ADVERSE SELECTION AND NON-TAKE INFERENCE

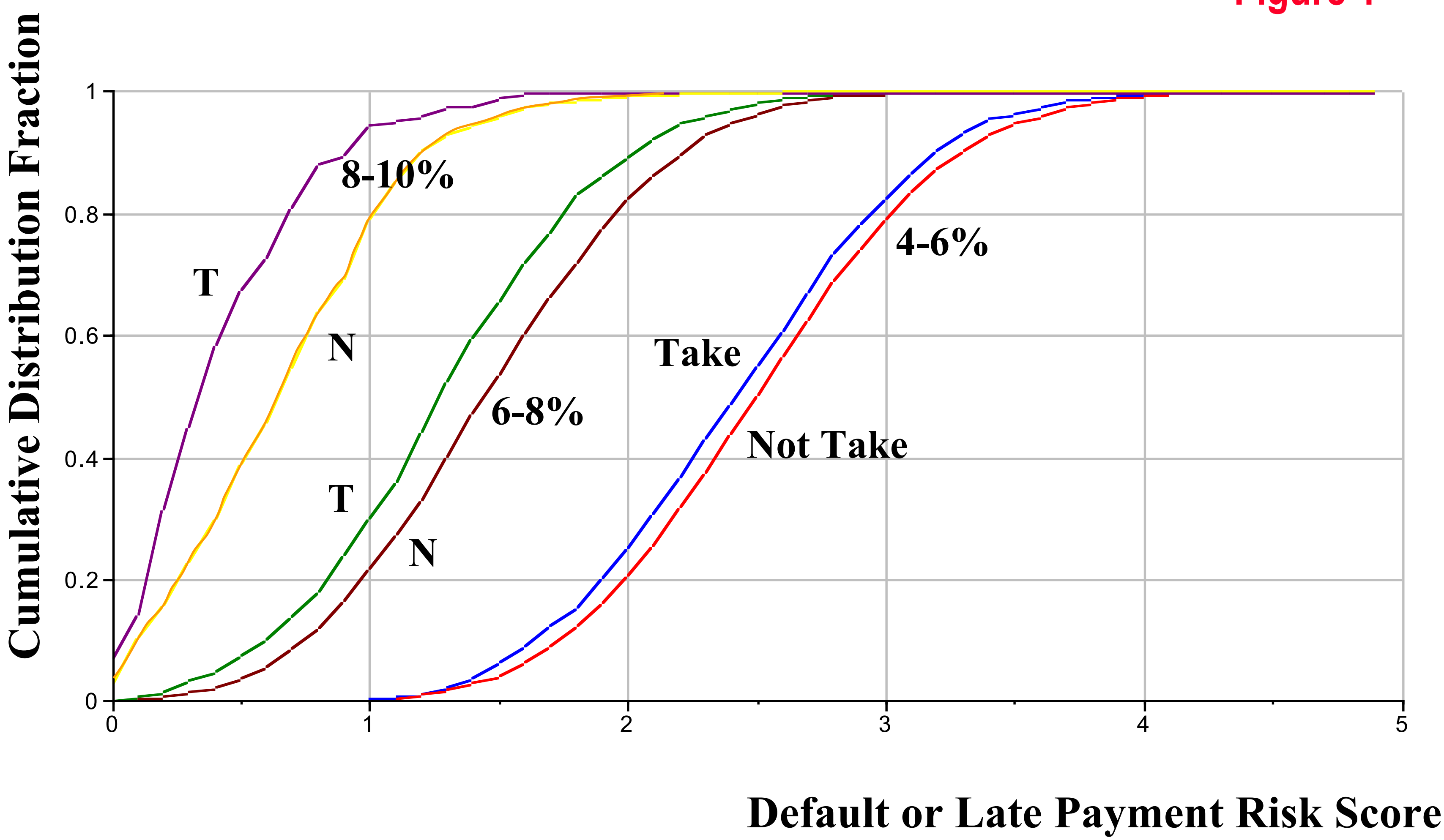
Robert Oliver, University of California, Berkeley, California
Aush Thaker, InfoCentricity, Inc. , Novato, California

1. **What is Adverse Selection? How to define, measure?**
2. **Traditional Measure: Shift in Score Distributions for Takes**
3. **Definition of No Adverse Selection**
4. **Conservation of Counts and Role of Non-Take Inference**
5. **Differential Risk and Response Elasticities:**
   Bayes' Rule and "No Free Lunch"
6. **Revising Risk Scores with woe on Take Rates**
7. **Summary**

TRADITIONAL VALIDATION OF ADVERSE SELECTION

Comparing Distributions of Scores in Take and Non-Take Populations for Three Price Tiers

Figure 1



0 .1 50 GRAPHFREQ (4 6 8 10) (0 5) (¯5 5) (14 7 8) TAKENOTTAKEREC DEV1512

REQUIREMENTS FOR BAD ADVERSE SELECTION (BAS) ?

