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Credit Card Pricing and Impact of Adverse Selection

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Contents

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background

- Credit cards are probably the most convenient form of credit of all competing financial assets that include both payment and credit devices (Ayadi, 1997).
- Variable rate loans have been legal since the early 1980s. In the credit card market, however, a standard rate continued to dominate until the early 1990s.
- The development of the internet and the telephone as new channels for loan applications has made the offer process more private to each individual (Thomas 2008).

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background

- However, Ausubel (1991) pointed out that competition in credit card pricing can result in adverse selection.
- Adverse selection occurs in a trading situation where one side processes information (the informed party) which is relevant to his trading partner who is the uninformed side.
- Adverse selection has already been investigated in the insurance industry and the second-hand cars market.
- In consumer lending market, Thomas (2008) points out that adverse selection is important in estimating the interaction between the quality of the applicant and the chance of them taking the loan.

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Auction Model of Credit card Solicitation

- Ausubel (1999) suggested that the auction model is a useful analogy to the credit card application process.
- Different issuer will have different score cards, and the resultant scores determine not only whether the issuer will accept that customer but nowadays also the interest rate that will be charged. -----This is what risk based pricing seeks to do.

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Auction Model of Credit card Solicitation

- In the “auction”, each lender i, obtains information on the applicant to obtain an application score s_i for that applicant.
- One can translate the score s_i to obtain p_i which is lender i’s probability that the applicant is a ‘Good’ and will not default on the account in a prescribed time horizon.
- However these estimates of probability of being Good are likely to have errors ε_i in them.

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Auction Model of Credit card Solicitation

- assume the applicant has a true probability of being good of $\tilde{p}$, and so $p_i = \tilde{p} + \varepsilon_i$.
- assume that the applicant will choose the credit card offer with the lowest interest rate and that all the firms are using the same risk based pricing approach.
- So the applicant will choose the firm whose probability $p_i = p^*$ of them being Good is the largest.
- This is an example of the winner's curse, in that the lender will have a higher assumed probability of the applicant being a good than is really the case.

Errors in probability of being Good

- We assume that lender i, estimates via the application score that the applicant's probability of being good is

$$p_i = \tilde{p} + \varepsilon_i$$

where the errors ε_i are independent random variable with distribution function $F(\cdot)$.

- We assume that $p_i^* = \max_{1 \le i \le N} \{p_i\}, \varepsilon_i^* = \max_{1 \le i \le N} \{\varepsilon_i\}$

are the probability of being Good and the error of the lender, whose credit card is chosen by the borrower

Errors in probability of being Good

- So the distribution of the true probability of being good of the applicant accepted by the lender who perceives the applicant's probability of being good to be , is given by,

$$G(t) = \Pr\{\tilde{p} \le t\} = \Pr\{p_i^* - \varepsilon_i^* \le t\} = \Pr\{p_i^* - t \le \varepsilon_i^*\}$$

$$= \Pr\{\max_{1 \le i \le N}\{\varepsilon_i\} \ge p_i^* - t\} = 1 - \Pr\{\max_{1 \le i \le N}\{\varepsilon_i\} \le p_i^* - t\}$$

$$= 1 - F(p_i^* - t)^N$$

So the density function of $\tilde{p}$

$$g(t) = N \cdot f(p^* - t) \cdot F(p^* - t)^{N-1}$$

Hence the expected value of is

$$Exp(\tilde{p}) = \int_{-\infty}^{+\infty} t \cdot g(t) \cdot dt = \int_{-\infty}^{+\infty} N \cdot t \cdot f(p^* - t) \cdot F(p^* - t)^{N-1} \cdot dt$$

Errors in probability of being Good

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- Defining $u = p^* - t$, we get

$$Exp(\tilde{p}) = \int_{-\infty}^{+\infty} (p^* - u) \cdot N \cdot f(u) \cdot F(u)^{N-1} \cdot du$$
$$= p^* \cdot \int_{-\infty}^{+\infty} N \cdot f(u) \cdot F(u)^{N-1} \cdot du - \int_{-\infty}^{+\infty} u \cdot N \cdot f(u) \cdot F(u)^{N-1} \cdot du$$

Hence $Exp(\tilde{p}) = p^* - b_N$

since $\int_{-\infty}^{+\infty} N \cdot f(u) \cdot F(u)^{N-1} \cdot du = \left[F(u)^N\right]$

and $b_N = \int_{-\infty}^{+\infty} u \cdot N \cdot f(u) \cdot F(u)^{N-1} \cdot du$

- The linear relationship in this equation is similar to that discussed in Phillips (2005) and Thomas (2008) where $Exp[\tilde{p}(r, p)] = p - dr, d > 0$ where it is called the linear probability adverse selection function.

