

# Asset Correlations for Credit Card Defaults

**Jonathan Crook**  
**Tony Bellotti**

**Credit Research Centre**  
**University of Edinburgh, UK**

## Abstract

The capital requirements formula within the Basel II Accord is based on a Merton one factor model and in the case of credit cards an asset correlation of 4% is assumed. In this paper we estimate the asset correlation for two datasets assuming the one factor model. The datasets relate to a national sample of credit card accounts and the second to a specific credit card portfolio. We show that the asset correlations assumed by Basel II are much higher than those observed in the datasets we analyse. We also segment one of the datasets and show the variation in asset correlations between risk segments. We also show the reduced importance of unobserved factors in explaining default patterns when observable, as opposed to unobservable, macroeconomic factors are included in the model. We then show the reduction in capital requirements that a typical lender would have if the values we estimated were implemented in the Basel Accord in place of the current values.

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Credit Research Centre, Business School, WRB 50 George Square, University of Edinburgh, Edinburgh EH8 9JY, UK

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Corresponding author: [j.crook@ed.ac.uk](mailto:j.crook@ed.ac.uk)

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## 1. Introduction

The aim of this paper is to present estimates of asset correlations for credit card portfolios. The recent difficulties of banks throughout the world has focussed attention on the amount and detailed nature of regulations that the banking sector operates under. Large banks in the G10 countries are obliged by their regulators to follow the Basel II Accord (Basel Committee on Banking Supervision: 2006) when deciding on the amount of capital to hold to protect their shareholders against operational, market and credit risk. Under the Accord banks may follow a standardised approach or an IRB approach to calculating their risk capital. Those following the latter, probably the majority of large banks, must calculate their capital requirements for different loan segments using an equation based on the Vasicek formula (Vasicek 1987) with different assumed asset correlation parameters for different types of loan portfolio. For many retail segments, such as mortgages and qualifying revolving retail exposures, the Accord assumes an asset correlation of a specific value: 15% for residential mortgages and 4% for revolving exposures. For other types of retail loans, and for other segments, for example corporate loans and loans to small and medium sized enterprises, the asset correlation depends on the long run probability of default for that portfolio, (as well as maturity for non retail exposures).

However the empirical work on which the assumed asset correlation values for mortgages and revolving credit portfolios were based has not been published and an embryonic literature, mainly examining corporate loans and loans to banks, finds asset correlation values much lower than the values implied by the Basel formula. For example Rösch (2003) estimated corporate asset correlations in Germany by modelling the number of bankruptcies over time. He found values around 0.86% when macroeconomic variables were omitted and 0.52% when they were included. Hamerle et al (2003a) found comparable values for many industries in the G7 countries. In other work Rösch (2005) found that corporate asset correlations varied according to rating class, where the class was allocated by an external ratings agency. In this work Rösch did not include borrower specific effects, however in Hamerle and Rösch (2006) this was attempted for a sample of German firms in manufacturing and

in commerce. The estimates were even smaller, declining from 0.6% to 0.04% when company specific variables were included. Estimates of asset correlations for consumer loans or for mortgages are rare. Rösch and Scheule (2004) considered three exposure classes, residential loans, credit cards and other consumer loans from US commercial banks. When, following the Basel Accord, they assume the default probabilities are constant over time, asset correlations of 1.0% for credit cards and 0.98% for real estate loans were estimated. When macroeconomic variables were included, the asset correlations fell to 0.66% and 0.28% respectively.

However there is no published work in which the asset correlations for consumer loan portfolios are estimated and where individual specific effects are included, and there is no published work that reports asset correlations for UK credit card portfolios. This is the aim of this paper. In principle one would expect that the more covariates that are used to explain the probability of default by a borrower the lower the computed asset correlation and empirical evidence for other sectors appear to confirm this. We use two datasets, one relating to individual borrowers and the second relates to all credit cards issued in the UK. We find asset correlation values that are very considerably below the value assumed in the Accord and that the correlation varies systematically with the riskiness of a borrower segment.

The structure of this paper is as follows. In the following section we specify the model and estimation strategy that we use. In section three we present our results and in section four we discuss some implications of them. Section five concludes.