For behavioral/demographic/financial data $\mathbf{x} \in \mathcal{X}$, loan rate $r \in \mathcal{R}$:

1. $p(B \mid T, \mathbf{x}) > p(B \mid \mathbf{x}) = \Pr\{loan\ default\ by\ borrower \mid \mathbf{x}\}$

2. $p(T \mid B, \mathbf{x}, r) > p(T \mid \mathbf{x}, r) = \Pr\{borrower\ Takes\ loan\ offer \mid \mathbf{x}, r\}$

3. *BAS* if **cdf** of Risk Scores of Takes dominates Non-Takes

4. *BAS* *iff* $\dfrac{\partial p(B \mid T, \mathbf{x}, r)}{\partial r} > 0$ {Phillips & Raffard (2009)}

5. Other ....

EQUALITY OF BAYES FACTORS ALWAYS HOLDS: Equation 7

$$\frac{p(B \mid T, \mathbf{x}, r)}{p(B \mid \mathbf{x}, r)} = \frac{p(T \mid B, \mathbf{x}, r)}{p(T \mid \mathbf{x}, r)}$$

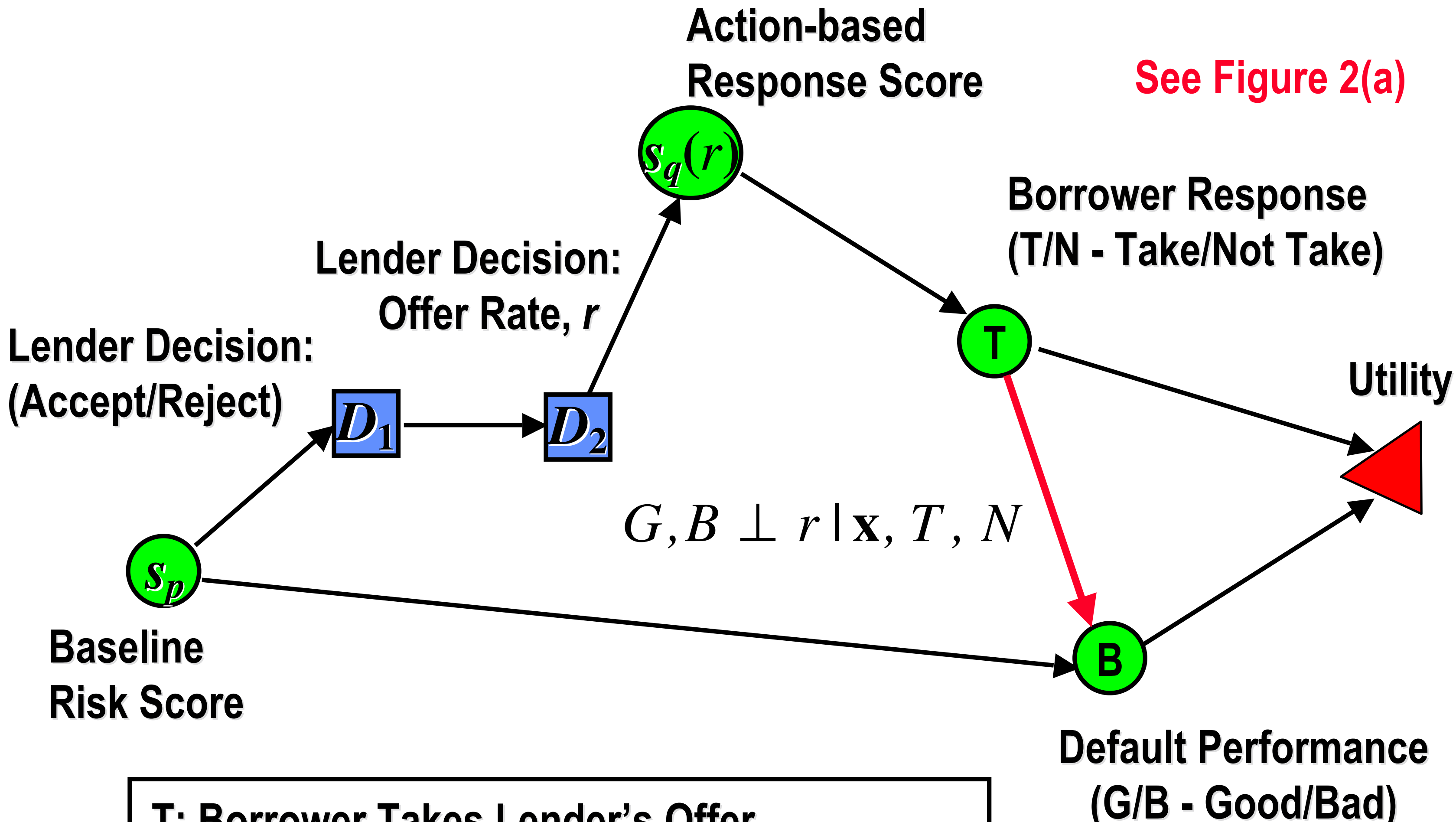
DEFINITION OF NO ADVERSE SELECTION, NAS: Equation 8

