Adverse Selection and Non-Take Inference with Coherent Risk and Response Scoring[1]

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Abstract

The authors offer a mathematical model for adverse selection based on borrower preferences for offers and the default (Bads) or non-default (Goods) status of booked accounts. We define the conditions for applicant risk and response to offers when there is no adverse selection (NAS); this definition allows us to estimate differential response rates for Good and Bad subpopulations and the Good/Bad odds for Takes, Non-Takes and Accepts. Observed performance from different price-risk offer segments allows us to compare price-driven risk elasticity and price-driven response elasticity in the presence of Good or Bad adverse selections; a special case applies when the borrower's capacity to repay is not an issue. We offer limited experimental results for different price-risk segments where risk and response scores are used to estimate changing borrower preferences. The critical role of Non-Take inference is presented.

Introduction

The concept of adverse selection arises in a number of economic and risk contexts where there is a belief but, unfortunately, limited experimental evidence in retail credit, that the borrowing consumer may have more relevant information about his or her own future default risk than is available to the lender at the time a loan offer is made. The presence of asymmetric or hidden information to participants in borrowing/lending negotiations is often used to explain why rational borrowers and lenders have different preferences that may not be recognized or properly assessed by the other party. Because a borrower may have personal information about conditions that make him or her more risky to the lender, he or she may not want to reveal that information and thereby jeopardize a favorable offer.

Ackerloff (1970) published his well-known paper on the market for lemons in the sale of second-hand autos where the seller knows more about the quality and condition of the automobile than does the prospective buyer. Many extensions and generalizations of games between buyer and seller with asymmetric information are described in Rasmussen, (1994). In credit risk, the notion that a "Bad" borrower is more likely to respond to a high-priced offer than a "Good" borrower may be justified on theoretical grounds or the presence of asymmetric information. Altman et al. (1998) define *Adverse Selection* (with reference to insurance in health plans) as "the tendency for sick (healthy) people to join plans at high (low) cost". The underlying thought is that of a subpopulation of individuals seeking insurance coverage; they "move into or out of" generous or overly restrictive plans. Ausubel, (1999), defines *Adverse Selection* in terms of the inferior risk characteristics of the pool of customers (borrowers) who accept an offer by comparing them with the customers who accept a "better" offer. Edelberg [2003] uses a two-period model to study the interaction between adverse selection and moral hazard and finds the counter-intuitive result that higher-risk borrowers often pay lower loan rates than lower-risk borrowers. Phillips (2009) defines *Direct Adverse Selection* as "A change in the distribution of risk score among funded loans as price is changed" and *Indirect Adverse Selection* as "The change in the loss behavior of borrowers with the same credit score as price is changed". Phillips and Raffard, (2010), argue that Adverse Selection exists if the derivative of the default rate (Bad rate) with respect to loan rate (price) is non-decreasing in the loan rate. As best we understand, none of these definitions define or estimate the number of adverse selections in the Take population although Altman makes it clear that, in the context of retail credit, increased counts of Goods or Bads may be the result of subsets of individuals being attracted to favorable (low price) or unfavorable (high-price) loans. Most of the analysis reported by Agarwal et al. (2010) compares the risk scores of borrowers who book lender offers with those that do not. Their results are based on the availability of proprietary data so that it has been possible to directly estimate the risk of borrowers who do not take the offers of the lender of interest but do accept alternative offers from other lenders in the marketplace. These different ways of describing adverse selection have the common feature that individuals who purchase insurance coverage or who make loans or purchase credit, have special circumstances or knowledge about their own situation that may not be available to the insurer/lender.

[1] Earlier version of this manuscript submitted February 4, 2011 to Journal of The Operational Research Society, U. K.

Financial experts have not always agreed on ways to disentangle adverse selections from ordinary borrower preferences or to incorporate adverse selection in ways that assist decision-makers in acquisition and management of credit risk and mortgage loans. In our limited experience many of the claims for and measurement of adverse selection are not well-founded for at least two reasons: scores used to make risk estimates seldom use input data or performance outcomes at the time price offers are made by the lender, that are relevant to prediction of adverse selection. Secondly, adverse selection is not formulated in terms of a (random) number of individuals that exhibit departures from cases where there is no adverse selection. If a lender offers a borrower a loan rate based on *delinquency scores* and then observes that the *number of defaults* in later time periods differs substantially from delinquency score predictions, adverse selection may not be the explanation for increased default rates as the increased rate may be due to use of an inappropriate score, a poorly calibrated prediction or a lack of clarity in distinguishing adverse selection from the normal aversion of most borrowers (both Good and Bad) to higher prices.

We denote the occurrence of a default as a Bad (B) and the occurrence of non-default as a Good (G). Goods and Bads are often used to denote other measures of performance such as late payment or fraud. We also denote the "acceptance of a lender's offer by a borrower" as the outcome of a random event with two possible outcomes, one of which is that the borrower "Takes" (T) the offer; the other possible outcome is "Not Take" (N). Theory, limited anecdotal and experimental evidence suggest that a Bad (B) borrower is more likely to Take high-priced offers than a G. B corresponds to default, a trapped state, not simply late payment, which may be a transient state. One measure that is often used in the literature to indicate the extent of adverse selection is the disparity between the cumulative distributions of risk (not necessarily default) scores for Take and Non-Take groups with low and high priced offers (See **Fig. 1**). We believe this measure is unsatisfactory, as one not cannot easily relate the differences in the score distributions to the risk performance or economic preferences of individual borrowers. We know of no study where there is an attempt to measure the count of adverse selections or compare the composition of booked borrowers with those that do not book.

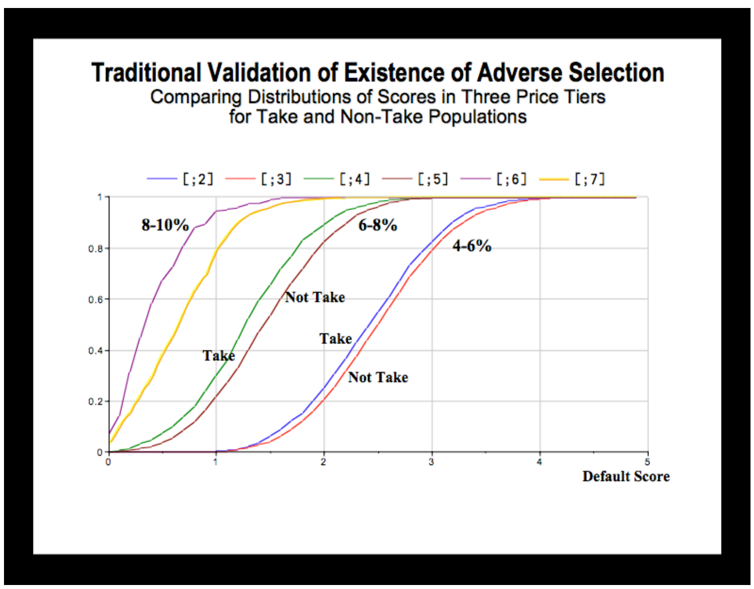

Figure 1: Comparison of Score Distributions for Takes and Non-Takes

An Example of Bad Adverse Selection

Table 1 illustrates an example that occurs when an offer is made to a sub-population of 1500 risk-acceptable borrowers. Cell entries are either observed counts or expectations derived from risk scores. The columns correspond to Good/Bad (Non-Default/Default) outcomes with the final column representing the observed number of Takes and Non-Takes. By contrast, the bottom row corresponds to borrowers who are acceptable for loan offers and the middle rows correspond to borrower Takes and Non-Takes. The lightly shaded grey cells in the second and third columns contain the expected number of counts based on estimates of risk assigned to each borrower. Typically, scores are calculated and available before responses to offers are known and are based on relevant (predictive) characteristics of individual borrowers. The PopOdds for the predicted number of Goods and Bads among Takes, Non-Takes and Accepts in the second and third column is 7.5:1. Cells with dark shades of grey correspond to actual counts of responses and non-responses as well as Good/Bad performance of those who book. It is worth noting that the observed counts of Bads in the Take group is approximately 40% higher than the predicted number; this results in a lower Good/Bad odds of 5:1 for the Take sub-population. It appears to be an indication of Bad adverse selection even when there is no comparison with the change in counts when higher or lower-priced offers are made to borrowers with similar risk assessments. Performance of Non-Takes who might have accepted offers with other lenders is generally unknown to the lender but it is possible that the Good/Bad count for Non-Takes, if they could be observed, would be different from that of the Take subpopulation. Although it is not obvious how one should estimate cell counts in the Non-Take and Accepts row after the Good and Bad counts among Takes have been observed, once the uncertain count in the number of Bads among Non-Takes is specified as Z_1, it is