### *Notational Conventions*

Throughout the paper we adopt the following conventions. We assume time is measured in discrete intervals. Let  $\mathbf{x}_i$  denote a vector of variables whose observed values are specific to individual  $i$  ( $i = 1 \dots N$ ), but that do not vary over time. Let  $\mathbf{x}_{it}$  denote a vector of variables whose observed values vary between individuals and between time periods ( $t = 1 \dots T$ ). Let  $\mathbf{z}_t$  denote a vector of variables whose observed values vary between time periods, but not between individuals. Let  $Y_t$  denote a variable whose values vary over time but which are not observable. We denote

individual variables in upper case and their realisations in lower case. We use  $l$  to denote an arbitrary lag. We write  $P(\bullet | Y_t = y_t)$  as  $p(y_t)$  for convenience.

## 2. The Model

The model follows Hamerle et al (2003b), Hamerle and Rösch (2006) and Rösch (2005). Consider the following to apply within a particular risk segment. Let  $A_{it}$  denote a borrower's assets and  $a_{it-1}$  denote the realisation of his asset's in a previous period. These may be in natural logs. Let the return on his assets be denoted  $R_{it}$ , so  $R_{it} = A_{it} - a_{it-1}$ . We assume  $R_{it}$  is an unobserved latent variable and is linearly related to its mean,  $\mu_{it}$ , an unobserved random time specific effect,  $Y_t$ , and a random component,  $\varepsilon_{it}$ . Thus we can write

$$R_{it} = \mu_{it} + bY_t + c\varepsilon_{it}, \quad (1)$$

where  $R_{it}$  is normally distributed with mean  $\mu_{it}$  and variance  $\sigma^2$ ;  $Y_t \sim N(0,1)$  and  $\varepsilon_{it} \sim N(0,1)$ . We assume  $Y_t$  and  $\varepsilon_{it}$  are independent and  $Y_t$  is serially uncorrelated. The variance of  $bY_t$  is  $b^2$  and that of  $\varepsilon_{it}$  is  $c^2$  and so, given independence of  $Y_t$  and  $\varepsilon_{it}$ ,  $\sigma^2 = b^2 + c^2$ . One implication of equation (1) is that conditional on  $Y_t$  values of  $R$  for any two cases,  $i$  and  $j$ , are independent. Another implication is that since  $R_{it}$  is unobserved we cannot estimate  $c$ .

Equation (1) may be standardised to give

$$\frac{R_{it} - \mu_{it}}{\sigma} = \frac{b}{\sigma} Y_t + \frac{c}{\sigma} \varepsilon_{it}. \quad (2)$$

Now suppose, following Gordy (2003) and Hamerle et al (2003b) we write equation (2) as

$$\frac{R_{it} - \mu_{it}}{\sigma} = \tilde{b} Y_t + \sqrt{1 - \tilde{b}^2} \varepsilon_{it}, \quad (3)$$

where  $\tilde{b} = \frac{b}{\sigma}$ . The correlation between the standardised values of  $R_{it}$  and  $R_{jt}$  is

$$\tilde{b}^2 = \left( \frac{b}{\sigma} \right)^2 \quad (\text{Schonbucher: 2000}).$$

Following Merton (1974) a borrower defaults when the value of his assets falls below a threshold level. If we denote this threshold as  $k_{it}$  the probability of default for borrower  $i$  is

$$\begin{aligned} P(A_{it} < k_{it}) &= P\left( \frac{R_{it} - \mu_{it}}{\sigma} < \frac{k_{it} - a_{it-1} - \mu_{it}}{\sigma} \right) \\ &= P\left( \tilde{b} Y_t + \sqrt{1 - \tilde{b}^2} \varepsilon_{it} < \alpha_{it} \right), \end{aligned} \quad (4)$$

where  $\alpha_{it} = \frac{k_{it} - a_{it-1} - \mu_{it}}{\sigma}$ . Thus the probability of default can be written as

$$P\left( \varepsilon_{it} < \frac{\alpha_{it} - \tilde{b} Y_t}{\sqrt{1 - \tilde{b}^2}} \right), \quad (5)$$

and conditional on the realisation  $Y_t = y_t$  this probability equals

$$p(y_t) = \Phi\left( \frac{\alpha_{it} - \tilde{b} y_t}{\sqrt{1 - \tilde{b}^2}} \right), \quad (6)$$

where  $\Phi$  denotes the standard normal cumulative distribution.