$$\frac{p(B \mid T, \mathbf{x}, r)}{p(B \mid \mathbf{x}, r)} = \frac{p(T \mid B, \mathbf{x}, r)}{p(T \mid \mathbf{x}, r)} = 1 \qquad G, B \perp T, N, r \mid \mathbf{x}$$

DEFINITION OF BAD ADVERSE SELECTION, BAS: Equation 9

$$\frac{p(B \mid T, \mathbf{x}, r)}{p(B \mid \mathbf{x}, r)} = \frac{p(T \mid B, \mathbf{x}, r)}{p(T \mid \mathbf{x}, r)} > 1 \qquad \mathbf{GAS}: \; < 1$$

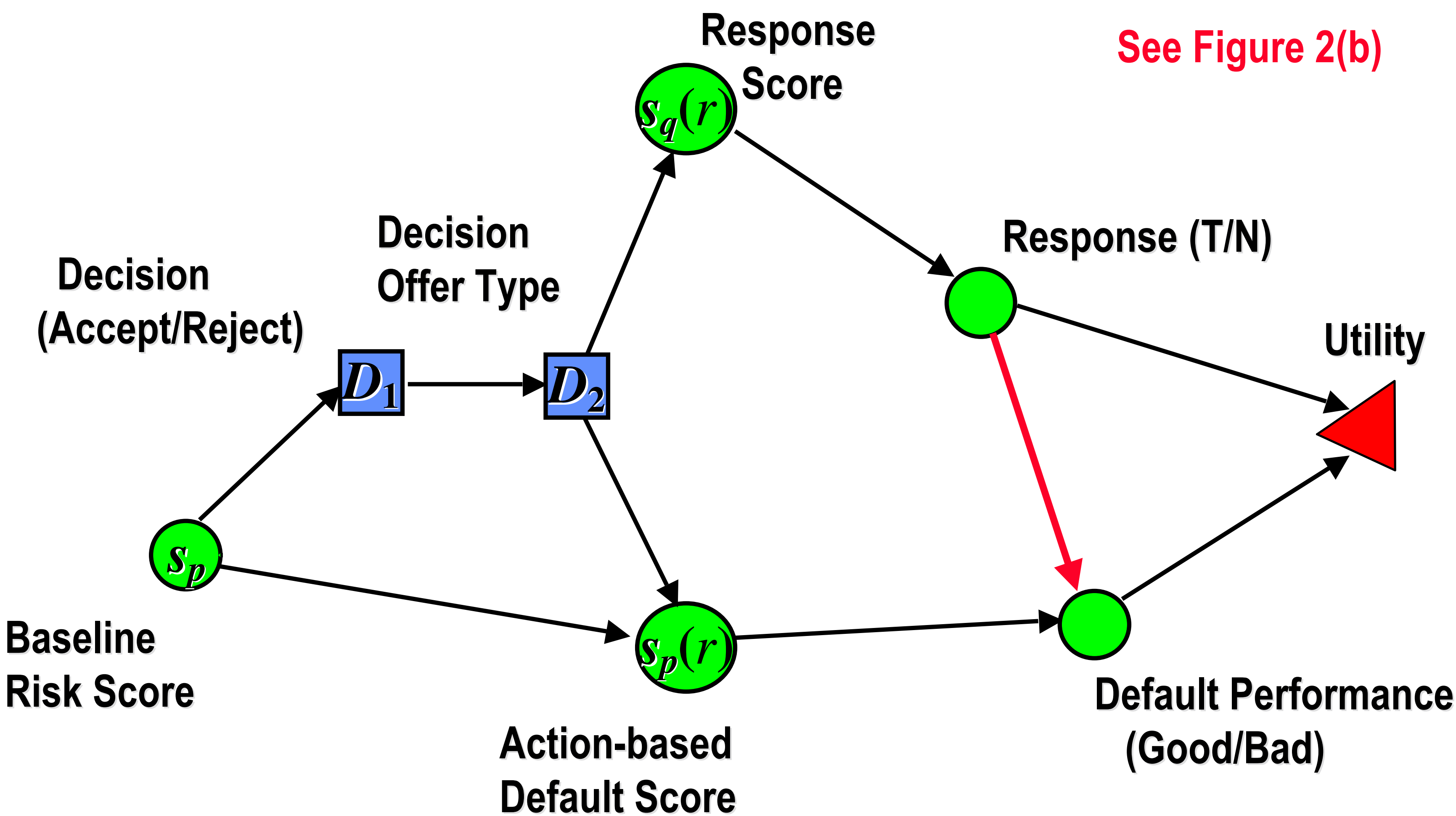
TWO-STAGE OFFER DECISION MADE BY LENDER BASED ON RESPONSE AND BASELINE DEFAULT SCORE