Errors in probability of being Good

If one assumes the error in the probability is a uniform distribution which spreads more the higher the rate charged, i.e. $[-dr, dr]$ then the above calculation given

$$b_N = \int_{-\infty}^{+\infty} u \cdot N \cdot f(u) \cdot F(u)^{N-1} \cdot du = \int_{-dr}^{+dr} \frac{u}{2dr} \cdot N \cdot \left(\frac{u+dr}{2dr}\right)^{N-1} \cdot du = dr \frac{(N-1)}{(N+1)}$$

The argument is that if the lender increases the interest rate, they are willing to accept applicants with a lower probability of being Good and this will lead to a wider range of errors

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Errors in the score

- The probability of the individual applicant being Good is not directly observed by issuers. Instead, the lenders collect data on previous borrowers with similar characteristics and translate this information into a credit score s for that applicant.

- To model this, suppose the customer with characteristics x will be given credit score of $s(x)$ which relates to the probability of the customer being good.

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Errors in the score

- Thomas (2009) imply how use logistic regression to build a score card leads to a log odds score where the relationship between the credit score and the probability of being Good is given by

$$s(x) = \log\left[\frac{p(x)}{1-p(x)}\right] \Rightarrow p(x) = \frac{e^{s(x)}}{1+e^{s(x)}}$$

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Errors in the score

- Suppose the errors that the lenders made are directly in the score they give the applicant, which translates into errors in the probability of the applicant being Good.
- Let $\tilde{s}$ be the "true" score of the applicant and this corresponds to $\tilde{p}$, the true probability of the applicant being Good.
- Assume lender i has a scorecard which gives that applicant a score s_i^*, where $\tilde{s} = s_i^* - \varepsilon_i$.
- Again we assume the applicant will choose the lender who gives the highest score since under risk based pricing this will lead to the offer which the lowest rate required on the credit card.
- Assume that there are N potential lenders and ε_i the scoring errors made by each lender are independent and have a common distribution with distribution function $F(\cdot)$.

Errors in the score

We can follow the calculations of the previous error type to get the following results.

$$G(t) = \Pr\{\tilde{p} \le t\} = \Pr\left\{\log\frac{\tilde{p}}{1-\tilde{p}} \le \log\frac{t}{1-t}\right\} = \Pr\left\{\tilde{s} \le \log\frac{t}{1-t}\right\}$$

$$= \Pr\left\{s_i * -\varepsilon_i * \le \log\frac{t}{1-t}\right\} = \Pr\left\{\log\frac{p*}{1-p*} - \log\frac{t}{1-t} \le \varepsilon_i *\right\}$$

since

$$\Pr\{\varepsilon_i * \ge y\} = \Pr\{\max[\varepsilon_1, \varepsilon_2, \varepsilon_3 ..., \varepsilon_N] \ge y\} = 1 - \Pr\{\max[\varepsilon_1, \varepsilon_2, \varepsilon_3 ..., \varepsilon_N] < y\}$$

$$= 1 - \prod_{i=1}^{N} \Pr\{\varepsilon_i < y\} = 1 - F(y)^N$$

We have $G(t) = 1 - F\left(\log\frac{p_i^*}{1-p_i^*} - \log\frac{t}{1-t}\right)^N$

Errors in the score

Then, the probability density function is

$$g(t)=\frac{N}{t\cdot(1-t)}f\left(\log\frac{p^*}{1-p^*}-\log\frac{t}{1-t}\right)F\left(\log\frac{p^*}{1-p^*}-\log\frac{t}{1-t}\right)^{N-1}$$

So, for the lender who has taken the applicant assuming his score is , then the expected true credit score of that applicant will be given by

$$E(\tilde{s})=E(s_i^*)-E(\varepsilon_i^*)=s_i^*-\int_{-\infty}^{\infty}yf(y)F(y)^{N-1}dy=s_i^*-b_N$$