	E[#Goods]	E[#Bads]	Observed #Goods	Observed #Bads	Totals
Takes (T)	264.7	35.3	250	50	300
Non-Takes (N)	1058.8	141.2	1200- Z_1	Z_1	1200
Total Accepts (A)	1323.5	176.5	1450- Z_1	50+ Z_1	1500

Table 1: Predicted and Observed Cell Counts for Goods/Bads, Takes/Not-Takes

easy to maintain conservation in all unshaded cells by expressing counts in terms of the observed responses and Good/Bad counts among Takes. In this case the odds of Good in the sub-populations of Take (T), Non-Take (N) and Accepts (A) are given by

$$o_T = \frac{250}{50} = 5, \qquad o_N = \frac{1200 - Z_1}{Z_1}, \qquad o_A = \frac{1450 - Z_1}{50 + Z_1}.$$

If Z_1 were equal to 200 then the odds of all three groups would be 5:1, i.e. no adverse selection. Smaller values of Z_1 lead to larger values of Good/Bad odds in the Accept population, and greater disparity between the odds of the Take and Non-Take populations.

Definition of Adverse Selection and NAS

Our characterization of the prediction-decision problem for assessing and making loan offers is shown in the extensive form influence diagrams of **Figs 2(a, b).** The origination of loans is a two-stage decision problem where a baseline risk score provides the lender a risk assessment of those potential borrowers that are deemed to be acceptable for offers. A second, and later decision by the lender, possibly as

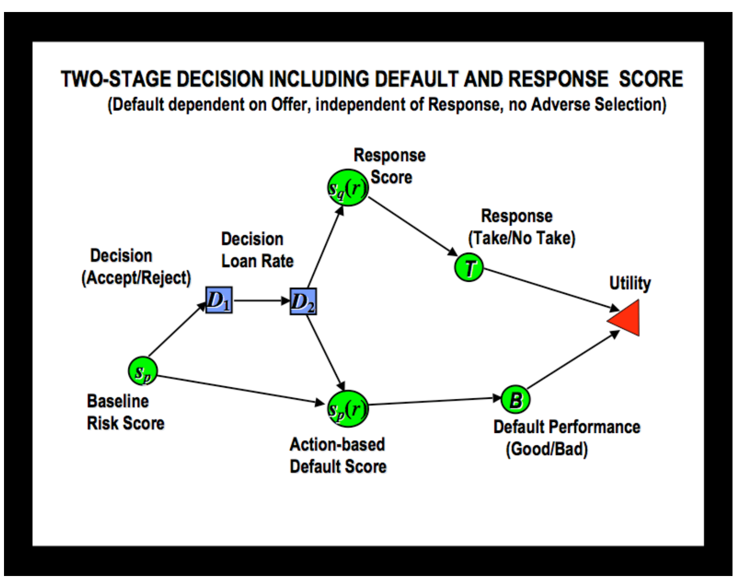

Fig. 2(a) - Offer Decisions, Responses and Defaults without Adverse Selection

the result of an optimization procedure based on business objectives and tradeoffs, matches the individual borrower to an offer; at that point in time the lender may also use an action-based default score as well as an action-based response score that predict borrower preferences and likely response and risk outcomes.

We assume that, in principle, three scores are available, two of which may be offer-dependent. In many cases where lenders use only one baseline risk score, the action-based score node in **Fig. 2(a)** is absent and there is a single arc connecting the baseline score node to the Default node. The Good/Bad outcomes of performance are conditionally independent of borrower response (Take/Not-Take), even though the risk and response scores may each depend on the offer rate, *r*. To mimic the conditional independence structure for loan offers in this figure think of a coin-tossing experiment in which one number and one outcome are stamped on each of two coins associated with a single borrower: the numbers are the prior probabilities of a Take and Bad, respectively. Independent tosses of each biased coin are made and the observed outcomes T or N and G or B are then stamped on the respective coins. The G or B designation identified on any coin associated with an N on the paired coin is covered with masking tape so that an external observer, say the lender, cannot discern the default/non-default status of Non-Takes. In the case of T coins, the G or B stamp on the paired coin is revealed to the lender.

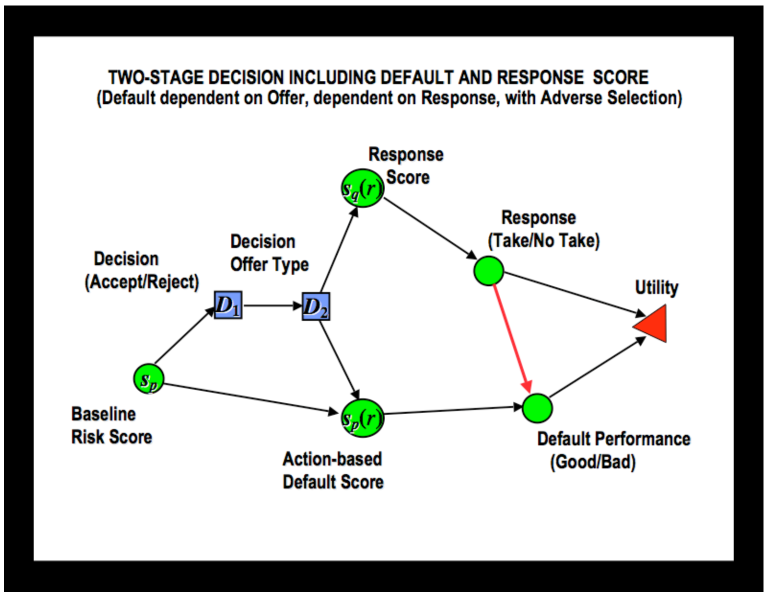

Fig. 2b - Offer Decisions, Responses and Defaults with Adverse Selection

In **Fig. 2(b)** the Good/Bad outcomes of performance depend on Take/Not Take outcomes of borrower response, including the special case where the only score used is the baseline score mentioned above. Most risk control departments are concerned about the implications of higher risk levels of those who respond to offers while marketing departments focus on response rates; thus there is a natural tradeoff and tension that occurs in the decision-making process. Limited experimental evidence suggests that the Bad rate for those who Take an offer is higher than it would be for those who do not; alternatively, the odds of a Good in the Take sub-population is lower than it would be for the Non-Take sub-population in the larger population of Accepts. Although there is general agreement about the existence of adverse selection there does not appear to be agreement on a precise definition on how it should be measured. To mimic the conditional dependence structure in **Fig. 2(b),** think of the earlier coin-tossing experiment having the same two biased coins associated with a single borrower. The first coin toss determines whether we get the outcome T or N. If it is a T the probability on the second coin is changed and then tossed with the result of a G or B outcome revealed to the lender. In the case of an N, the probability on the second coin is also changed; it is tossed but the G or B outcome is **not revealed** to the lender. The G or B outcome on the paired coin that results from a T outcome is always revealed to the lender.