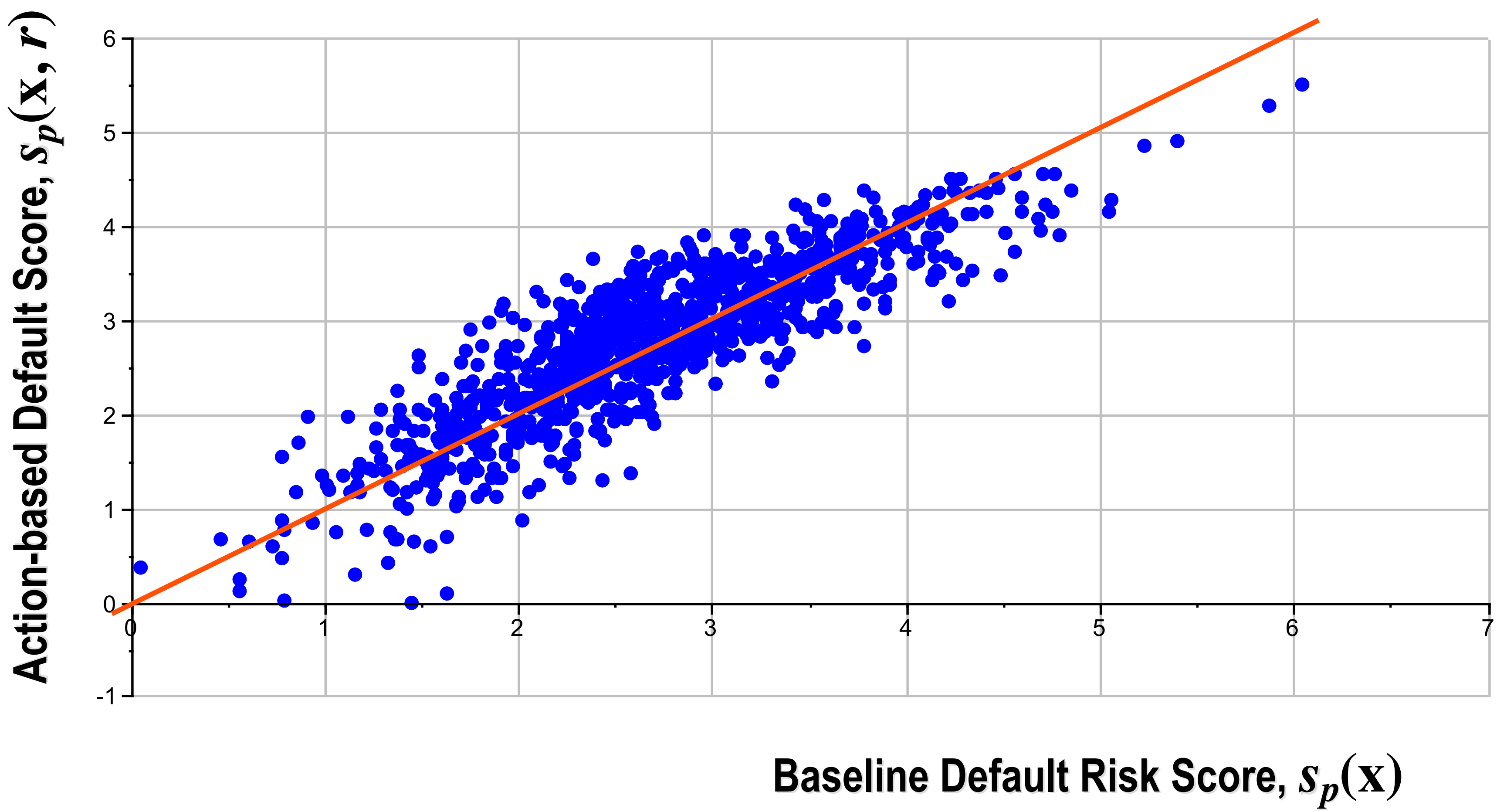
T: Borrower Takes Lender's Offer
B: Borrower Defaults on Loan

