

Investment Decision in a New Credit Score System¹

Gyorgy Varga
FCE Consulting/Brazil
varga@fce.com.br

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Comments are welcome!

This article provides a framework to decide upon the timing to purchase a new Credit Score System. This is an irreversible investment hardly useful anywhere else, and can always postpone its purchase. Therefore the investment in a new CSS is financially similar to a call option, whereby the investment is the exercise price and the option premium is the value to postpone the investment. We use the real option modeling to decide on the timing to purchase a new system. The results suggest that only the cost of a new system and its average time decay are relevant.

1. Introduction

The Credit Scoring System (CSS) is a tool to automate and facilitate the credit concession especially to a large number of clients. It has been a fundamental tool to decrease the cost of credit concession and certainly responsible for the huge credit concession provided by banks (or any credit provider, for instance, a retail store). The CSS aims to classify credit applicants as good or bad payers based on characteristics of each applicant. In general those models are created with techniques from econometrics or pattern recognition analysis. The results are not perfect and the model quality is gauged by their errors. Shifting economic conditions, population changes, model flaws and several other reasons impose errors and time decay on the current CSS. Banks and any user of CSSs are constantly measuring and reevaluating their systems to obtain an acceptable total error. Although a model with lower total error is always desired it comes with a cost.

The investment decision in a new CSS shall depend on the total error of the current system, the error of a new system and its cost. A bank has always the option to postpone the decision of buying a new CSS. According to Thomas et al. (2002) page 161, this is an open question; there is no framework to decide upon the timing to purchase a new CSS.

This article provides a framework to decide upon the timing to buy a new CSS. The investment in a new CSS is irreversible and once done can hardly be used somewhere else. A CSS user can always postpone its purchase, he has the option to acquire a new CSS at a later time, profiting for the possibility of new information to arrive and change the need for a new CSS. Therefore the investment in a new CSS is financially similar to a call option, whereby the investment is the exercise price and the option premium is the value to postpone the investment. We assume that CSS errors grow (the system decays) with time and at any time the errors can be bigger or smaller than expected. Therefore we treat the system errors as a random variable and the main source of uncertainty in the credit concession problem. We define a stochastic process for this random variable and calculate the option value of postponing the purchase of a new CSS. We basically apply the real options technique to provide a framework for this problem. In this framework the variability of the CSS error will have a great impact on the timing to purchase a new CSS.

Another technique to decide on the timing would be the NPV rule, which is not adequate because it does not measure the value of the option to acquire the CSS at a future time. A positive NPV is not enough to decide on a new CSS, this value should be bigger than the option to invest at a future time.

The real option approach has been applied to the problem of investment decision on new software by Sullivan et al. (1999), which is a very similar problem of deciding upon a new CSS. The real option approach to the investment decision is extensively described in Dixit and Pindyck (1994).

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The next section defines a framework for the credit decision, in the third section we show the real option model for new CSS purchase, in the fourth section we perform a sensitivity analysis and conclude in the last section.

2. Credit Decision

The environment of our model is one where the credit provider, for simplicity the bank, maximizes its profit. In order to maximize, the bank has to decide on the amount v to lend². We assume that the bank has a total of d to lend out and pays r on the total money available d , and charges from each client $r+s$.

The CSS cost I , and can generate two types of errors. It can forecast as bad an applicant which in fact is a good payer (usually called error type I), we call the population percentage of the goods that are misclassified as α . Another error is to forecast as good an applicant which in fact is a bad payer (usually called error type II); we call the population percentage of the bad payers that are misclassified as β . Therefore the total system errors are:

$$[\alpha p + \beta(1 - p)] \quad (1)$$

where p is the percentage of the population that are good payers.

We assume that at the current market spread (s) the demand for loans from individuals considered good is bigger than the amount available d :

$$[(1 - \alpha_t)p + \beta_t(1 - p)]x_t > d$$

The credit decision is:

$$\begin{aligned} & \text{Max } [(1 + r + s)(1 - \alpha_t)p - \beta_t(1 - p)]v_t - d(1 + r) - I \\ & \{v_t\} \\ & \text{subject to } [(1 - \alpha_t)p + \beta_t(1 - p)]v_t \leq d. \end{aligned} \quad (2)$$

At each period t the bank should choose the optimum credit conceded v_t . As long as the expected gain is positive, the optimum credit concession is the maximum possible d , otherwise it is zero.

The sequence $\{\alpha_t, \beta_t\}_{t \geq 1}$ defines the CSS quality or their total errors. The values of $\{\alpha_t, \beta_t\}$ range from 0 to 1, its maximum errors of 1 imply that all loans have been given to bad payers. From a CSS without forecasting power, one would expect both errors been 0.5, this is the error of a pure random model. In fact, borrowers can learn how to gamble with the system and one can end up with a CSS with errors below 0.5.