Equation (3) assumes that the standardised return does not vary systematically over time; instead it varies only randomly according to  $Y_t$  and  $\varepsilon_{it}$ . This is the assumption of Basel 2. But now suppose that the standardised return, and so the probability that case  $i$  defaults, is affected by observable states of the economy, possibly lagged. For example if interest rates increase we may expect that an average individual is less able

to repay his outstanding loans. If, following Hamerle et al (2003b and 2006), we assume that the mean of the return depends linearly on observable macroeconomic variables, as well as characteristics of a borrower, we can replace equation (1) by

$$R_{it} = \beta_0 + \mathbf{x}_i^T \boldsymbol{\beta} + \mathbf{x}_{it}^T \boldsymbol{\gamma} + \mathbf{z}_t^T \boldsymbol{\delta} + bY_t + c\varepsilon_{it}, \quad (7)$$

where  $\beta_0$  is a constant and  $\boldsymbol{\beta}, \boldsymbol{\gamma}$  and  $\boldsymbol{\delta}$  are vectors of parameters to be estimated. By substitution equation (6) becomes

$$P(A_{it} < k_{it} | \mathbf{x}_i = x_i, \mathbf{x}_{it} = x_{it}, \mathbf{z}_t = z_t, Y_t) = \Phi \left( \frac{\alpha_{it} - x_i^T \tilde{\boldsymbol{\beta}} - x_{it}^T \tilde{\boldsymbol{\gamma}} - z_t^T \tilde{\boldsymbol{\delta}} - \tilde{b} Y_t}{\sqrt{1 - \tilde{b}^2}} \right). \quad (8)$$

where  $\tilde{\boldsymbol{\beta}} = \boldsymbol{\beta} / \sigma$ ,  $\tilde{\boldsymbol{\gamma}} = \boldsymbol{\gamma} / \sigma$  and  $\tilde{\boldsymbol{\delta}} = \boldsymbol{\delta} / \sigma$ .

To parameterise this model we follow Hamerle et al (2006) equation (8) and also assume  $\alpha_{it}$  is a constant. There the log-likelihood is derived by supposing that we observe a particular default pattern across borrowers in period  $t$ , and finding an expression that equals the probability of observing that pattern, conditional on the realisation  $y_t$  of  $Y_t$ . Thus if we observe a default pattern  $(d_{it}, d_{2t}, \dots, d_{Nt})$  where  $d_{it} = 1$  denotes that borrower  $i$  in period  $t$  defaults, the log-likelihood is

$$LL = \sum_{t=1}^T \ln \left( \int_{-\infty}^{+\infty} h(y_t) \phi(y_t) dy_t \right), \quad (9)$$

where  $h(y_t) = \prod_{i=1}^{N_t} P_{it}(\bullet | Y_t = y_t)^{d_{it}} (1 - P_{it}(\bullet | Y_t = y_t))^{1-d_{it}}$  and  $\phi$  denotes the standard normal density function.

Essentially equation (8), together with the default condition, is a mixed fixed effects - random effect probit model where the fixed effects are  $\mathbf{x}_i^T \tilde{\boldsymbol{\beta}} / \sqrt{1 - \tilde{b}^2}$ ,  $\mathbf{x}_{it}^T \tilde{\boldsymbol{\gamma}} / \sqrt{1 - \tilde{b}^2}$  and  $\mathbf{z}_t^T \tilde{\boldsymbol{\delta}} / \sqrt{1 - \tilde{b}^2}$  and the random effect is

$(\tilde{b} / \sqrt{1 - \tilde{b}^2}) Y_t$ . Notice that unlike more conventional panel models the random effect is over time not over cases. Correspondingly the integration in the maximum likelihood function (equation 9) is over the random effect that varies over time.