TWO-STAGE DECISION WITH ACTION-BASED DEFAULT AND RESPONSE SCORE

(Offer-dependent Default Risk with Bad Adverse Selection)



TAKE, Non-Prepay VAL2012

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RR←TNTPNPRECORDS VAL2012  ⎕CHART 1000 2↑(3⊃RR)[;6  7] ]
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COMPARING PREDICTED COUNTS WITH OBSERVED RESPONSES AND GOODS/BADS

Table 1

	E[#Goods]	E[#Bads]	Observed #Goods	Observed #Bads	Totals
Takes (T)	1530.0	127.8	1508	150	1658
Non-Takes (N)	3331.0	273.7			3605
Total Accepts by Lender	4862.0	401.5			5263

Table 1: Predicted and Observed Cell Counts for Goods/Bads, Takes/Not-Takes

(4 6) (2 3) (¯4 4) (14 7 8) EXPOBSADVSEL and EXPADVSEL DEV1612

COUNTS OF OBSERVED GOODS/BADS AND RESPONSES

	Goods	Bads	Totals
Takes (T)	250	50	300
Not Takes (N)			1200
Total Accepts (A)			1500

COUNTS OF OBSERVED GOODS/BADS AND RESPONSES WITH NON-TAKE INFERENCE COUNT, Z

	Goods	Bads	Totals
Takes (T)	250	50	300
Not Takes (N)	1200-Z	Z	1200
Total Accepts (A)	1450-Z	50+Z	1500

See Table 1

Z is the inferred number of Bads among the Non-Takes

Reallocation of Adverse Counts, Y, to obtain NAS

Tables 2, 3 and 4

	Goods	Bads	Totals
Takes (T)	250	50-Y	300-Y
Not Takes (N)	1200-Z	Z+Y	1200-Y
Total Accepts (A)	1450-Z	50+Z	1500

NAS: $\frac{250}{50-Y} = \frac{1200-Z}{Z+Y}$, $Y(Z) \geq 0$ is number of Bad Adverse Selects

Counts of Observed (Inferred) Goods/Bads
(Action Based Default Score Segment 2-3 in VAL2012)

See Table 6

3-5% APR Premium	Observed/Inferred #Goods	Observed/Inferred # Bads/Goods (Adverse Selects)	Totals
Non-Prepays	547	36 (Y= - 5.3)	583
Non-Takes and Prepays	305 0	235.5 (Z)	328 6
Accepts	359 7	271.5	386 9

5-7% APR Premium	Observed/Inferred #Goods	Observed/Inferred # Bads/Goods (Adverse Selects)	Totals
Non-Prepays	493	76 (Y= +31.8)	569
Non-Takes and Prepays	538 2	450.5 (Z)	583 2
Accepts	587 5	526.5	640 1

