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Asset Correlations for Credit Card Portfolios

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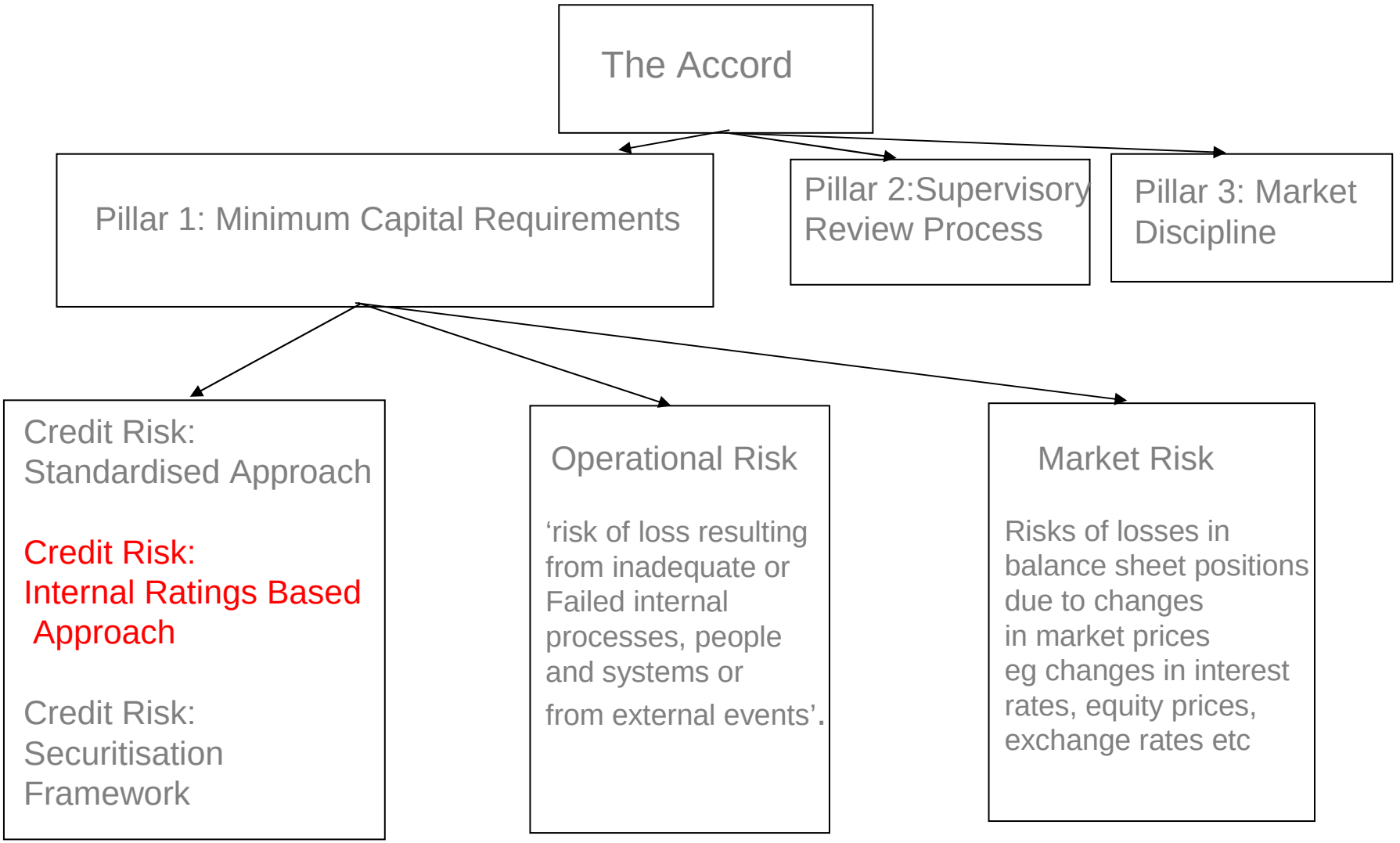
Agenda

- Outline of Basel 2
- Estimation for account level data
- Estimation for aggregate level data
- Results
- Conclusions

Basel 2 Capital Accord



RWA for Revolving Exposures

Required for institution as a whole : RWA > Capital

Risk weighted assets = capital requirements for (Market risk + Operational risk) x 12.5

+ sum of risk weighted assets over different types of credit risk.