In terms of probabilities of being Good this becomes

$$E(\log\left(\frac{\tilde{p}}{1-\tilde{p}}\right))=E(\log\left(\frac{p^*}{1-p^*}\right))-E(\varepsilon_i^*)=E\left(\log\left(\frac{p^*}{1-p^*}\right)\right)-\int_{-\infty}^{\infty}yf(y)F(y)^{N-1}dy=E\left(\log\left(\frac{p^*}{1-p^*}\right)\right)-b_N$$

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Errors in the score

In the case when the score errors are uniformly distributed from $-dr$ to dr, where again is the interest rate charged one gets

$$E\left(\log\left(\frac{\tilde{p}}{1-\tilde{p}}\right)\right) = E\left(\log\left(\frac{p^*}{1-p^*}\right)\right) - E(\varepsilon_i^*) = \log\left(\frac{p^*}{1-p^*}\right) - dr\frac{N-1}{N+1}$$

which is strongly related to the linear log odds selection function suggested in (Thomas 2008) and (Phillips 2005).

Impact of Adverse Selection on Risk-based Pricing

- Risk-based pricing means that the interest rate charged on a loan to a potential borrower depends on the lender's view of the borrower's default risk.
- In particular we are interested in the fact that these winner's curse selection errors are a form of adverse selection since they increase as the interest rate charged increases.
- We use a rate model which looks at the profitability of deciding whether to lending one unit to an applicant (see Thomas 2008).

Impact of Adverse Selection on Risk-based Pricing

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- In the previous section we described how the winner's curse can lead to a linear or logistic relationship between these two probabilities.

- In this case, the lender is assuming that the probability of the lender being Good is that probability p^* that would

$E(\tilde{p}) = p^* - er$, and $E(\log\frac{\tilde{p}}{1-\tilde{p}}) = \log\frac{p^*}{1-p^*} - er$

- In general we denote this relationship as $\tilde{p}(r, p^*)$

Impact of Adverse Selection on Risk-based Pricing

- We assume that the risk free rate at which the lender can borrower the money is r_F and the loss given default (the percentage of defaulted loan finally lost) is l_D .
- If the lender charges a rate r to an applicant whose probability of being Good is p, then the take probability, (the chance the applicant will accept such a loan) is $q(r, p)$

Impact of Adverse Selection on Risk-based Pricing

- If the lender believes the borrower has a probability p of being Good, then the lender believes the expected profit if a rate $r(p)$ is charged to be

$$EP(r,p) = q(r,p)[(r(p) - r_F)\cdot p - (l_D + r_F)\cdot(1-p)]$$

- In order to find the optimal interest rate, we differentiate this equation with respect to r and set the derivate to zero, to find when the profit is optimised. This gives a risk based interest rate of

$$r^*(p) = r_F + (l_D + r_F)\cdot\frac{1-p}{p} - \frac{q(r,p)}{\partial/\partial r\, q(r,p)}$$

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Impact of Adverse Selection on Risk-based Pricing

The reality through is that the lender's estimate of the probability of the borrower being Good is p^*, where the true probability is $\tilde{p}$.

- The optimal profit the lender would possibly obtain from such a borrower if the lender had the correct view of the borrower's probability of being Good would be

$$EP_{opt}\left[r^*(\tilde{p}),\tilde{p}\right]=q\left(r^*(\tilde{p}),\tilde{p}\right)\cdot\left[\left(r^*(\tilde{p})-r_F\right)\cdot\tilde{p}-\left(l_D+r_F\right)\cdot\left(1-\tilde{p}\right)\right] \quad 1)$$

- However, the lender's estimate of the borrower's probability of being Good is , and so what the lender expects the profit to be is

$$EP_{\exp}\left[r^*\left(p^*\right),p^*\right]=q\left(r^*\left(p^*\right),p^*\right)\cdot\left[\left(r^*\left(p^*\right)-r_F\right)\cdot p^*-\left(l_D+r_F\right)\cdot\left(1-p^*\right)\right] \quad 2)$$

even though the borrower's true probability is $\tilde{p}$

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Impact of Adverse Selection on Risk-based Pricing

- In fact, the borrower will not live up to this expectation and the true expected profit the lender will get is

$$EP_{true}\left[r^*\left(p^*\right),\tilde{p}\right]=q\left(r^*\left(p^*\right),\tilde{p}\right)\cdot\left[\left(r^*\left(p^*\right)-r_F\right)\cdot\tilde{p}-\left(l_D+r_F\right)\cdot\left(1-\tilde{p}\right)\right] \qquad 3)$$

Numerical Examples

- Consider two examples, the first is where the take function and the adverse selection function are linear in form and the second is where both are logistic in form. Take functions of those two forms were discussed in Phillip (2005). In both cases we assume the risk free rate is 5%, $r_F = 0.05$ and the loss given default $l_D = 0.5$.