In the discussions that follow we assume that the terms of the loan are entirely captured by the loan rate, *r* or the premium over the risk-free rate although the ideas put forth in this paper, can be generalized to the case where **r** is itself a vector. We use **x,** a vector, to denote behavioral, financial, demographic characteristics and other relevant data to define the unconditional probability of a Bad in the Accept population as

$$p(B \mid \mathbf{x}, r) = \Pr\{Bad \mid data\ \mathbf{x},\ offer\ r\} \qquad \mathbf{x} \in \mathcal{X},\ r \in \mathcal{R} \qquad \textbf{(1)}$$

where $\mathbf{x} \in \mathcal{X},\ r \in \mathcal{R}$ denotes appropriate sets of predictive and loan offer rate data. For the prospective borrower the appropriate data could easily include privileged or private information that is not in $\mathcal{X}$. The probability of a Bad conditional on a Take is

$$p(B \mid T, \mathbf{x}, r) = \Pr\{Bad \mid Take,\ data\ \mathbf{x},\ offer\ r\} \qquad \textbf{(2)}$$

To obtain the marginal probability in **(1)**, Bads among non-Takes (N) must also be considered because

$$p(B \mid \mathbf{x}, r) = p(B \mid T, \mathbf{x}, r)p(T \mid \mathbf{x}, r) + p(B \mid N, \mathbf{x}, r)p(N \mid \mathbf{x}, r) \qquad \textbf{(3)}$$

always holds. Fahner, (2010)) and others have suggested that the existence of adverse selection among Bads is an inequality, which states that the conditional default probability among Takes is higher than the unconditional default probability for acceptable borrowers:

$$p(B \mid T, \mathbf{x}, r) > p(B \mid \mathbf{x}, r). \qquad \textbf{(4)}$$

Because **(3)** and **(4)** hold, coherence requires that

$$p(G \mid N, \mathbf{x}, r) > p(G \mid \mathbf{x}, r)\ . \qquad \textbf{(5)}$$

Let us now consider response rates, obviously of great interest to lenders and organizations offering different credit products; experts such as Gerbino and Rosenberger, (2005) have suggested that in the presence of adverse selection a Bad borrower is more likely to take the offer than a Good or a randomly selected member of the Accept population. By contrast with **(4)**, we have

$$q_B(\mathbf{x}, r) \triangleq p(T \mid B, \mathbf{x}, r) > p(T \mid \mathbf{x}, r) \triangleq q(\mathbf{x}, r)\ . \qquad \textbf{(6)}$$

The probability statements in **(4)** and **(6)** are equivalent because Bayes' Rule tells us that the connection between the conditional probability of a Bad given a Take and the conditional probability of a Take by a Bad is equality of the Bayes' factors:

$$\frac{p(B \mid T, \mathbf{x}, r)}{p(B \mid \mathbf{x}, r)} = \frac{p(T \mid B, \mathbf{x}, r)}{p(T \mid \mathbf{x}, r)}. \qquad \textbf{(7)}$$

We note that **(7)** always holds and lays the foundation for defining No Adverse Selection (**NAS**) as the equality of prior (before a Take) and posterior (after a Take) probabilities,

$$\textbf{NAS:} \quad \frac{p(B \mid T, \mathbf{x}, r)}{p(B \mid \mathbf{x}, r)} = \frac{p(T \mid B, \mathbf{x}, r)}{p(T \mid \mathbf{x}, r)} = 1 \qquad \textbf{(8)}$$

This condition corresponds to **Fig. 2(a)** where the arc between the T and B node is absent. Obviously, *G* can be substituted for *B* in the definition of **NAS**. Bad Adverse Selection **(BAS)** in **(4)** is defined as the case where both Bayes' factors in **(7)** are greater than one, i.e.

$$\textbf{BAS:} \quad \frac{p(B \mid T, \mathbf{x}, r)}{p(B \mid \mathbf{x}, r)} = \frac{p(T \mid B, \mathbf{x}, r)}{p(T \mid \mathbf{x}, r)} > 1\,. \qquad \textbf{(9)}$$

Thus, when borrowers who Take offers have higher probabilities of default than those to whom offers are made, an equivalent statement is that the Take probability for Bads is higher than it is for the entire group. Good adverse selection **(GAS)** is represented by the reverse inequality, which occurs when a disproportionately large number of Good borrowers are attracted to under-priced loans or insurance policies (See Altman, (1998)). Anecdotal evidence suggests that BAS (adverse selection for Bads) is greater with high-priced loan offers to high risk borrowers while GAS is likely to be greater with low-priced loans for low risk borrowers. Notice, also, that **(8)** and **(9)** hold even when the left-hand Bayes' factor does not depend on the loan rate, often referred to as the situation where borrowers have the capacity to pay and the prior probability of default (before booking) is independent of the loan rate.

Neither **(8)** nor **(9)** requires a comparison of different loan rates or that one offer is in some sense inferior to another*, only that risk and response outcomes are conditionally dependent so that posterior probabilities of Good/Bad outcomes differ from their priors.* By contrast, Ausubel, (2009) states that there is experimental evidence of adverse selection from inferior offers while Phillips and Raffard, (2010) use a definition that incorporates "…. price changes with the same credit score …". They explicitly describe a measure for Total Adverse Selection that is the sum of expected default rate derivatives over the profile of risk scores of booked accounts held constant (Indirect Adverse Selection) and expected default rates of booked accounts due to changes in the profile of risk scores (Direct Adverse Selection). Such definitions require an examination of the derivatives of individual probabilities of borrower default or, as a minimum, the comparison of *different price offers:*

$$p(B \mid T, \mathbf{x}, r') > p(B \mid T, \mathbf{x}, r) \qquad r' > r \qquad \textbf{(10)}$$

The difficulty with using **(10)** for a definition of adverse selection is that because of the price change, it becomes difficult to distinguish between two different effects that may be taking place simultaneously: the offer at rate *r* is more attractive and preferred by most borrowers (both Good and Bad) to one at a higher rate because, typically, there is increased demand for less expensive loan offers by *all* borrowers. At the same time the internal composition or rearrangement of the relative fraction of Goods and Bads within the Take or Non -Take populations may be changing because of the presence and degree of adverse selection. It is not immediately obvious how one should discriminate between increased preferences for less expensive offers and increased attractiveness related to privileged information.

If we are certain that **NAS** holds in **Table 1** then the PopOdds of Takes, Non-Takes and Accepts are equal and the counts would have to be adjusted for each Z_1 as shown in **Table 2**. For purposes of discussion let us assume we know there are Bad adverse selections in the count of Takes in the first row. For a given value of Z_1 we would have to reduce the number of Bad adverse selections, Y_1, to obtain the NAS condition; thus Y_1, the difference between the count of observed Bads when adverse selection is present and the number of Bads among Takes when there is no adverse selection equals the number of Bad adverse selections. In order to maintain conservation we must also adjust three other cells by an internal reallocation of Bad and Total responses; note that counts in some cells depend only on Z_1, some only on Y_1, but only one cell has a count which depends on both and equals $Z_1 + Y_1$. In this simple example there is BAS when $0 \le Y_1 \le 50$.

	Goods	Bads	Totals
Takes (T)	250	$50\text{-}Y_1$	$300\text{-}Y_1$
Not Takes (N)	$1200\text{-}Z_1$	$Z_1 + Y_1$	$1200 + Y_1$
Total Accepts (A)	$1450\text{-}Z_1$	$50 + Z_1$	1500

Table 2: Counts of Goods/Bads, Takes/Not Takes under NAS

From the **NAS** definition in **(8)**,

$$o_A = \frac{1450 - Z_1}{50 + Z_1} = \frac{1200 - Z_1}{Z_1 + Y_1} = \frac{250}{50 - Y_1}. \qquad \textbf{(11)}$$

Solving for Y_1 in terms of Z_1 yields the count of Bads in the Take row that would result in NAS for each Z_1,

$$Y_1(Z_1) = \frac{50 o_A - 250}{o_A} = \frac{60,000 - 300 Z_1}{1450 - Z_1}. \qquad \textbf{(12)}$$

Conservation of counts must always be satisfied in **Table 2**. We get the surprising result that when BAS is thought to be present, NAS could only have been achieved when the total number of responses is less than or equal to the booked accounts and the size of the reduction depends on our initial estimate for Z_1. Clearly, Non-Take inference is an essential requirement for the estimation of NAS. In our case we want to infer Bads among Non-Takes, rather than the traditional Reject Inference which is to infer Bads that might have resulted had the lender accepted members of the Reject sub-population.