For (non defaulted) revolving credit exposures

Risk-weighted assets = K x 12.5 x EAD

Capital Requirement $(K) = LGD.\Phi\left(\frac{\Phi^{-1}(PD) + \rho.\Phi^{-1}(0.999)}{\sqrt{1-\rho}}\right) - PD.LGD$

Asset correlation (ρ) = 0.04

Φ = cumulative distribution function for a standard normal random variable
PD = probability of default
LGD = Loss given default
EAD=Exposure at default

Previous Estimates of Asset Correlations

Corporate

Hamerle & Rosch (2006) acct level, 1991-2000, N=53,280 firms, **Bankruptcy,** Germany

	Without MEVs	**With credit score and MEVs**
Manufacturing	0.58%	0.043%
Commerce	0.14%	0.014%

Hamerle et al (2003) 5 industrial sectors in each of G7 countries, aggregate data, 10 to 20 year time spans. **'defaults'**

Canada	0.24% - 0.60%	France	0.11% - 0.71%	Germany	1.51%-0.61%
GB	1.08% -1.44%	Italy	0.23%-0.44%	Japan	0.35%-1.97%
USA	0.43% - 2.27%				

Retail

Rosche & Scheule (2004) aggregate data, US **charge-off rates**
No MEVs (3MEVs)

Residential	0.98% (0.28%)	Credit card	1.02% (0.66%)	Other	0.73% (0.44%)
1991-2001		1985-2001		1985-2001	

Some Approaches

Credit Portfolio View

$$P_{it} = F\left(\alpha_t + \mathbf{x}_i^T \boldsymbol{\beta}_A + \mathbf{x}_{i(t-l)}^T \boldsymbol{\beta}_B + \mathbf{z}_{a_i+t-l}^T \boldsymbol{\beta}_M\right)$$

Correlation: through macroeconomic variables (MEVs)

Credit Metrics

$$R_{it} = \sqrt{\rho} Y_t + \sqrt{1-\rho}\varepsilon_{it}$$

Correlation: fixed ρ

$$P_{it} = \Phi\left(\frac{k - \sqrt{\rho} Y_t}{\sqrt{1-\rho}}\right)$$

The Model 1

Let R_{it} denote return on assets so $R_{it} = A_{it} - a_{it}$

Assume $R_{it} = \mu_{it} + bY_t + c\varepsilon_{it},$

$$R_{it} \sim N(\mu_{it}, \sigma^2); \quad Y_t \sim N(0,1); \quad \varepsilon_{it} \sim N(0,1);$$

Y_t and ε_{it} are independent; Y_t is serially uncorrelated.

Standardise R_{it} to give $\frac{R_{it} - \mu_{it}}{\sigma} = \frac{b}{\sigma} Y_t + \frac{c}{\sigma} \varepsilon_{it}$ but now write this as

$$\frac{R_{it} - \mu_{it}}{\sigma} = \tilde{b} Y_t + \sqrt{1 - \tilde{b}^2}\, \varepsilon_{it}$$

Following Merton (1974) a borrower defaults when his assets fall below a threshold level k_{it} so probability of default is

$$P(A_{it} < k_{it}) = P\left(\frac{R_{it} - \mu_{it}}{\sigma} < \frac{k_{it} - a_{it-1} - \mu_{it}}{\sigma} \right)$$

$$= P(\tilde{b} Y_t + \sqrt{1 - \tilde{b}^2}\, \varepsilon_i < \alpha_{it}) \quad \text{where } \alpha_{it} = (k_{it} - a_{it-1} - \mu_{it})/\sigma$$

The Model 2

Conditional on the realisation $Y_t = y_t$ the probability of default is

$$p(y_t) = \Phi\left(\frac{\alpha_{it} - \tilde{b} y_t}{\sqrt{1-\tilde{b}^2}}\right) \qquad (1)$$

New assumption: $R_{it} = \beta_0 + \mathbf{x}_i^T \boldsymbol{\beta}_A + \mathbf{x}_{i(a_i+t-l)}^T \boldsymbol{\beta}_B + \mathbf{z}_{a_i+t-l}^T \boldsymbol{\beta}_M + bY_t + c\varepsilon_{it}$