One can parameterise such a model by maximising the LL function where  $\frac{\tilde{b}}{\sqrt{1 - \tilde{b}^2}}$  is the variance of the random effect and the asset correlation between any two borrowers within the same segment is

$$\rho = \frac{\left( \frac{\tilde{b}}{\sqrt{1 - \tilde{b}^2}} \right)^2}{\left( \frac{\tilde{b}}{\sqrt{1 - \tilde{b}^2}} \right)^2 + \left( \frac{\sqrt{1 - \tilde{b}^2}}{\sqrt{1 - \tilde{b}^2}} \right)^2} = \tilde{b}^2. \quad (10)$$

The value of  $\left( \frac{\tilde{b}}{\sqrt{1 - \tilde{b}^2}} \right)$  can be estimated as part of the maximum likelihood estimation and so we can calculate the asset correlation using equation (10). We parameterised equations (6) and (8) using maximum likelihood and estimated the standard error of rho using the delta method of Billingsley (1986).

Alternatively one can obtain estimates of the asset correlation in a portfolio using aggregated data instead of data for individual borrowers. Here we closely follow the models of Gordy and Heitfield (2002), Rösch (2003) and Rösch (2005). Define

$$D_t = \sum_{i=1}^{N_t} d_{it} \text{ where } d_{it} \text{ is defined above and } N_t \text{ is the number of active borrowers in}$$

period  $t$ . Assume equation (1) explains returns. By assumption, conditional on the realisation  $Y_t = y_t$ , the returns of any two borrowers within a segment are independent. Therefore the distribution of the number of defaults in period  $t$ , conditional on the realisation of  $Y_t$ , is binomial with parameters  $(N_t, p(y_t))$  where  $p(y_t)$  is given by equation (6). The probability of  $D_t$  defaults in period  $t$  may be written as

$$l(D_t | Y_t) = \binom{N_t}{D_t} p(y_t)^{D_t} (1 - p(y_t))^{N_t - D_t}. \quad (11)$$

By assumption  $Y_t$  and  $\varepsilon_{it}$  are serially independent so the unconditional marginal log-likelihood (which we denote LL) can be found by integrating equation (11) over all possible values of  $Y_t$ , taking the product over time periods  $t=1 \dots T$  and taking logs. This gives

$$LL = \sum_{t=1}^T \ln \left( \int_{-\infty}^{+\infty} \binom{N_t}{D_t} p(y_t)^{D_t} (1 - p(y_t))^{N_t - D_t} \phi(y_t) dy_t \right). \quad (12)$$

Alternatively, if we assume returns are explained by equation (7) rather than by equation (1), then  $p(y_t)$  in equation (12) may be replaced by  $p(y_t, z_t)$ , which is equation (8) with  $\beta$  and  $\gamma$  restricted to a vector of zeros.

For an individual risk segment equation (12) allows one to parameterise equation (6) (or equation (8) with the above restrictions) using merely data on the number of defaults, the number of active accounts and observable time varying covariates. As Rösch (2005) states, if time varying macroeconomic variables are included in equation (6) the proportion of the variance of  $R_{it}$  that is explained by the random effect will be lower than if the macroeconomic variables were omitted.

We parameterised equation (6) using the NLMIXED procedure in SAS. We used Gaussian adaptive quadrature (Pinheiro and Bates: 1995) to compute the integral over the time varying marginal effects and the delta method of Billingsley (1986) to compute the approximate standard errors for rho. Both are standard procedures in SAS.

### 3. Results

#### 3.1 Account Level Data

We use two datasets. The first dataset is a randomly selected sample of approximately 200,000 credit card accounts that relate to a single credit card issued by a financial institution. We define default as missing a third due monthly payment or the account being written off, and the time of default is the first month in which either event happens. The observations cover 87 consecutive months from the late 1990s to the mid 2000s. The dataset is unbalanced: accounts were opened at different points in time. Some defaulted in the observation period, others did not. Due to software constraints we took a random sample of 20% of the accounts to estimate the parameters of equation (6) for the portfolio as a whole. The results are shown in Table 1. These show the estimated asset correlation value of 0.396%. This is considerably below the value of 4.00% given in the June 2006 version of the Basel II formula for revolving accounts not in default. The Basel Accord states that the risk weight formula is to apply to segments of equal risk within a portfolio, ('risk buckets'). In practice risk segments are often identified by borrowers operating in the same country or, in the case of corporate loans, in the same industry. In the case of credit cards one could segment a portfolio according to very many variables, for example income level, occupation, address region, age and so on. To investigate the effect of a segmentation we divided the portfolio into risk groups where each group consisted of borrowers in the same decile of generic risk score. Each decile therefore consisted of approximately 20,000 accounts. The results of estimating equation (6) for selected deciles is shown in Table 2. The results show a systematic relationship between asset correlation and generic risk score. At higher risk score deciles (lower risk) the asset correlation is smaller than at higher risk score deciles (higher risk). In decile 10, the asset correlation was not statistically different from zero.