When adverse selection is present the marginal and differential (conditional) Take rates are given by

$$q = \Pr\{T\} = \frac{300}{1500} = 0.20 \qquad q_G \triangleq \Pr\{T \mid G\} = \frac{250}{1450 - Z_1} \qquad q_B \triangleq \Pr\{T \mid B\} = \frac{50}{50 + Z_1}$$

but with NAS the Take rate for Bads and the marginal Take rate are equal and depend on both Y_1 and Z_1:

$$q = \Pr\{T\} = \Pr\{T \mid B\} = \Pr\{T \mid G\} = \frac{50 - Y_1(Z_1)}{50 + Z_1} = \frac{300 - Y_1(Z_1)}{1500}$$

As an example consider the case when Z_1= 200 and $Y_1 = 0$ the PopOdds of all rows equals 5:1 and we have NAS. The Take rates of Goods and Bads are equal to one another and to the marginal rate, q=0.20. On the other hand, when o_A=10, we find from**(11)** and **(12)** that Z_1=86.4, Y_1=25 which means that adverse selection accounts for about half the number of observed Bads in the Take population. With adverse selection the fraction of Bads among Takes is 1/6=0.167 but is less among Non-Takes because

$$p(B \mid N) = \frac{Z_1}{1200} = \frac{86.4}{1200} = 0.072\,. \qquad \textbf{(13)}$$

In this case, conditional Take rates for the Good and Bad subpopulations differ greatly from one another and are given by

$$q_G = \frac{250}{1363.6} = 0.183 \qquad q_B = \frac{50}{136.4} = 0.367\,. \qquad \textbf{(14)}$$

Higher Priced Offer

Assume that a new higher-priced offer is made to the same population. If the loan rate in **Tables 1** and **2** was r and the increase in the loan price is Δr, the new rate is $r+\Delta r$. It is found that the observed Good/Bad counts are 160 and 40 as shown in **Table 3** so that the Take count is reduced to 200 and the number of Non-Takes is increased to 1300. The observed counts in top row and right-most column in **Table 3** correspond to the case where Y terms are zero and there are no entries in the unshaded cells. Subscript 2 denotes the higher priced offer. These new counts are due to two distinct effects: the reduced appeal of the higher priced offer to all borrowers in combination with what appears to be a disproportionate increase in the fraction of observed defaults (Bads), i.e. the adverse selects.

	Goods	Bads	Totals
Takes	160	40-Y_2	200- Y_2
Not Takes	1300- Z_2	Z_2+ Y_2	1300+ Y_2
Total Accepts	1460- Z_2	40+ Z_2	1500

Table 3: Counts of Goods/Bads, Takes/Not Takes without Adverse Selection in Higher Priced Offer

Let us assume, for the moment, that the default risks of the borrowers in the Accept population are unaffected by the change in the offer rate with a PopOdds for Accepts given by o_A used in **Table 2 -** this would correspond to the case where all borrowers in the Accept group are unaffected by the higher loan rate and have the capacity to pay. When we allocate the count of Goods and Bads in the Non-Take row in **Table 3** (as we did in **Table 2**) we find, that the count of Bads among Takes is always 10 larger for the higher-priced offer; furthermore, the difference in the count of Bad adverse selections between the high priced and low priced offer can be positive or negative.

Using the same calculations as in **(11)**, the Bads and the number of adverse selections among Takes for both offers are shown in **Table 4** for different values of the Accept PopOdds, o_A.

Accept PopOdds, o_A	Non-Take Bads, Z_1	Adverse Selection, Y_1	Non-Take Bads, Z_2	Adverse Selection, Y_2	Change Y_2- Y_1
20:1	21.4	37.5	31.4	32	-5.5
10:1	86.4	25	96.4	24	-1
9:1	100	22.2	110	22.2	0
8:1	116.7	18.8	126.7	20	1.2
7:1	137.5	14.3	147.5	17.1	2.8
6:1	164.3	8.3	174.3	13.3	5
5:1	200	0	210	8	8
4:1	250	-12.5	260	0	12.5

Table 4: Adverse Selection Counts for Low and High Priced Offer with Different Accept PopOdds

Let us assume that the PopOdds for Accepts is o_A=10, and that the loan rate of offer 1 is 6% and offer 2 is 7%, i.e. an increase of 100 basis points. A discrete version of price-response elasticity (negative of the traditional elasticity used in the economics literature) for the conditional Take rate of Goods and Bads can be easily calculated. The low price offer Take probabilities are obtained from **(14)** and the high price offer Take probabilities can be calculated from 2nd row (from top) of **Table 4**. Thus we obtain the Good/Bad elasticities:

$$\varepsilon_B^{(T)} = \frac{\Delta q_B}{\Delta r}\frac{r}{q_B} = -6\frac{0.073}{0.367} = -1.20 \quad > \quad \varepsilon_G^{(T)} = \frac{\Delta q_G}{\Delta r}\frac{r}{q_G} = -6\frac{0.066}{0.183} = -2.16 \qquad \textbf{(15)}$$

In this example the price response elasticity for Bads is larger than for Goods which means that the percentage decrease in response rates for Goods is greater (in absolute value) than for Bads. The marginal price-response elasticity is therefore

$$\varepsilon^{(T)} = \frac{\Delta q}{\Delta r}\frac{r}{q} = \varepsilon_G^{(T)} p(G \mid T) + \varepsilon_B^{(T)} p(B \mid T) = -2.16(0.833) - 1.20(0.167) = -2.00\,.$$

Differential Elasticities

To show how differential Take rates are influenced by the preferences of borrowers and the presence of adverse selection we define a price-risk elasticity in addition to the traditional and well-known response ("price-volume") elasticities for Takes. Even though it is slightly more complicated, it is important to use a notation that adheres to the standard convention for conditional dependence and independence. *Price-risk elasticity* is the percentage change in probability of default as a function of small percentage changes in price or loan rate. We denote the conditional *price-response elasticity* of Takes or Non-Takes (superscript *T* or *N*) with a *B* or *G* subscript. Because most of these elasticities cannot be measured directly there is a need for Non-Take inference to establish the magnitude of differential Take rates in distinct Good/Bad subpopulations. The conditional price-response elasticities are:

$$\varepsilon_B^{(T)} \triangleq \frac{\partial q_B(\mathbf{x},r)}{\partial r}\frac{r}{q_B(\mathbf{x},r)}\,, \quad \varepsilon_G^{(T)} \triangleq \frac{\partial q_G(\mathbf{x},r)}{\partial r}\frac{r}{q_G(\mathbf{x},r)}\,, \qquad \textbf{(16)}$$

with a similar definition for Non-Takes. The unconditional (marginal) price-response elasticity for Takes is

$$\varepsilon^{(T)} \triangleq \frac{\partial p(T \mid \mathbf{x},r)}{\partial r}\frac{r}{p(T \mid \mathbf{x},r)} = \frac{\partial q}{\partial r}\frac{r}{q(\mathbf{x},r)} = \frac{\partial \ln q(\mathbf{x},r)}{\partial \ln r} \qquad \textbf{(17)}$$

With this definition, price-response elasticity is non-positive for Takes and non-negative for Non-Takes.