Linear Relationship Model

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- For the linear, take probability or response rate function define

$$q(r,p)=\min\{\max[0,1-b(r-r_L)+c\cdot(1-p),1]\} \quad \text{for} \quad 0\le p\le 1$$

- Then the optimal interest rate is

$$r^*(p)=r_F+(l_D+r_F)\cdot\frac{1-p}{p}-\frac{1-b\cdot(r-r_L)+c\cdot(1-p)}{b}$$

We choose the value with $r_L=0.04$, $b=2.5$, and $c = 2$

- If we assume the relationship between $\tilde{p}$ and p^* is given by a uniform error, then we get a linear relationship between $\tilde{p}$ and p^*.So that, from previous section, we have

$$\tilde{p} = p_i^* - dr(\frac{n-1}{n+1}), \quad d > 0$$

Linear Relationship Model

- and if we assume $N = 500$, $d = 0.15$ then $\tilde{p} = p_i^* - 0.1494 \cdot r$
- The results of applying these relationships in 1), 2) and 3) lead to the results in Table 1:

$\tilde{p}$	EP_{true}	EP_{opt}	p^*	EP_{exp}
0.3	-0.05638	0.0000	0.3	0.0000
0.4	0.000257	0.018773	0.4	0.018
0.5	0.035762	0.044867	0.5	0.044204
0.6	0.061176	0.067491	0.6	0.065896
0.7	0.081096	0.085253	0.7	0.085078
0.8	0.091737	0.093626	0.8	0.094044
0.9	0.09291	0.094666	0.9	0.094861
0.94	0.093676	0.095306	0.94	0.095365
0.96	0.09391	0.09548	0.96	0.0955
0.98	0.094048	0.095559	0.98	0.095561

Table 1: Results of a linear probability adverse selection function.

