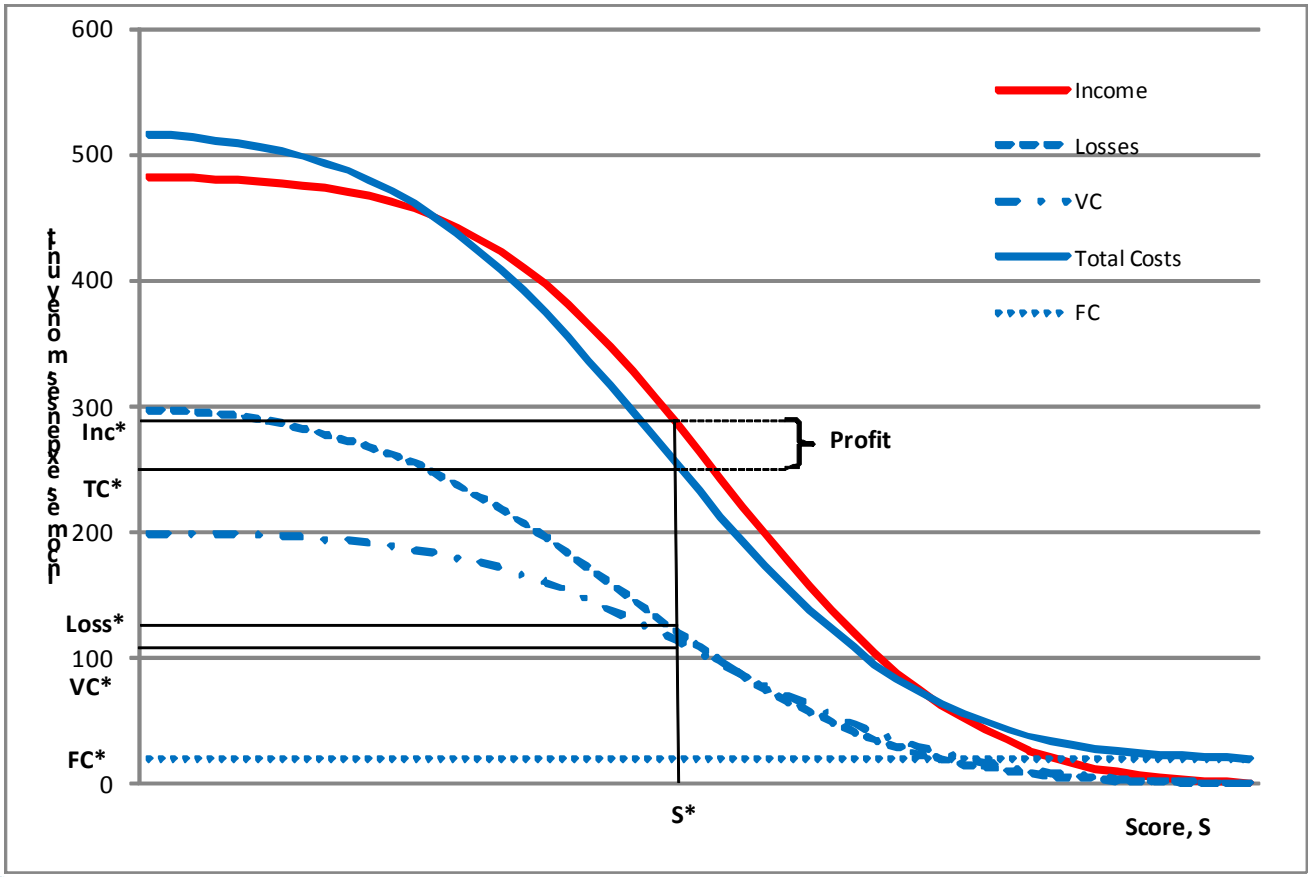
Credit Portfolio Profitability Model (2/4)



- Cumulative Application Flow and Bad Rate distributions by cut-off score are the core of optimization model

Credit Portfolio Profitability Model (3/4)

- Dependence of Income and Expenses on cut-off score on Portfolio level
- Total costs = Fixed Cost + Variable Cost + Losses due to defaults