Marginal *price-risk elasticity* is defined as:

$$\delta^{(B)} \triangleq \frac{\partial p(B \mid \mathbf{x}, r)}{\partial r} \frac{r}{p(B \mid \mathbf{x}, r)} = \frac{\partial \ln p(B \mid \mathbf{x}, r)}{\partial \ln r}. \qquad \textbf{(18)}$$

Just as price-response elasticities have been used to measure the change in borrower preferences for percentage increases in rate, risk elasticities can be thought of as the percentage change in "Goodness" or "Badness" for percentage increases in rate. While the marginal price-risk elasticity refers to the default rate of members of the Accept population, the conditional price-risk elasticities represent defaults within Take and Non-Take groups. The roles of superscripts and subscripts are reversed:

$$\delta_T^{(B)} \triangleq \frac{\partial p(B \mid T, \mathbf{x}, r)}{\partial r} \frac{r}{p(B \mid T, \mathbf{x}, r)} \qquad \delta_N^{(B)} \triangleq \frac{\partial p(B \mid N, \mathbf{x}, r)}{\partial r} \frac{r}{p(B \mid N, \mathbf{x}, r)} \qquad \textbf{(19)}$$

Price-risk elasticity for Takes is positive with BAS and negative with GAS. Because of the equality of Bayes' factors in **(7)**, unequal differential Take rates for Goods and Bads exist if and only if there are unequal differential Good/Bad default rates for Take and Non-Take sub-populations. From symmetry arguments it is easy to see that there are a total of six risk and six response elasticities for Takes, Non-Takes and Accepts,

$$\mathbf{E} = \begin{bmatrix} \varepsilon_G^{(T)} & \varepsilon_B^{(T)} & \varepsilon^{(T)} \\ \varepsilon_G^{(N)} & \varepsilon_B^{(N)} & \varepsilon^{(N)} \end{bmatrix} \qquad \mathbf{\Delta} = \begin{bmatrix} \delta_T^{(G)} & \delta_T^{(B)} \\ \delta_N^{(G)} & \delta_N^{(B)} \\ \delta^{(G)} & \delta^{(B)} \end{bmatrix}, \qquad \textbf{(20)}$$

where superscripts in **E** are associated with rows and subscripts with columns, the opposite being true with **Δ.** Note, also, that the final column in the former and the bottom row in the latter refers to the marginal elasticities (without subscripts). The vector of price-response elasticities for Takes corresponds to the top row of **E** and the price-risk elasticities for Bads correspond to the rightmost column of the **Δ** matrix in **(20)**. One case of special interest occurs when **(18)** is zero which means that the bottom row of **Δ** is zero.

In much of what follows and in most of the practical risk and credit scoring applications that we are familiar with, default predictions do not explicitly incorporate offer or rate terms which means that the lenders are not concerned about the borrower's ability or "capacity to pay". When the capacity to pay is not an issue the probability of default for Accepts is independent of the loan rate; this represents a special case that will yield simplified formulas for risk and response elasticities. We should mention that some models in the literature assume the presence of Bad adverse selection when action-based default scores include a negative term proportional to the loan offer rate, *r*. In our models adverse selection only depends on whether the added information from an observed Take does or does not change the prior default score or probability and whether risk score profiles in the Take and Non-Take populations are affected; as an example, the use of a baseline score which is conditionally independent of offer terms, *r*, may nevertheless lead to BAS (GAS) in the Take subpopulation if a large number of potential "Bads" ("Goods") among Accepts are disproportionately attracted to offers.

Conservation of Price-Risk and Price-Response Elasticities

To capture the relationship between Take and Default rates we again use Bayes' Rule. The posterior conditional probability of a Bad given a Take and the conditional probability of a Take by a Bad are given by

$$p(B \mid T, \mathbf{x}, r) = \frac{p(T \mid B, \mathbf{x}, r) p(B \mid \mathbf{x}, r)}{p(T \mid \mathbf{x}, r)} = p(B \mid \mathbf{x}, r) \frac{q_B(\mathbf{x}, r)}{q(\mathbf{x}, r)} \qquad \textbf{(21)}$$

If the Take probability of the Bad subpopulation is the same as the unconditional response rate, the last ratio equals one, the probability of a Bad in the Take population is equal to the probability of a Bad for the population of Accepts and we have NAS. On the other hand, if we have BAS we might expect the derivative of **(21)** to increase with *r*.

It is straightforward to express the partial derivative of the posterior probability of a Bad in **(21)** with respect to the loan rate.

$$\frac{\partial p(B \mid T,\mathbf{x},r)}{\partial r} = \frac{\partial}{\partial r}\frac{q_B(\mathbf{x},r)p(B \mid \mathbf{x},r)}{p(T \mid \mathbf{x},r)}$$
$$= \frac{1}{p(T \mid \mathbf{x},r)}\left(p(B \mid \mathbf{x},r)\frac{\partial q_B(\mathbf{x},r)}{\partial r} + q_B(\mathbf{x},r)\frac{\partial p(B \mid \mathbf{x},r)}{\partial r} - \frac{q_B(\mathbf{x},r)p(B \mid \mathbf{x},r)}{p(T \mid \mathbf{x},r)}\frac{\partial p(T \mid \mathbf{x},r)}{\partial r}\right) \quad \textbf{(22)}$$

The middle term inside the large parenthesis is proportional to the change in the Bad probability of accepted borrowers. Although this term may, in some cases, be unaffected by changes in the loan rate, in general, its presence and influence must be assessed. By factoring out common terms we can write **(22)** in terms of the difference between price-response elasticity for all respondents and the conditional price risk elasticity for Bads as well as a term which is proportional to the ratio of the conditional to the unconditional Bad probability.

By factoring out the three terms on the rhs of **(21)** from **(22)** we obtain the equivalent expression

$$\frac{\partial p(B \mid T,\mathbf{x},r)}{\partial r} = \frac{1}{r}\frac{q_B(\mathbf{x},r)p(B \mid \mathbf{x},r)}{p(T \mid \mathbf{x},r)}\left(\varepsilon_B^{(T)} - \varepsilon^{(T)} + \frac{\partial p(B \mid \mathbf{x},r)}{\partial r}\frac{r}{p(B \mid \mathbf{x},r)}\right)$$
$$= \frac{p(B \mid T,\mathbf{x},r)}{r}\left(\varepsilon_B^{(T)} - \varepsilon^{(T)} + \frac{\partial p(B \mid \mathbf{x},r)}{\partial r}\frac{r}{p(B \mid \mathbf{x},r)}\right). \quad \textbf{(23)}$$

On dividing both sides by the factor to the left of the large parenthesis, we obtain a conservation equation for the deviations in conditional elasticities from their marginals, namely

$$\delta_T^{(B)} - \delta^{(B)} = \varepsilon_B^{(T)} - \varepsilon^{(T)}. \quad \textbf{(24)}$$

(24) always holds and is independent of whether the borrower does or does not have the capacity to pay. The left-hand side is the deviation of the conditional price-risk elasticity for Bads among Takes from its marginal and is equal to the deviation in the conditional price-response elasticity for Takes among Bads from its marginal. By symmetry it follows that there are four Take/Non-Take price-risk counterparts to the traditional price-response elasticities; each conservation equation corresponds to the paired differences between conditional risk and response elasticities.

Expressions similar to **(22)** can be derived for the rate of change of the Good probability where G replaces B in the conditional statements.

$$\frac{\partial p(G \mid T,\mathbf{x},r)}{\partial r}\frac{r}{p(G \mid T,\mathbf{x},r)} - \frac{\partial p(G \mid \mathbf{x},r)}{\partial r}\frac{r}{p(G \mid \mathbf{x},r)} = \varepsilon_G^{(T)} - \varepsilon^{(T)}. \quad \textbf{(25)}$$

Because the conditional Good/Bad probabilities must sum to one, their partial derivatives sum to zero which means that one can convert the derivatives for Good probabilities to equivalent expressions involving Bad probabilities. Adding **(25)** to **(23)** and simplifying terms yields the result that the price-risk elasticity for Takes consists of two terms, one being proportional to the difference in the differential price-response elasticities, the other being proportional to the price-risk elasticity for Accepts,

$$\delta_T^{(B)} = \frac{\partial p(B \mid T,\mathbf{x},r)}{\partial r}\frac{r}{p(B \mid T,\mathbf{x},r)} = (\varepsilon_B^{(T)} - \varepsilon_G^{(T)})p(G \mid T,\mathbf{x},r) + \delta^{(B)}\left(\frac{p(G \mid T,\mathbf{x},r)}{p(G \mid \mathbf{x},r)}\right). \quad \textbf{(26)}$$

If borrowers have the capacity to pay, the price-risk profiles of Accepts is unaffected by the loan rate; thus, the rightmost term in **(26)** vanishes. If borrowers do not have the capacity to pay and the action-based default score is explicitly dependent on the loan rate, there is an additional contribution from loan rate changes to the marginal default probabilities in the lender's Accept population. If there is no adverse selection and the posterior to prior probability of default equals one we recover a result that has a similar structure to **(24)**.