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Linear Relationship Model

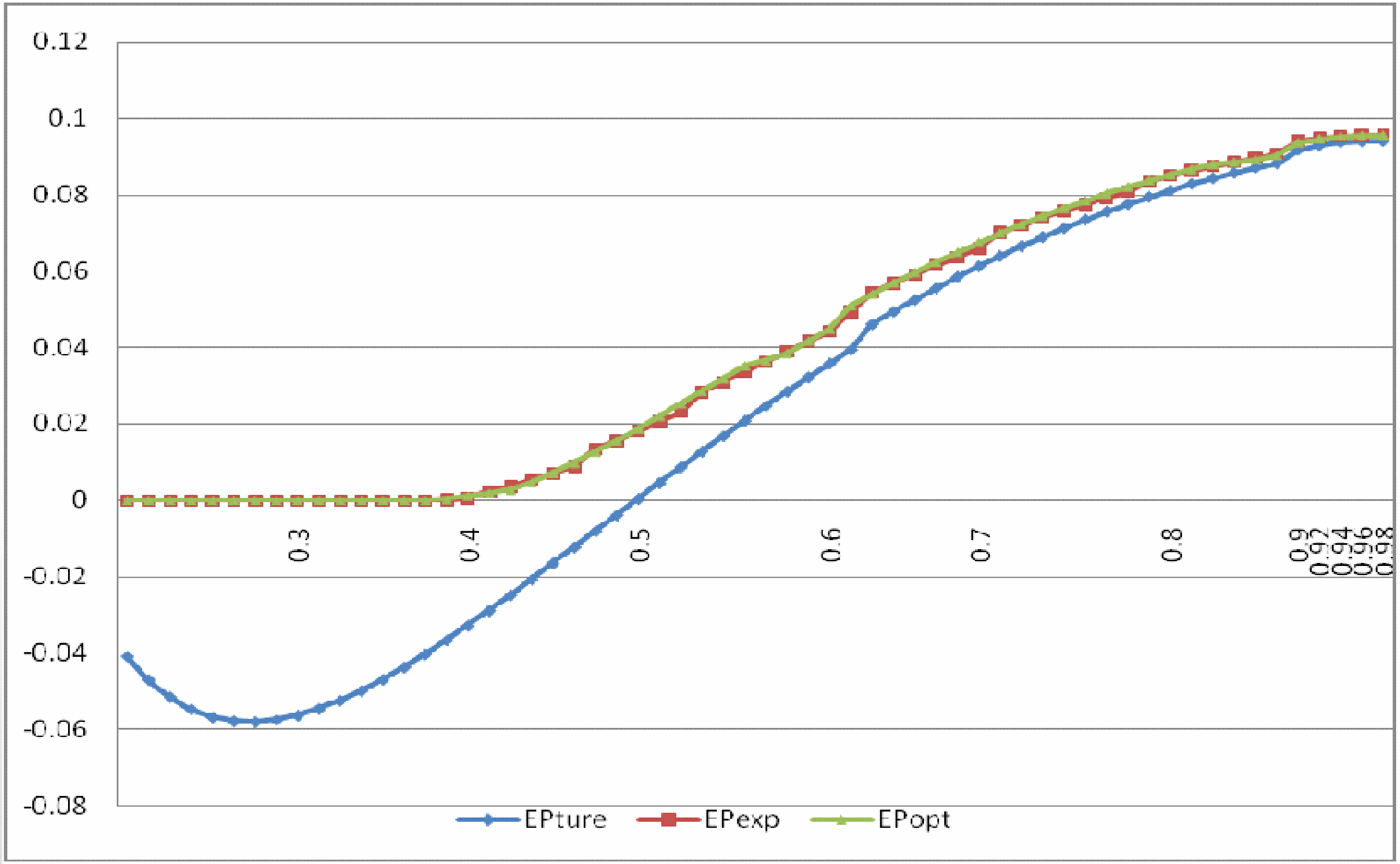

Figure 1: Plot of results of a linear model.

Logistic Model

- The logistic risk-based response function or take rate is

$$q(r,p)=\frac{e^{a-br-cp}}{1+e^{a-br-cp}}$$

- The optimal interest rate in this case is

$$r^{*}(p)=r_F+(l_D+r_F)\cdot\frac{1-p}{p}-\frac{1+e^{a-br-cp}}{b}$$

Using the cost structure of the previous case with risk-free rate being 0.05 and the loss given default l_D being 0.5, we assume the parameters for the logistic response rate function are $a=54$, $b=32$,and $c=50$.

Logistic Model

- Assume the relationship between $\tilde{p}$ and p^* is linear in the log odds corresponding to the situation in section 2 where the error is in the score. We assume that it is given by

$$\log\frac{\tilde{p}}{1-\tilde{p}}=\log\frac{p^*}{1-p^*}-dr(\frac{n-1}{n+1}),d>0$$

$$\Rightarrow \tilde{p} = \frac{p}{(1-p)\cdot e^{dr\frac{N-1}{N+1}}+p}$$

We take the values $N=500$ and $d=4$ so that error between the log odds is $\tilde{p}=\frac{p}{(1-p)\cdot e^{3.98r}+p}$

Logistic Model

- The results of applying these relationships in 1), 2) and 3) lead to the results in Table 2

$\tilde{p}$	EP_{true}	EP_{opt}	p^*	EP_{exp}
0.3	-0.256505902	0.000108971	0.3	0.000108973
0.4	-0.179800649	0.04958824	0.4	0.049588431
0.5	-0.120515218	0.103989042	0.5	0.103988875
0.6	-0.046397856	0.146820976	0.6	0.145201731
0.7	0.00021474	0.15051339	0.7	0.150765621
0.8	0.035509349	0.132797723	0.8	0.133179458
0.9	0.060262605	0.090140422	0.9	0.091745987
0.92	0.060924634	0.078527762	0.92	0.075977514
0.94	0.058631672	0.067225836	0.94	0.065011196
0.96	0.053439103	0.056521742	0.96	0.053937944
0.98	0.046080056	0.046653585	0.98	0.048451123

Table 2: Results of a linear log odds probability adverse selection.