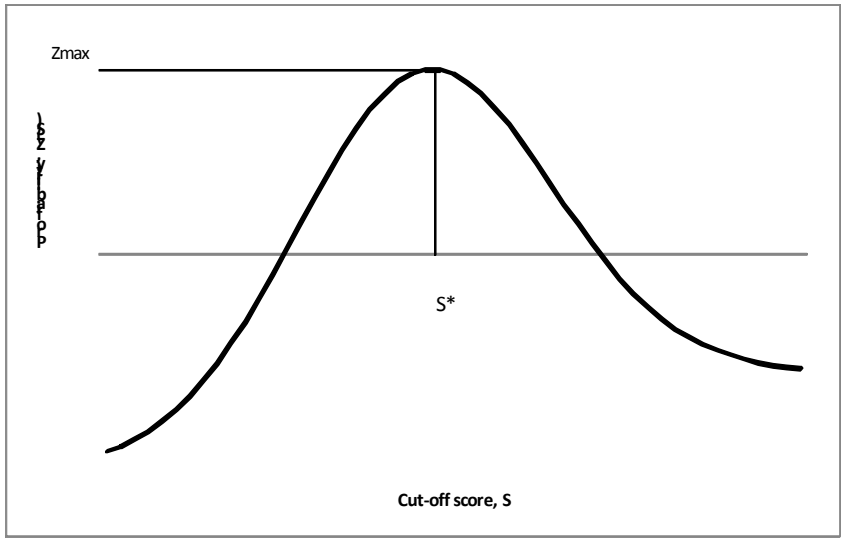
Credit Portfolio Profitability Model (4/4)

- Dynamic maximization criteria *Z* of the credit portfolio profitability is defined by the following functional:

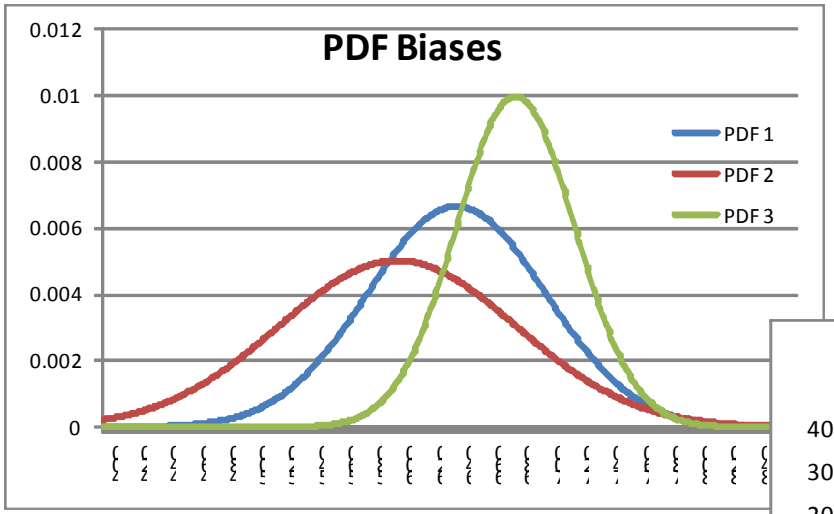
$$Z(S;t0,t1)=\int_{t0}^{t1} AR(t;S(t))\cdot AF(t)\cdot\begin{pmatrix} APR(t)\cdot(1-BR(S(t))\cdot LGD(t))\cdot AvgL(t)- \\ -BR(S(t))\cdot LGD(t)\cdot AvgLBad(t)- \\ -Exp(t)\cdot AvgL(t) \end{pmatrix} dt \Rightarrow \max$$

where

- *t0* start time
- *t1* end time
- *S* cut-off score
- AR acceptance rate defined by cumulative population distribution function
- *APR* Annual Percentage Rate
- *AF* Application Flow function
- *AvgL* average size of granted loan
- *AvgLBad* average size of bad loan in portfolio
- *Exps* total expenses per 1 m.u.
- *LGD* Loss Given Default