Bad Adverse Selection when Borrowers have the Capacity to Pay

In many practical applications of credit risk decisions, the lender only uses a baseline score such as a Bureau score or a proprietary internal score based on past performance and relevant predictors and does not find the need to include the loan rate as a predictor of Good/Bad. This coincides with the assumption that there is no "Capacity" effect, i.e. the loan rate offer does not, in and of itself, directly influence the probability of default even though there may be an indirect linkage to loan rate resulting from the Take and Non-Take preferences of borrowers. It is interesting that this takes us back to one of the three components of the original "Three C's Rule" referring to Character, Capacity and Collateral (Lewis, (1992)), where, traditionally, decisions were based on making judgments abut the ability or capacity of the borrower to repay loans as promised.

As we have already mentioned there is an important special case when a change in the loan rate does **not** affect the probability of a Bad or Good in the population of borrowers acceptable to lenders; in other words prices only influence the internal reallocation of Goods and Bads within the Take and Non-Take sub-populations but not the overall risk of the Accept population. Derivatives of the marginal Bad rate and price-risk elasticity (without a condition on Take (T) or Non-Take (N)) can now be set equal to zero:

$$\delta^{(B)} = \delta^{(B)}(\mathbf{x}, r) = \frac{\partial p(B \mid \mathbf{x}, r)}{\partial r} \frac{r}{p(B \mid \mathbf{x}, r)} = 0 \qquad \textbf{(27)}$$

Even though this simplifying condition does not always hold we emphasize that adverse selection can still result from the internal reallocation of Goods and Bads within Takes. In such cases the final term on the right hand side of **(26)** vanishes so that the rate of change of the risk of default (conditional Bad probability for Takes) is inversely proportional to the loan rate and directly proportional to the product of default/non-default probabilities and the difference between the differential price-response elasticities:

$$\frac{\partial p(B \mid T, \mathbf{x}, r)}{\partial r} = (\varepsilon_B^{(T)}(\mathbf{x}, r) - \varepsilon_G^{(T)}(\mathbf{x}, r)) \frac{p(B \mid T, \mathbf{x}, r) p(G \mid T, \mathbf{x}, r)}{r} \qquad \textbf{(28)}$$

The condition is again on the Take group, not the population of all borrowers as a whole. Assume that the probability of a Good is positive. An equivalent statement to **(28)** is that price-risk elasticity for Takes is proportional to the product of the Good probability and the difference in the conditional price-response elasticities is now:

$$\delta_T(\mathbf{x}, r) = (\varepsilon_B^{(T)}(\mathbf{x}, r) - \varepsilon_G^{(T)}(\mathbf{x}, r)) p(G \mid T, \mathbf{x}, r) > 0 \quad iff \quad \varepsilon_B^{(T)}(\mathbf{x}, r) > \varepsilon_G^{(T)}(\mathbf{x}, r) \ . \qquad \textbf{(29)}$$

Thus the positivity of **(29)** is the result of greater price-response sensitivity for Bads than for Goods and is zero only when conditional elasticities are equal and there is no Adverse Selection - note that if the probability of a Good among the Takes is zero, we must have either BAS or NAS. If the inequality is reversed the statement holds for Goods. Furthermore, the price-response elasticity for the group of borrowers who take the lender's offer is the unconditional expectation of the conditional Good and Bad price-response elasticities:

$$\varepsilon^{(T)}(\mathbf{x}, r) = \varepsilon_G^{(T)}(\mathbf{x}, r) p(G \mid T, \mathbf{x}, r) + \varepsilon_B^{(T)}(\mathbf{x}, r) p(B \mid T, \mathbf{x}, r) \qquad \textbf{(30)}$$

We emphasize that, in general, **(28)** and **(29)** do not hold (e.g. as can occur when there is a Capacity effect); when they do, conditional and marginal risk elasticities can be expressed in terms of their response counterparts. From **Tables (2) and (3)** when o_A=10 we obtain (recall that our response elasticities are the negative of the traditional definitions):

$$\mathbf{E} = \begin{bmatrix} -2.16 & -1.20 & -2.00 \\ 0.49 & 0.70 & 0.50 \end{bmatrix} \qquad \boldsymbol{\Delta} = \begin{bmatrix} -0.16 & 0.80 \\ -0.015 & 0.195 \\ 0 & 0 \end{bmatrix}$$

Using the notation in **(24)**, we confirm that $\varepsilon_B^{(T)} - \delta_T^{(B)} = -(1.20 + 0.8) = -2.0 = \varepsilon^{(T)}$. With BAS, the response elasticity of Bads is larger than that of Goods while conditional default risk elasticities are negative for Goods and positive for Bads.

Risk and Response Scores

Thus far, the definitions of adverse selection and elasticities have not explicitly required either risk or response scores illustrated in the graphs of **Figs. 1** and **2**. While they were originally developed and used to measure relative risk performance of borrowers they are often used as guidelines to help design loan offers. that simultaneously recognize borrower risk, preferences and attractiveness of customized loan structures. Unfortunately, there are a number of difficulties that complicate the design of risk-based pricing policies. The first is the identification of timely and relevant characteristics that influence risk and response scores, a second is the specification of the conditional independencies that influence outcomes and a third is the degree to which adverse selection may contaminate the original assessments.

We define the **baseline log odds default score** as the score when no offer rates or terms are included. It is well-known that the score splits into two additive components: one term depending on the log of population odds, the other on the weight of evidence or log of information odds which itself measures the relative importance of the Good and Bad profiles:

$$s_p = s_p(\mathbf{x}) \triangleq \ln o(G|\mathbf{x}) = \ln \frac{p(G \mid \mathbf{x})}{p(B \mid \mathbf{x})} = \ln \frac{p(G)}{p(B)} + \ln \frac{f(\mathbf{x} \mid G)}{f(\mathbf{x} \mid B)}. \qquad \textbf{(31)}$$

When the default score includes the new information associated with each offer, whose attractiveness to the borrower is uncertain until it has been Taken or Not-taken, we modify the definition in **(31)** so that the action-based score is defined as

$$s_p(r) = s_p(\mathbf{x}, r) \triangleq \ln o(G|\mathbf{x}, r) = \ln \frac{p(G \mid \mathbf{x}, r)}{p(B \mid \mathbf{x}, r)} \qquad \mathbf{x} \in \mathcal{X},\ r \in \mathcal{R} \qquad \textbf{(32)}$$

The key role that risk and response scores provide is that (i) over many decades they have exhibited remarkable stability and reliability in providing trustworthy assessments of well-defined risk outcomes and (ii) a well-calibrated scalar score is a sufficient statistic for large vectors of behavioral, demographic and financial data. A well-calibrated default score provides as much information as is available from the original data on which the score was based so that

$$p(B \mid \mathbf{x}, r) = p(B \mid \mathbf{x}, r, s_p(r)) = p(B \mid s_p(r)) \qquad \mathbf{x} \in \mathcal{X},\ r \in \mathcal{R}\ . \qquad \textbf{(33)}$$

In what follows it should be understood that the score s_p is shorthand for the baseline $s_p(\mathbf{x})$ and $s_p(r)$ for the action-based score $s_p(\mathbf{x}, r)$. These scores are scalars even though **x** is usually a high dimensional vector and *r* might include financial information other than the loan rate. An example of a baseline risk score would be a Bureau score where *r* is not explicitly included. Although records for the latter indirectly contain important financial information of a borrower the details of financial terms associated with a particular loan are not included. A Bureau score should not substitute for a default score. In general, baseline and action-based scores are computed at different times, with different data and with different performance outcomes in the life cycle of the origination process. In the case of a finite number of different offers, there are as many default scores, as there are offers plus one, the latter being a baseline score. In comparing baseline and action-based scores as defined in **(31)** and **(32)** we understand that a baseline score augmented by a loan rate is not the same as a score based on the original data augmented by loan rates:

$$p(B \mid \mathbf{x}, r) = p(B \mid s_p(r)) \neq p(B \mid s_p, r). \qquad \textbf{(34)}$$

In the right hand side the data available to the lender would consist of a two-element vector: the baseline score and loan rate whereas on the left-hand side the relevant behavioral/demographic/financial data as well as loan rate is available in construction of the scorecard.
We should recognize that the score weights for attributes, other than the loan rate, associated with an action-based score may be non-linear functions of the loan rate so that two individuals with different behavioral/demographic/financial data may have the same baseline score but may have very different action-based scores even when the loan rate to each individual is identical. Borrowers with the same baseline score can have larger or smaller action-based scores; thus, default scores and probabilities of default that include the effect of the rate *r* may be larger or smaller than those associated with the baseline. The fact that the coefficient of a continuous variable, *r*, is negative in a regression of ln odds is not a guarantee that the probability of a Bad always decreases with increasing loan rate, *r*!