Table 3 shows the effect of additional covariates on the estimated value of  $\rho$ . We consider two alternative equations; one including a variable that varies over cases and over time: balance divided by credit limit and a equation which omits such a variable. As expected, in both cases when more covariates are added the value of  $\rho$  declines.

The equations suggest some collinearity because when balance divided by credit limit is included, many of the other variables become insignificant.

However a caveat to this section is in order. The account level dataset spans only 87 months. The second dataset spans a longer period.

### 3.2 Aggregate Data

The second dataset is quarterly time series data relating to the repayment performance of all credit card issuers in the UK and was kindly supplied by APACS. The data period is from 1990 Q2 to 2007 Q4 and so spans a much longer period than the account level data.. This dataset does not show the number of accounts that defaulted in each time period, but instead shows the number of accounts that were overdue by various periods, for example by one month, two months and so on. We also had the number of accounts that were written off in each quarter as well as the number of active accounts. We defined a default event as being when an account became three months overdue. The relationship between these concepts can be written as:

$$D_t - R_t = O_t - O_{t-1} + W_t, \quad (13)$$

where  $D_t$  is the number of accounts that became three months overdue in month  $t$ ;  $R_t$  denotes the number of accounts that were more than 3 months overdue in  $t-1$  that repaid sufficient amounts to be just 3 months overdue in period  $t$ ;  $O_t$  denotes the number of accounts that were three or more months overdue in period  $t$ ; and  $W_t$  denotes the number of accounts that were written off in period  $t$ . The dataset contained information on  $O_t$  and  $W_t$  and prior research experience by the authors suggests that the number of accounts that are 3 months overdue that repay enough to be just three months overdue is very small and so we assume that  $R_t = 0$ . If this assumption is untrue then we are modelling  $D_t - R_t$ , the net change in the number of accounts 3 months overdue between  $t$  and  $t-1$  rather than  $D_t$ . The time series of the number of defaults as a percentage of active accounts is shown in Figure 1. The data show a discernable upward trend since around May 1997, albeit with some

perturbations. Table 4 shows the results of parameterising equation (6). These suggests that the asset correlation of the portfolio of credit cards made available by UK issuers is around 1.8% when estimated in this way. This is considerably higher than the 0.4-0.6% which estimates based on the borrower level dataset suggested. Both sets of values are considerably lower than the values assumed by the Basel 2 formula.

#### 4. Implications

Considering both sets of results the values of asset correlations that we have estimated are similar to those found by other researchers. For example, values around 1% for credit cards in the US were found by Rösch and Scheule (2004). Hamerele et al (2006) found values around 0.6% for manufacturing and 0.1% for Commerce in Germany in the 1990s. However Rösch (2005) is an exception that finds asset correlations for US corporates around 5% when modelling defaults for three separate risk grades, BB, B and CCC. We also found that higher risk segments had higher asset correlations. This was not found by Rösch (op cit) who found that whilst the CCC grade had the highest correlation the lowest was for the B grade. The reduction in the value of rho when other covariates are included in the model is expected and consistent with previous studies.

The Basle II Accord gives the capital requirement,  $K$ , for a risk segment of a portfolio of qualifying revolving retail exposures as

$$K = \Phi \left( \frac{\Phi^{-1}(PD) + \rho \Phi^{-1}(0.999)}{\sqrt{1 - \rho}} \right), \quad (14)$$

where PD is the mean probability of default. Table 5 shows the implications for capital requirements for differing PDs and asset correlations. It can be seen that if the mean PD is around 3% and the asset correlations of 0.4% or even 0.6% were applied, the capital requirement, as a factor of LGD would be approximately 25% of that required by the Accord with its given asset correlation of 4%. The over capitalisation

is larger if the mean default rate is lower. For example if the mean default rate was 1% the capital requirement would be around 23% of the value required under the Accord.