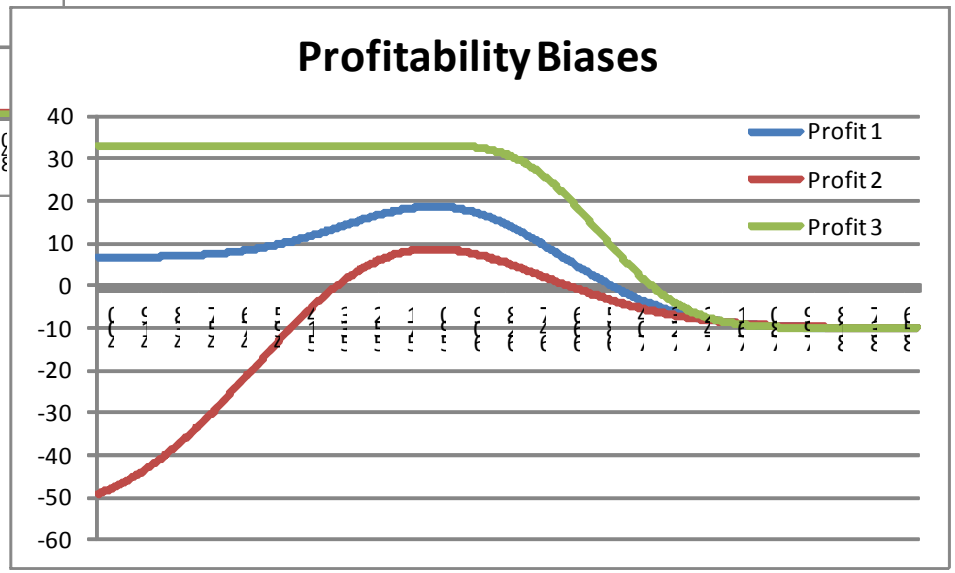


Distributions Biases



At different period of time the population distribution function changes mean and standard deviation

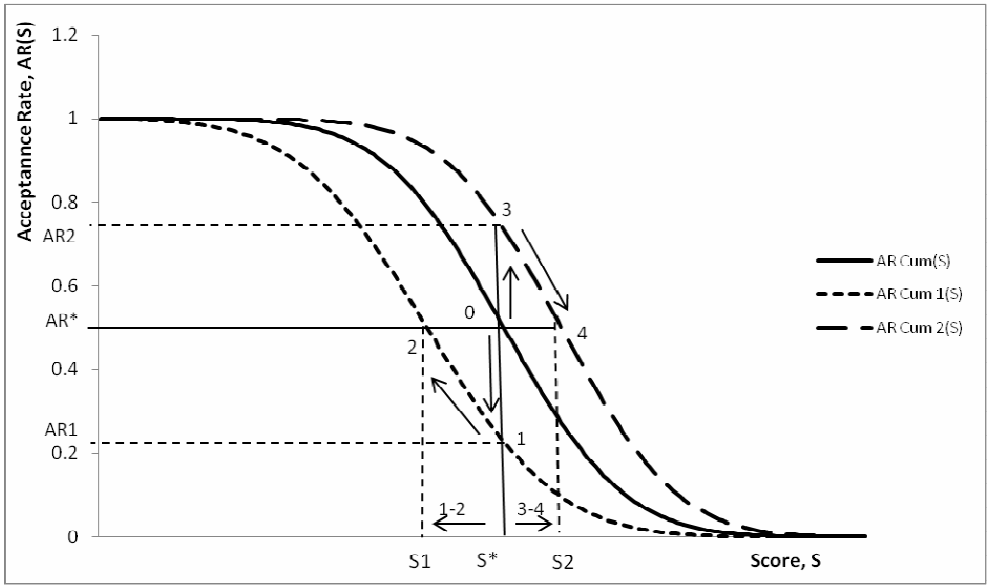
As result the Profitability function changes its shape



- Population Distribution Function shifts and as a result Profitability Functional biases (Bad Rate distribution is stable by calibrated score)

Cut-off stabilization problem

- Because of the economic environment fluctuations, problem of the dynamic control over the credit portfolio characteristics needs to be solved.
- The displacements of the point (S*, AR*) from position 0 to positions 2 and 4 results from shifts of the Cumulative Acceptance Rate curve to AR Cum1 and the AR Cum2 respectively.



- To stabilize the system performance, classic control theory techniques can be used, *e.g.* feedback controller with the gain factors. For example, the cut-off score at time t+1 can be defined as:

$$S(t+1) = \left(AR(t; S(t)) - AR^*(t)\right)\alpha + S^*$$

- Thus, adjusted cut-off score depends on current Acceptance Rate function *AR(t; S(t))*, the target Acceptance Rate *AR*(t)*, initial optimal cut-off *S**, and gain factor α.