Eq (1) becomes

$$P(A_{it} < k_{it} \mid \mathbf{x}_i = x_i, \mathbf{x}_{it} = x_{it}, \mathbf{z}_t = z_t, Y_t) = \Phi\left(\frac{\alpha_{it} - \mathbf{x}_i^T \tilde{\boldsymbol{\beta}}_A - \mathbf{x}_{ia_i+t-l}^T \tilde{\boldsymbol{\beta}}_B - \mathbf{z}_{a_i+t-l}^T \tilde{\boldsymbol{\beta}}_M - \tilde{b} Y_t}{\sqrt{1-\tilde{b}^2}}\right) \qquad (2)$$

The Model 3

Following Hamerle (2006) log-likelihood of observing the default pattern $(d_{1t}, d_{2t}, \ldots d_{Nt})$

$$L(\theta) = \sum_{t=1}^{T} \ln\left(\int_{-\infty}^{+\infty} \prod_{i=1}^{N_t} P_{it}(\bullet \mid Y_t = y_t)^{d_{it}} (1 - P_{it}(\bullet \mid Y_t = y_t))^{1-d_{it}} \phi(y_t) dy_t \right)$$

Equations (1) and (2) are mixed effects-random effects probit models

Asset correlation between *i* and *j* in same segment is $\rho = \tilde{b}^2$

where $\frac{\tilde{b}}{\sqrt{1-\tilde{b}^2}}$ is the variance of the random effect.

Model: Aggregate Data

Let $D_t = \sum_{i=1}^{N_t} d_{it}$

By assumption, conditional on the realisation $Y_t = y_t$, the returns on 2 borrowers are independent. So distribution of no of defaults, conditional on the realisation of Y_t is binomial, parameters $(N_t, p(y_t))$

Likelihood of D_t defaults in period t is

$$L(D_t \mid Y_t = y_t) = \binom{N_t}{D_t} p(y_t)^{D_t} (1 - p(y_t))^{N_t - D_t}$$

Unconditional marginal likelihood is

$$LL = \sum_{t=1}^{T} \ln \left(\int_{-\infty}^{+\infty} \binom{N_t}{D_t} p(y_t)^{D_t} (1 - p(y_t))^{N_t - D_t} \phi(y_t) dy_t \right)$$

Result: Borrower Level

Account Level Data

Approx 40,000 credit card accounts covering 87 months ending mid 2000s

Const	sd$\left(\frac{\tilde{b}}{\sqrt{1-\tilde{b}^2}}\right)Y_t$	ρ
na	0.0630**	0.00396**

Results Borrower Level

Decile	sd $\left(\frac{\tilde{b}}{\sqrt{1-\tilde{b}^2}}\right)Y_t$	ρ
1	0.0803**	0.0064**
3	0.0590**	0.0035**
5	0.0506**	0.0026**
7	0.0466**	0.0022*
10	0.0334**	0.0011