Our finding that higher asset correlations occur in higher risk bands might be concerning to lenders because it is the higher risk band borrowers who are most likely to default. Nevertheless the asset correlations, even in the highest risk band in our data, were not as large as the assumed correlation in the Accord, and it is the assumptions of the Accord that are currently enforced by regulators.

A further implication is that equation (14) is effectively equation (6) with  $\alpha_{it}$  assumed constant for the segment and the value of  $Y_t$  set at  $\Phi^{-1}(0.999)$ , its extreme expected value on 0.1% of occasions. With an assumed values of  $\rho$  and  $p(y_t)$  one can estimate a mean PD from this equation. This mean PD is unaffected by changes in  $Y_t$  because the value of  $Y_t$  has been fixed. The estimated value of PD is therefore a through the cycle estimate and this estimation method has been advocated by for example Forrest (?) as a method of estimating a TTC PD for portfolios with few, if indeed, any observed defaults. However if the value of  $\rho$  is assumed to be 4% when in fact it is merely 25% of this value then the TTC PD will be considerably in error. Those who wish to use this method might be wise to consider lower values of  $\rho$ .

## 5. Conclusions

Although the Basel II Accord requires banks who wish to follow an IRB approach to estimating their capital requirements to use an asset correlation for credit card loans of 4%, our results suggest that this is considerably larger than the asset correlations that occur in practice. Our results suggest a value of around half of the required amount. This finding adds to those of other studies which have concentrated on corporate loans so that this conclusion applied to both corporate and consumer loans. We have found that asset correlations are greater for riskier credit card borrowers and that there may be noticeable differences between the correlations of different lenders.

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**Table 1**  
**Borrower Level Dataset Parameterisations of Equation (6)**

|           | Const | sd ( $Y_t$ ) | $\rho$    |
|-----------|-------|--------------|-----------|
| Coeff     | na**  | 0.0630**     | 0.00396** |
| SE        |       | (0.0093)     | (0.0012)  |
| Chi Sq(1) |       |              | 45.35     |

No Groups = 87

Av no of obs per group 19,879

Na = not available for confidentiality reasons.

\* denotes significance at 5%; \*\* denotes significance at 1%

The significance of the standard deviation is determined by dividing the coefficient by its standard error and assuming this has an approximate t-distribution.

Ref: \stata\p3p11.log

**Table 2**  
**Parameterisations of Equation (6) for Score deciles**

|          |           | Const    | $\text{sd} \left( \frac{\tilde{b}}{\sqrt{1-\hat{b}^2}} . Y \right)$ | $\rho$   | Av no of<br>obs per group<br>(N x T) |
|----------|-----------|----------|---|----------|--------------------------------------|
| Decile 1 | Coeff     | -2.575** | 0.0803**  | 0.0064** | 8,089                                |
|          | SE        | (0.0113) | (0.0108)  | (0.0017) |                                      |
|          | Chi Sq(1) |          |   | 67.59    |                                      |
| Decile 3 | Coeff     | -2.756** | 0.0590**  | 0.0035** | 9,695                                |
|          | SE        | (0.0097) | (0.0097)  | (0.0011) |                                      |
|          | ChiSq(1)  |          |   | 25.64    |                                      |
| Decile 5 | Coeff     | -2.902** | 0.0506**  | 0.0026** | 10,759                               |
|          | SE        | (0.0098) | (0.0117)  | (0.0012) |                                      |
|          | Chi Sq(1) |          |   | 9.03     |                                      |
| Decile 7 | Coeff     | -3.024** | 0.0466**  | 0.0022   | 11,238                               |
|          | SE        | (0.0106) | (0.0148)  | (0.0014) |                                      |
|          | Chi Sq(1) |          |   | 3.88     |                                      |
| Decile 9 | Coeff     | -3.333** | 0.0334  | 0.0011   | 9,598                                |
|          | SE        | (0.0165) | (0.0011)  | (0.0029) |                                      |
|          | Chi Sq(1) |          |   | 0.16     |                                      |

No groups=87.