Assets & Liabilities Management System modelling (1 / 4)

- Let's set the A&LM System as differential equations with delays.
- The loans debit turnover is the demands function and is defined by application flow as

$$L_{Dt}(U_L, F_L; t) = AF(U_L(t), F_L(t))$$

- The loans credit turnover is defined as

$$L_{Cr}(U_L, F_L, S; t) = \int_0^m \left((L_{Dt}(U_L, F_{L,}; t-\tau)) \cdot \left(1 + \frac{i_L(t-\tau)}{rl_{t-\tau}}\right) \cdot (1-BR) \right) d\tau$$

- where
- $i_{L(t-\tau)}$ is credit interest rates at the moment of time t−τ;
- m is maximum term of credit;
- τ is time lag (delay) which reflects the term of the loan;

- The loans credit turnover function includes the risk component, which is defined as (1−BR) and corresponds to the payback credit amount.

Assets & Liabilities Management System modelling (2/4)

- Hereinafter we consider one of the approaches to Assets & Liabilities Management System modelling. The dynamic of the system status X (current funds of the bank) based on the accounting balance is defined by differential equation with time delay τ at the moment of time t:

Demand functions:

$D_{Cr}(U_D, F_D)$ – deposits credit turnover (inflow);

$D_{Dt}(U_D, F_D)$ – deposits debit turnover (outflow);

$L_{Dt}(U_L, F_L)$ – loans debit turnover (inflow);

$L_{Cr}(U_L, F_L)$ – loans credit turnover (outflow)

External Factors:

F_D, F_L are the vectors of external factors impact (GDP, marginal savings index, average market indexes and rates, etc)

Controls:

U_D, U_L are vectors of control for deposits and credits (interest rates, marketing costs, branch network development etc.)

Expenses functions:

$IB(t)$ is the interbank turnover (as a regulator of overliquidity in the model);

$C(L_{Dt}, D_{Cr})$ are funding, operational and administrative costs functions

$$\dot{X}(t) = D_{Cr}(U_D, F_D; t) - \int_0^n \left(D_{Dt}(U_D, F_D; t-\tau) \cdot \left(1 + \frac{i_D(t-\tau)}{rd_\tau} \right) \right) d\tau + L_{Cr}(U_L, F_L; t) -$$

$$- L_{Dt}(U_L, F_L; t) + IB(t) - C(L_{Dt}(t), D_{Cr}(t))$$

Assets & Liabilities Management System modelling (3/4)

- Let's introduce new control into the differential equation – cut-off *S*. The volume of granting loans depends on acceptance rate as well as on pricing and advertising controls.

Demand functions:

$D_{Cr}(U_D, F_D)$ – deposits credit turnover (inflow);

$D_{Dt}(U_D, F_D)$ – deposits debit turnover (outflow);

$L_{Dt}(U_L, F_L)$ – loans debit turnover (inflow);

$L_{Cr}(U_L, F_L)$ – loans credit turnover

External Factors:

F_D, F_L are the vectors of external factors impact (GDP, marginal savings index, average market indexes and rates, etc)

Controls:

U_D, U_L are the vectors of control for deposits and credits (interest rates, marketing costs, branch network development etc.);

S – cut-off score

Expenses functions:

IB(t) is an interbank turnover (as a regulator of overliquidity in the model);

$C(L_{Dt}, D_{Cr})$ are the funding, operational and administrative costs functions

$$\dot{X}(t) = D_{Cr}(U_D, F_D; t) - \int_0^n \left(D_{Dt}(U_D, F_D; t-\tau) \cdot \left(1 + \frac{i_D(t-\tau)}{rd_\tau} \right) \right) d\tau + L_{Cr}(U_L, F_L, S; t) -$$

$$- L_{Dt}(U_L, F_L, S; t) + IB(t) - C(L_{Dt}(t), D_{Cr}(t))$$

Assets & Liabilities Management System modelling (4/4)