Logistic Model

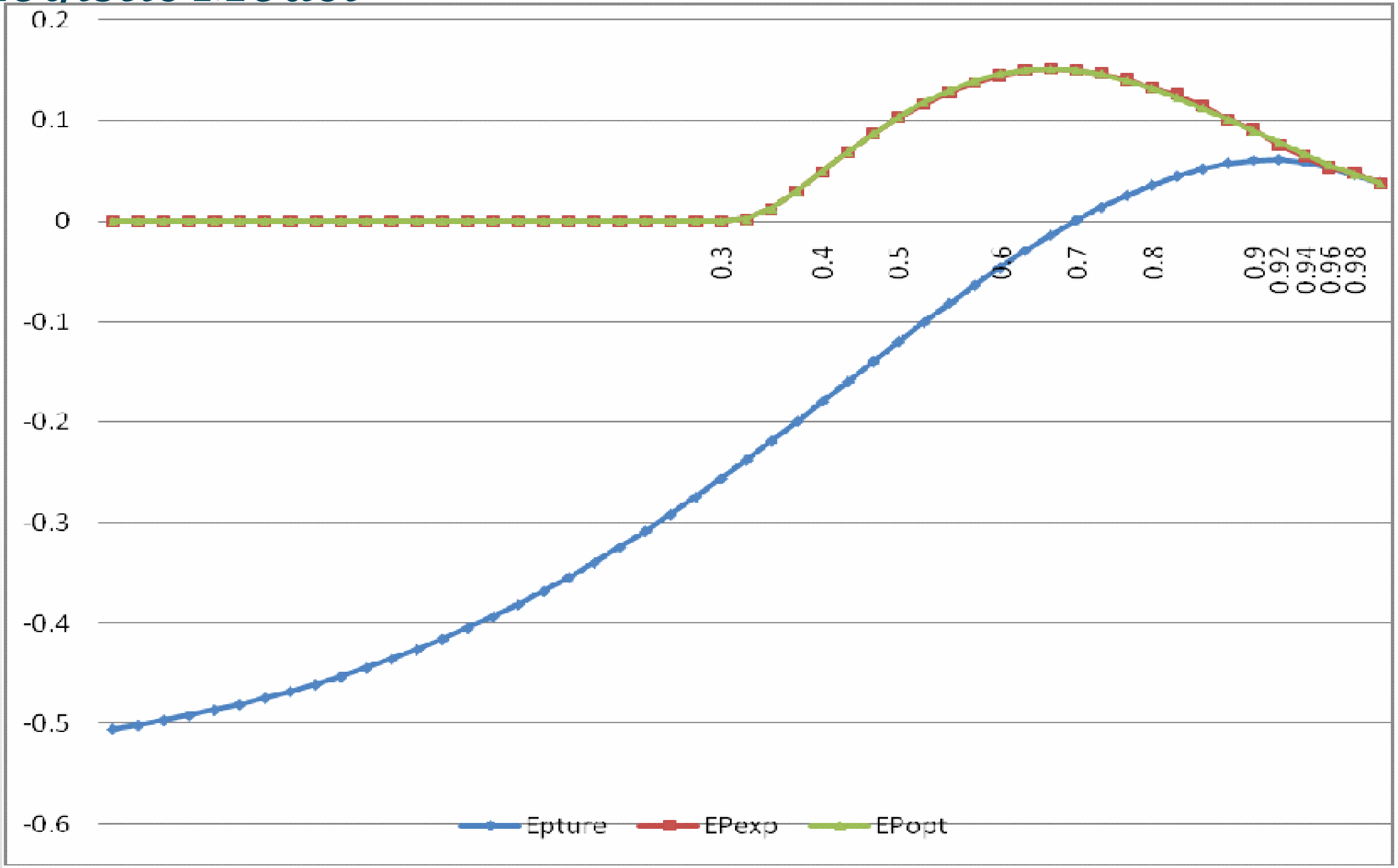

Figure 2: Plot of results of a linear log odds probability adverse selection.

Conclusions

- We show how modelling the way a borrower selects a loan as an auction, means that winner's curse leads to adverse selection.
- We show that the relationships between the actual default risk of a borrower and the lender's perceived view of this risk are very simple ones, whatever the distribution of the errors the lender makes.
- By building a simple model of the profit a lender makes from a loan, we are able to examine the effect of these adverse selection errors.

Conclusions

- One way out of this dilemma is for the lender to allow for the fact he will misrepresent the risk of the borrower, when calculating the optimal rate to charge.
- The difficultly with this is that the population who take the loan depend critically on the variable rates being offered, and one of the strengths of variable pricing is that one can vary the rates to respond to charges in the market.

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