In **Fig. 3** below we show a scatter diagram for the baseline and action-based default scores of several hundred borrowers with the diagonal line corresponding to equality in both scores. The concept of risk or default scores extends to response scores; there is seldom any need for a baseline response score as the primary influence on response is the loan rate or price. When T and N denote Take and Non-Take borrower outcomes we have

$$s_q(r) = s_q(\mathbf{x}, r) \triangleq \ln \frac{p(T \mid \mathbf{x}, r)}{p(N \mid \mathbf{x}, r)}. \qquad \textbf{(35)}$$

As with risk scores, we will assume that response scores represent sufficient statistics for the prediction of Take and Not-Take outcomes. It is clear that risk and response scores in **(32)** and **(35)** are dependent not only because they each depend on *r* but because there may be several common characteristics in the **x** vector that affect both scores. Specifying the odds or score of an event is equivalent to specifying its probability; if the probability of a Bad in the denominator of **(31)** or **(32)** decreases, the numerator must increase so that both odds and score increase. This convention is in keeping with the majority of credit risk scores provided by commercial vendors where a larger score implies a less risky borrower; we adopt the same convention for response scores.

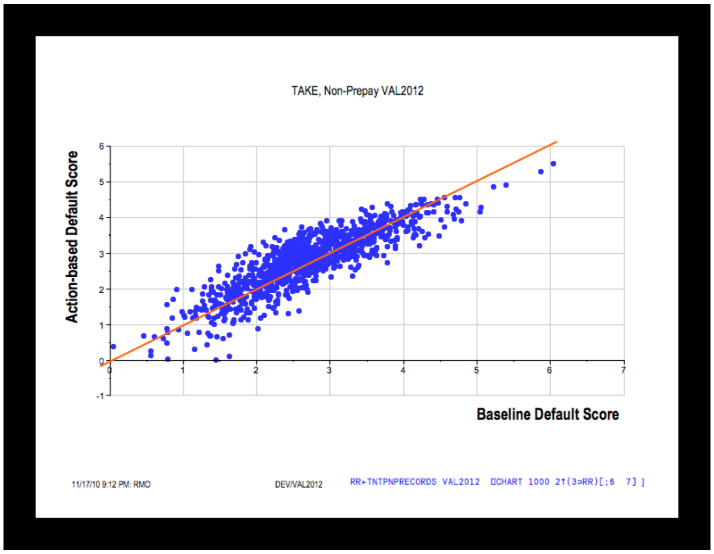

Figure 3 Scatter Diagram for Baseline and Action-Based Scores.

In the ID of **Figure (2a)** the probability of a Bad (default) is affected by the offer rate but not by the response or non- response of borrowers to these offers. We use two scores, one being an action-based default score $s_p(r)$, the other being a baseline score which or may not even be a default score but which, nevertheless, may influence the loan rate in offers the lender is willing to make. In this Figure, *B* (*G*) and *T* (*N*) are **conditionally independent** given *r* and **x** and there is no adverse selection (**NAS**):

$$p(B \mid T, \mathbf{x}, r) = p(B \mid \mathbf{x}, r) = p(B \mid s_p(r)) \qquad B, G \perp T, N \mid \mathbf{x}, r \qquad \textbf{(36)}$$

Figure (2b) illustrates the case where the probability of a Bad is directly influenced by the Take outcome as well as the offer rate, the former also being **directly** influenced by the offer rate. Thus, there are two possible paths, one direct, one indirect, for the influence of loan rate on the Good/Bad outcome, a feature that would appear to make sense in distinguishing between direct and indirect adverse selection if that is an important consideration. In either case we know that **BAS** yields

$$p(B \mid T, \mathbf{x}, r) = p(B \mid T, s_p(r)) > p(B \mid s_p(r))$$

Obviously, Non-Take inference plays a critical role in assessing the number of Bads among Non-Takes and Accepts so, difficult as it may be, greater understanding and experimentation with this important topic should be encouraged. It is instructive and valuable to revisit the closely related topic of Reject Inference in Classification and Credit scoring in Hand and Henley, (1997) (italics for our substitutions):

> "…Typically it (*the risk score*) is the set of people who were classified as good risks by an earlier score-card. .... If the new score-card is based on a superset of the characteristics used in the original score-card then the true classes in the reject *(Not Take)* region are missing, but those in the accept *(Take)* region are not. In this case, the available data can be used to construct an accurate model, without taking into account the rejected *(Not Take)* cases, but only over the 'accept' *(Take)* regions of the space, as defined by the original classifier. Extrapolation over the former reject *(Not Take)* region is then needed, *(provided we believe that preferences by borrowers for loan rates from the new scorecard are "similar" to those with the old)*. … . Improved classification could be produced if information was available in the reject *(Not Take)* region - if some applicants who would normally be rejected were accepted *(Taken)*."

As suggested by Hand and Henley what is urgently needed is access to new information (characteristics) among the Non-Take populations. For example, the knowledge as to whether a borrower did or did not have another loan offer would be a valuable first step as would postmortem "after-the-fact" surveys of borrowers who turned down one lender at a given rate but booked with a different lender at the same or a possibly different rate. An increasingly important but expensive data acquisition strategy would be to track the longitudinal behavior and performance of borrowers who do not take the offers of one lender but do take offers from other lenders in the marketplace. This requires extensive tracking capabilities and possible agreements among competitive lenders. Apparently such data was available to Agarwal, S., S. Chomsisengphet, and C. Liu (2010) in their study but other than repeatedly documenting the cumulative distribution functions for scores of Takes and Non-Takes, they made no effort to model or quantify the number or degree of adverse selections. Unlike the classification schemes for Reject Inference where one can run small side-experiments to accept borrowers, ordinarily classified as Rejects, in order to gain behavioral and performance information, this opportunity is not available in the analysis of adverse selections of Non-Takes because an enticement that encourages a borrower to respond to offers is, by itself, influencing, probably reducing, the "price" and rate of the loan.

Default Score Revisions for Take and Non-Take Sub-populations

We can compare the relative odds of Goods among Takes as suggested by Good, (1961) which, from **(7)**, expresses the posterior odds of a Good among Takes as the prior odds times a ratio which depends on differential Take rates:

$$\frac{p(G \mid T, \mathbf{x}, r)}{p(B \mid T, \mathbf{x}, r)} = \frac{p(G \mid \mathbf{x}, r)}{p(B \mid \mathbf{x}, r)} \times \frac{p(T \mid G, \mathbf{x}, r)}{p(T \mid B, \mathbf{x}, r)} = \frac{p(G \mid \mathbf{x}, r)}{p(B \mid \mathbf{x}, r)} \times \frac{q_G(\mathbf{x}, r)}{q_B(\mathbf{x}, r)} \; . \qquad \textbf{(37)}$$

The ratio of interest in **(37)** is the relative Take rates of Goods and Bads. What the Bayes' factors tell us is that the Good/Bad odds of the Take group is the Good/Bad odds for the entire population multiplied by the price dependent ratio of Take rates for Goods and Bads. The adjustment is the weight of evidence in favor of Take rates for Goods against Bads at each offer price and default score. By including this weight of evidence in favor of Goods who take the offer, the posterior default score inherits the influence of differential response rates:

$$s_p(r \mid T) \triangleq \ln \frac{p(G \mid T, \mathbf{x}, r)}{p(B \mid T, \mathbf{x}, r)} = \ln\left(\frac{p(G \mid \mathbf{x}, r)}{p(B \mid \mathbf{x}, r)} \times \frac{q_G(\mathbf{x}, r)}{q_B(\mathbf{x}, r)}\right) = s_p(r) + \ln \frac{q_G(\mathbf{x}, r)}{q_B(\mathbf{x}, r)} \qquad \textbf{(38)}$$