\* denotes significance at 5%; \*\* denotes significance at 1%

The significance of the standard deviation is determined by dividing the coefficient by its standard error and assuming this has an approximate t-distribution

Ref: stata\p3p14.log, stata\p3p15.log, stata\p3p16.log, stata\p3p19.log

**Table 3**  
**Borrower dataset: Parameterisation of Equation (8)**

|  | <b>Coeff</b> | <b>z-stat</b>  | <b>Coeff</b> | <b>z-stat</b>   |
|--|--------------|----------------|--------------|-----------------|
| Const  | -2.626       | -171.67        | -3.870       | -109.85         |
| Income<br>(Base Age: 18-24)                  | -0.000       | -0.030         | 0.000        | 0.84            |
| Age 25-29                                    | -0.073       | -3.82**        | 0.052        | 2.14**          |
| Age 30-33                                    | -0.103       | -5.10**        | -0.005       | -0.20           |
| Age 34-37                                    | -0.078       | -3.86**        | 0.028        | 1.08            |
| Age 38-41                                    | -0.138       | -6.33**        | -0.023       | -0.84           |
| Age 42-47                                    | -0.112       | -5.46**        | 0.015        | 0.59            |
| Age 48-55                                    | -0.200       | -9.15**        | -0.039       | -1.42           |
| Age 56 plus<br>(Base: non self<br>-employed) | -0.286       | -11.78**       | 0.016        | 0.05            |
| Self Employed                                | 0.60         | 3.62**         | 0.052        | 2.51**          |
| Time with bank                               | -0.001       | -16.54**       | -0.0004      | -6.42**         |
| No of cards                                  | -0.008       | -1.47          | -0.010       | -1.73           |
| Balance/credit limit                         |              |                | 1.809        | 66.90**         |
| $\Delta$ Interest rate                       | -0.117       | -1.89          | 0.123        | 1.55            |
| $\Delta$ Unemployment                        | 0.003        | 2.06**         | 0.002        | 1.23            |
| $\Delta$ Real earnings                       | 1.230        | 2.54**         | 0.427        | 0.70            |
| $\Delta$ House price                         |              |                | -0.000       | -1.92           |
| $\Delta$ FTSE                                |              |                |              |                 |
| $\Delta$ Total Credit                        |              |                | -0.000       | -0.79           |
| Sd( $Y_t$ )                                  | 0.047        | 5.53**         | 0.050        | 5.22            |
| $\rho$                                       | 0.0022       | Chi sq 20.38** | 0.0024       | Ch sq (1)=15.85 |

\* denotes significance at 5%; \*\* denotes significance at 1%

The significance of the standard deviation is determined by dividing the coefficient by its standard error and assuming this has an approximate t-distribution

**Table 4**  
**Aggregate Level Dataset Parameterisations of Equation (6)**

|       | Const    | sd ( $Y_t$ ) | $\rho$   |
|-------|----------|--------------|----------|
| Coeff | -2.642** | 0.1379**     | 0.0187** |
| SE    | 0.0164   | 0.0116       | 0.0031   |

No Groups (time periods) = 71

Av no of obs per group = 1

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\* denotes significance at 5%; \*\* denotes significance at 1%  
 The significance of the standard deviation is determined by dividing the coefficient by its standard error and assuming this has an approximate t-distribution.

Ref: \APACS\2.html

**Table 5**  
**Capital Requirements as a Multiple of LGD implied by Basel II Accord**

| <b>PD</b>   | $\rho$      | <b>Capital Requirement</b> |
|-------------|-------------|----------------------------|
| 0.01        | 0.004       | 0.006373                   |
| 0.01        | 0.006       | 0.008163                   |
| <i>0.01</i> | <i>0.04</i> | <i>0.030621</i>            |
| 0.02        | 0.004       | 0.011299                   |
| 0.02        | 0.006       | 0.014391                   |
| <i>0.02</i> | <i>0.04</i> | <i>0.051418</i>            |
| 0.03        | 0.004       | 0.015635                   |
| 0.03        | 0.006       | 0.019844                   |
| <i>0.03</i> | <i>0.04</i> | <i>0.068735</i>            |

Basel assumed parameters in italics.

### Defaults on UK Credit Cards