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(DATA← (3 5 7) (2 3) (¯4 4) (14 7 8) EXTRACTRECORDS VAL2012)(3 7 EXPOBSADVSEL2 DATA)(1/6/11)
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DIFFERENTIAL RISK AND RESPONSE ELASTICITIES

Response:

$$\varepsilon^{(T)} \triangleq \frac{\partial p(T \mid \mathbf{x}, r)}{\partial r} \frac{r}{p(T \mid \mathbf{x}, r)} = \frac{\partial p / p}{\partial r / r}$$

$$\varepsilon_B^{(T)} \triangleq \frac{\partial p(T \mid B, \mathbf{x}, r)}{\partial r} \frac{r}{p(T \mid B, \mathbf{x}, r)}$$

Risk:

$$\delta^{(B)} \triangleq \frac{\partial p(B \mid \mathbf{x}, r)}{\partial r} \frac{r}{p(B \mid \mathbf{x}, r)}$$

$$\delta_T^{(B)} \triangleq \frac{\partial p(B \mid T, \mathbf{x}, r)}{\partial r} \frac{r}{p(B \mid T, \mathbf{x}, r)}$$

See Equations 16 through 19

Six Response Elasticities: Three for Takes
Six Risk Elasticities: Three for Bads

Response

$$\mathbf{E} = \begin{bmatrix} \varepsilon_G^{(T)} & \varepsilon_B^{(T)} & \varepsilon^{(T)} \\ \varepsilon_G^{(N)} & \varepsilon_B^{(N)} & \varepsilon^{(N)} \end{bmatrix}$$

Risk

$$\mathbf{\Delta} = \begin{bmatrix} \delta_T^{(G)} & \delta_T^{(B)} \\ \delta_N^{(G)} & \delta_N^{(B)} \\ \delta^{(G)} & \delta^{(B)} \end{bmatrix}$$

See Equation 20

No Free Lunch Equation

Use Bayes' Rule:

$$p(B \mid T,\mathbf{x},r) = \frac{p(T \mid B,\mathbf{x},r)}{p(T \mid \mathbf{x},r)}\, p(B \mid \mathbf{x},r)$$

Derivative with respect to loan rates:

$$\frac{\partial}{\partial r} p(B \mid T,\mathbf{x},r) = \frac{\partial}{\partial r} \frac{p(T \mid B,\mathbf{x},r)\, p(B \mid \mathbf{x},r)}{p(T \mid \mathbf{x},r)}$$

$$= \frac{p(B \mid T,\mathbf{x},r)}{r} \left(\varepsilon_B^{(T)} - \varepsilon^{(T)} + \frac{\partial p(B \mid \mathbf{x},r)}{\partial r} \frac{r}{p(B \mid \mathbf{x},r)} \right)$$

yields the risk-response exchange:

$$\delta_T^{(B)} - \delta^{(B)} = \varepsilon_B^{(T)} - \varepsilon^{(T)}$$

Equation 24

The change (deviation) in the price-risk elasticity for Bads among Takes equals the change (deviation) in the price-response elasticities for Takes among Bads.

Response Updates to Prior Risk Scores

See Equation 38

$$s_p(r \mid T) \triangleq \ln \frac{p(G \mid T, \mathbf{x}, r)}{p(B \mid T, \mathbf{x}, r)} = s_p(r) + \ln \frac{q_G(\mathbf{x}, r)}{q_B(\mathbf{x}, r)}$$

See Equation 40

$$\Delta s_p \triangleq s_p(r \mid T) - s_p(r \mid N) = \ln \frac{q_G(\mathbf{x}, r)}{q_B(\mathbf{x}, r)} - \ln \frac{1 - q_G(\mathbf{x}, r)}{1 - q_B(\mathbf{x}, r)}$$

May help explain the shift in default score distributions for Take/Not-Take populations in Figure 1

SUMMARY

Equality of Bayes' Factors shows that Bad Adverse Selection of those who Take are equivalent to Increased Take rates for Bads.

Bayes' factor of 1 defines NAS. Non-Take Inference is directly incorporated in estimates of number of Adverse Selections. This allows asymmetric information available to borrowers to be reflected in their preferences and differential responses.

Price-risk elasticities act in concert with price-response elasticities; there are six elasticities of each type and four conservation equations for exchange of risk and response elasticities. No Free Lunches.

Posterior Take/Non-Take Risk scores (and differences between differential Response Rates) can be estimated from weight of evidence on Take/Non-Take rates.