If the Take probability for Goods is less than for Bads their ratio is less than one and the second log term is negative which is equivalent to BAS, adverse selection of Bads. When this occurs with a higher price-offer the default score for the higher priced offer is less than the default score for the lower priced offer. When the ratio is greater than one, we have GAS. Thus, the default score conditional on a Take is larger or smaller than the action-based default score for the borrower depending on whether the ratio of differential Take probabilities is larger or smaller than one and its log is greater than or less than zero. Similar comments apply to Non-Takes. The importance of the conditional Take rates is apparent when one realizes that the rate of change of the posterior Good/Bad odds with respect to *r* equals zero iff

$$\frac{\varepsilon_G^{(T)}}{\varepsilon_B^{(T)}} = \frac{q_G(\mathbf{x}, r)}{q_B(\mathbf{x}, r)} . \qquad \textbf{(39)}$$

There are some applications where the Take/Not Take correction factors in **(38)** are almost entirely due to price effects so that a useful model for the ratio of Good/Bad response rates can be obtained without any requirement for a response score. One can also calculate the posterior score in **(38)** conditioned on Non-Takes rather than Takes; on subtracting the former from the latter, we find that the prior score cancels and we are left with

$$\Delta s_p = s_p(r \mid T) - s_p(r \mid N) = \ln \frac{q_G(\mathbf{x}, r)}{q_B(\mathbf{x}, r)} - \ln \frac{1 - q_G(\mathbf{x}, r)}{1 - q_B(\mathbf{x}, r)} \; , \qquad \textbf{(40)}$$

which may explain the gaps in the cumulative score distributions in **Figure 1**. For the numerical results reported in the top portion of **Table 6**, the update to the default score is

$$\ln \frac{q_G(\mathbf{x}, r)}{q_B(\mathbf{x}, r)} = \ln \frac{547/3597}{36/272} = 0.14 \, ,$$

i.e. a positive increase in the score. The difference in the posterior scores is therefore

$$s_p(r \mid T) - s_p(r \mid N) = \ln \frac{q_G(\mathbf{x}, r)}{q_B(\mathbf{x}, r)} - \ln \frac{1 - q_G(\mathbf{x}, r)}{1 - q_B(\mathbf{x}, r)} = 0.14 + 0.02 = 0.16\,.$$

For the bottom half of Table 6 with BAS the reduction in the posterior scores is $s_p(r \mid T) - s_p(r \mid N) = -0.69 - 0.09 = -0.78$.

Estimation of Scores and Experimental Results from Validation Samples

The results that are reported include an analysis of a proprietary database of mortgage applications that included prime and sub-prime paper. The results reported here are concerned with fixed-rate 1st mortgages although preliminary results suggest that similar conclusions can be drawn from other loan types. All risk and response scores used in this study were based on data available at the time the offer decision was made; the action-based default/non-default and response/no response scores depend on offer rates as well as the premia of quoted rates over LIBOR-3.

The sample time frame used for scorecard models built in this study was calendar year 2004. The Take/No Take performance was obtained from application records. The Good/Bad outcomes among Takes were available for a period of two to three years. Booked loans over 120 days past due, foreclosed, or bankrupt were tagged as defaults, i.e., Bads, while the others were tagged as Goods. The resulting sample contained over 50,000 records, of which 40% were held out for validation. Scorecards predicting Good/Bad and Take/No Take performances were built. As mentioned earlier a significant number of the Takes were Pre-pays; Good/Bad inference was used for the combined population of Prepays and Non-Takes as defaults could not arise from the Prepays. A typical default scorecard for our mortgage loan origination models included:

rate premium (interest rate less LIBOR-3mo)
loan to value ratio
FICO score
income
assets (type and size)
employment status
term of loan requested
loan type
home price appreciation (HPI)
back-end ratio

The "rate premium" was adjusted for the most recently available LIBOR3 rate prior to a loan's application date; this was an attempt to immunize the models from rate changes taking place over a twelve-month window. There was not a material difference in the observation date of the "rate premium" variable and the observation date of the other predictors. HPI variables attempted to capture possible speculative motivations of the borrowers. HPI values used are the quarterly, state-level HPI values published by the US Federal Housing Finance Agency. As expected, the "rate premium" variable was the strongest variable in the default scorecard models. Not surprisingly, "FICO score" and "loan to value" variables were also strong predictors of risk. Predictors for the typical response scorecard included:

rate premium (interest rate less LIBOR-3mo)
loan amount
loan type
% chg in HPI (1qtr)
broker fee amount
FICO score
down payment amount
property type
back end ratio
borrower years @ residence
borrower years @ job
front_end_ratio

The single most important variable in the response scorecards was the loan rate premium, the same characteristic that appeared in the default scorecards. Although score weights were not optimized over these performance measures the K-S, AUC and Divergence values of the development samples that we used are

Score	K-S	AUC	DIV
Baseline "good"/"bad" score	33%	0.72	0.81
Action-based "good"/"bad" score	36%	0.74	0.92
Baseline "take"/"no take" score	18%	0.62	0.45
Action-based "take"/"no take" score	23%	0.66	0.57

Table 5: Performance measures for Development Sample Scorecards

In the accounts that we analyzed we were faced with large numbers of Prepay events, i.e. early termination of the loan contract with full payment of all outstanding interest and unpaid balances. Thus, there are four rather than three rows (states) that might have been considered in **Tables 1** and **2**: Non-prepays, Takes who Prepay, Non-Takes and Accepts. Only default and response scorecards were developed although it might make sense to extend this framework to include a Prepay scorecard. The need for inference of Goods and Bads extends to all four states but in our development of scorecards we replaced the Take group by the Non-Prepay state and combined the Prepays with the Non-Takes to keep the format consistent with the theoretical model in this paper. The availability of loan rates, Takes, Non-Takes and Prepay data has made it possible to obtain Non-Take inferences for Goods and Bads in the combined groups.

Table 6 below examine records in a validation sample contained in a risk segment with baseline default scores in the intervals (2.0-3.0) and two different price tiers with loan rate premiums (3-5%) and (5-7%). This risk segment corresponds to an average default rate of approximately 7%. The numbers in parentheses in the Non-Prepay/Bad cells are our estimate of the number of adverse selections for BAS (positive) or the change in the Bad count for GAS (negative numbers).

3-5% Rate Premium	Observed/Inferred #Goods	Observed/Inferred #Bads (Adverse Selects)	Totals
Non-Prepays	547	36 (-5.3)	583
Non-Takes and Prepays	3050	235.5	3286
Accepts	3597	271.5	3869

5-7% Rate Premium	Observed/Inferred #Goods	Observed/Inferred #Bads (Adverse Selects)	Totals
Non-Prepays	493	76 (31.8)	569
Non-Takes and Prepays	5382	450.5	5832
Accepts	5875	526.5	6401

Table 6: Observed/ Inferred Cell Counts for Goods/Bads, Takes/Not-Takes for one risk, two rate segments

Table 6 indicates that with the higher priced loan rate premium the number of Bad Adverse selections is slightly less than half of the Bad counts among Takes; there are possibly a small number of Good adverse selections in the lower priced tier but further analysis would have to be made to decide whether this number is large enough to be significant. It should be noted that different analyses using either a superior scoring technology and/or more informative data should be able to obtain improved estimates of adverse selections and risk/response elasticities.

Summary

The authors have defined (i) conditions for no adverse selection and have provided (ii) a simple theoretical model to compare counts in the presence of adverse selections with counts expected when there is no adverse selection. We also define (iii) price-risk elasticity and derive conservation equations that reveal the probabilistic exchanges between risk and response preferences as loan rates are changed. By comparing theoretical predictions with observed response and risk outcomes, we offer (iv) limited experimental results for different price-risk segments where default risk and response scores are used to quantify borrower preferences and the magnitude of adverse selections. We identify (v) the critical role of Non-Take inference with the hope that these results provide (vi) further incentives for statistical testing and experimentation and a better understanding of the role and magnitude of adverse selection in marketing and risk assessment within the credit loan and mortgage industry.

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