- The demand function is changed by new control introduction.
- The loans debit turnover is the demands function and is defined as

$$L_{Dt}(U_L, F_L, S; t) = AF(U_L(t), F_L(t)) \cdot \int_{S(t)}^{S\max} ar(s) ds$$

- The loans credit turnover is defined as

$$L_{Cr}(U_L, F_L, S; t) = \int_0^m \left((L_{Dt}(U_L, F_{L,} S; t-\tau)) \cdot \left(1 + \frac{i_L(t-\tau)}{rl_{t-\tau}}\right) \cdot \left(1 - \frac{\int_{S(t)}^{S\max} ar(s) \cdot br(s) ds}{\int_{S(t)}^{S\max} ar(s) ds}\right) \right) d\tau$$

where

- $i_{D(t-\tau)}$, $i_{L(t-\tau)}$ are deposit and credit interest rates at the moment of time t−τ;
- S is cut-off score;
- m is maximum term of credit;
- τ is time lag (delay) which reflects the term of the loan.

The Expected Losses are included into the L_{Cr} function.

Optimization model integration approach

- As a result of the research, the **integral** Credit Portfolio Dynamic Model and Assets & Liabilities Management System Model were developed and the ***gross profit functional*** was introduced, which is defined as

$$P(t1)=\int_{t0}^{t1}\left(\begin{array}{l}\int_{0}^{m}\left(\left(L_{Cr}\left(U_L,F_L,S;t-\tau\right)\right)\cdot\left(\frac{i_L(t-\tau)}{rl_\tau}\right)\right)d\tau- \\ -\int_{0}^{n}\left(D_{Dt}\left(U_D,F_D;t-\tau\right)\cdot\left(\frac{i_D(t-\tau)}{rd_\tau}\right)\right)d\tau+IB(t)r_{IB}(t)-C(L(t),D(t))\end{array}\right)dt\Rightarrow\max$$

where

  - L_{Cr} and D_{Dt} are the loans credit turnover and deposits debit turnover,
  - i_L and i_D are the loans and deposits interest rates respectively,
  - τ is a lag (delay), equal to the term of the loan;
  - S is a cut-off score;
  - *C(L(t),D(t))* is a cost function;
  - U_L, U_D are the controls;
  - F_L, F_D are the external factors.

- Solving the maximization problem based on this functional, is the matter of variations calculus, though the functional is too complicated for the analytical techniques.

Summary: Business Implementation

- What is the best strategy to increase portfolio profitability?
  - Increase interest rate – margin growth (if demand is inelastic)
  - Decrease interest rate – application flow growth (if demand is elastic)
  - Advertising campaign/actions – application flow growth
- All this strategies can bias the population distribution.
- What is the decision and consequences?
  - Increase acceptance rate – increase/fixed bad rate;
  - Fixed acceptance rate – increase/decrease bad rate (it depends on distribution shift)
- Scenario analysis and simulation can help to answer this question and find the most optimal strategy of controls.
- If you set cut-offs then review the future funds availability.
- Loss now – gain in future!

Summary: Further Steps

- The ALM system, based on the **dynamic models**, in this particular case employing delayed differential equations for the bank's activities simulation, is one of the most suitable approaches to effective risk-based management.
- *There are **three main directions*** for application and further research:
  - Investigation of the ***macroeconomics factors'*** impact on the retail banking and development of appropriate control strategy for the portfolio on decision making process level; credit portfolio and bank liquidity ***stress-testing*** and the ***anticrisis strategies*** development (controls and decisions);
  - ***Optimal dynamic control modelling*** based on *full cycle scoring systems* and *dynamic flow models* for the multi-target management problems of:
    - profit maximization in the short- and long-term period,
    - retention of the market share and/or portfolio quality on the defined level;
    - challenger strategy testing
  - ***Theoretical and conceptual approach to the integration*** of:
    - credit portfolio control model, based on *full cycle scoring systems* (response, application, behavioural, collection, attrition etc.),
    - *ALM system,* based on dynamic flow model, at the entity level.
- **Simulation process** is a productive way to find the optimal solution to complex maximization problem with system of limitations.

Thank you for your attention!

- Osipenko Denis,
  Retail Risk Management, 'Raiffeisen Bank Aval' JSC, Kiev, Ukraine

- E-mail:
  - denys.osipenko@aval.ua
  - denis.osipenko@gmail.com