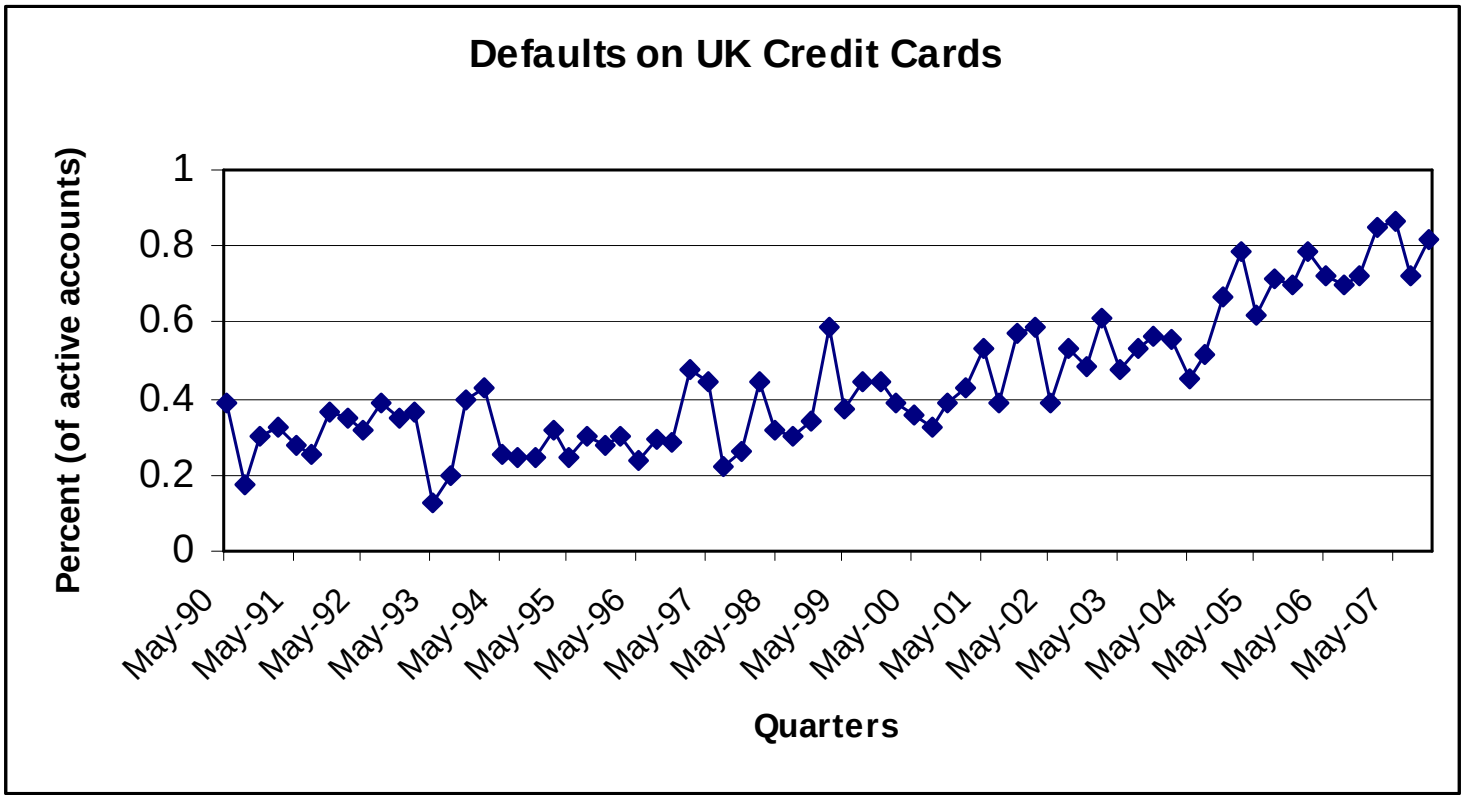
Asset correlation declines as risk declines

Parameterization Model 2

	Coeff Appln Model	Coeff Behavl Model
Constant	- **	-**
Income	-	+
(Base Age: 18-24)		
Age 25-29	-**	+**
Age 30-33	-**	-
Age 34-37	-**	+
Age 38-41	-**	-
Age 42-47	-**	+
Age 48-55	-**	-
Age 56 plus	-**	+
(Base: non self-emp)		
Self Employed	+ **	+**
Time with lender	-**	-**
No of cards	-	-
Balance/credit limit		+**
ΔInterest rate	-	+
ΔUnemployment	+**	+
ΔReal earnings	+**	+
ΔHouse price		-
ΔTotal Credit		-
Sd(Y_t)	**0.047****	**0.050****
ρ	0.0022 $\chi^2(1)=20.38$**	0.0024 $\chi^2(1)=15.85$**

* denotes significance at 5%; ** denotes significance at 1%

Number of Defaults on UK Credit Cards



Results Aggregate Level

Data for all credit cards issued in UK. Source APACS. Quarterly 1990Q1 to 2007Q4

Const	sd $\left(\frac{\tilde{b}}{\sqrt{1-\tilde{b}^2}}\right)Y_t$	ρ
-2.642**	0.1379**	0.0187**

No of time periods: 87

Capital Requirements as a Multiple of LGD implied by Basel II Accord

Range of Hypothetical PDs

PD	ρ	Capital Requirement
0.01	0.004	0.006373
0.01	0.006	0.008163
0.01	***0.040***	***0.030621***
0.02	0.004	0.011299
0.02	0.006	0.014391
0.02	***0.040***	***0.051418***
0.03	0.004	0.015635
0.03	0.006	0.019844
0.03	***0.040***	***0.068735***

Basel (2006) assumed parameters in red.

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Conclusion

We have estimated asset correlations for credit cards to be between 1 and 2% which is much less than that assumed by Basel II (4%).

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