3. CSS Investment Decision

The investment decision faced by the bank is whether to purchase a new CSS, which depends on the performance of the CSS that they currently own. The higher the CSS errors more incentive the bank will have to purchase a new one. The errors are summarized by $\{\alpha_t, \beta_t\}$ and are uncertain.

In our model all sources of uncertainty come from these two variables. Any new CSS is defined by a flat cost of I , initial errors of $\{\alpha_0, \beta_0\}$, and a sequence of errors $\{\alpha_t, \beta_t\}$. The CSS decay is characterized by both errors growing at a rate $\mu > 0$ and volatility σ per period. For simplicity, we also assume that the errors are unrelated to any other variable from the credit decision even the proportion (p) of good payers within the population.

² The credit concession problem can have a different formulation than (2), in fact, it should be done as a result of a portfolio optimization (see Baensens and Van Gestel 2009).

A positive growth rate (μ) leads to a higher error, which characterizes the CSS decay. There is also an upper limit at $\{1,1\}$, which is the maximum error and a lower limit at $\{0,0\}$ which is the minimum error. We assume a lognormal diffusion process for both errors. This avoids negative values for the errors but not the upper boundary of 1, but once this boundary is far from the initial values, we will simply neglect this constraint. These two lognormal processes are approximated by a trinomial tree according to Amin (2001) as shown in figure 1. The jumps (upward and downward) and their probabilities (q) are chosen to match the growth rate, volatility of the errors and their boundaries.

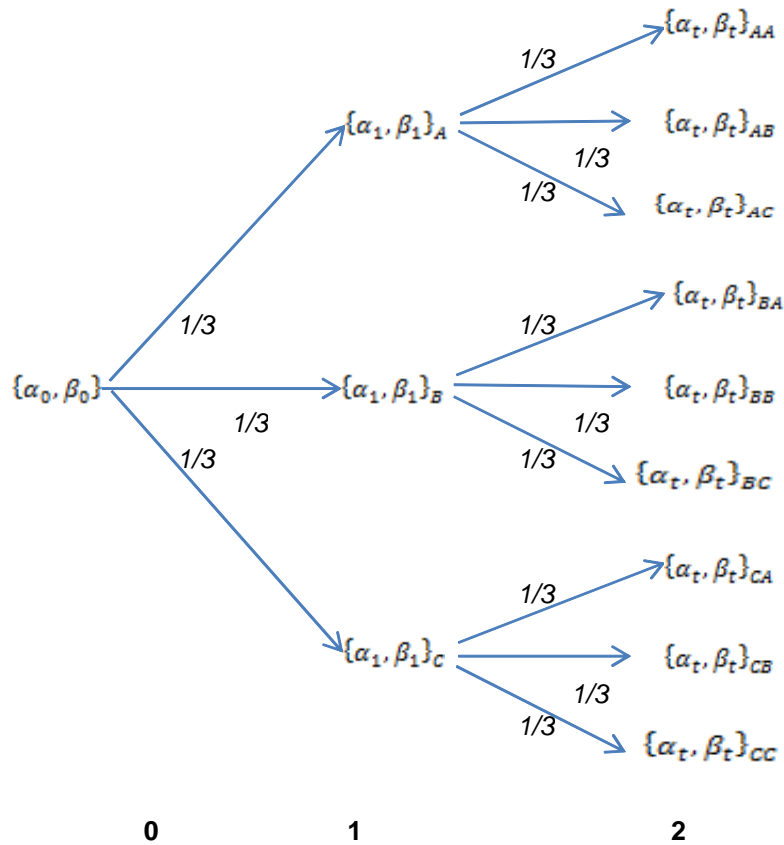


Figure 1. Trinomial tree representing the error's dynamic.

Then there is the question of timing the acquisition of a new CSS. The purchase of a new CSS will on average diminish the errors, which is always good, but there is a cost to do it and the decision can be postponed. We evaluate this decision model and calculate the value of the option to postpone this decision. The best decision is the one that is most valuable.

If the errors increase, the value of a new CSS is even bigger but if not, postponing the purchase of a new CSS is not so attractive. The value of waiting to invest must be compared to the current loss of not having a new CSS. This value depends on the variability of the error and the discount rate. A bank may maintain a poor performance CSS due to a possible improvement in the future. The errors should increase persistently in order to trigger the purchase of a new CSS.

We assume that at any period, the bank can purchase a new CSS paying I . Any new CSS has the same initial error $\{\alpha_0, \beta_0\}$ and the same dynamic decay, regardless of when it is purchased.

The decision is also path dependent; a higher error in any moment may trigger the decision to buy a new CSS if the error in past periods also increased.

To illustrate the application of the model we develop the following example: A bank has a total amount (d) of 100 available to lend, the cost of a new CSS is 2, the spread is 10% per period, the interest rate is 6% per period and the proportion of good payers 80%. A new CSS has initial errors of {10%, 10%}, their average decay per period is 15%, its standard deviation 10% and its correlation 50%. The trinomial tree for this CSS error is in figure 2.

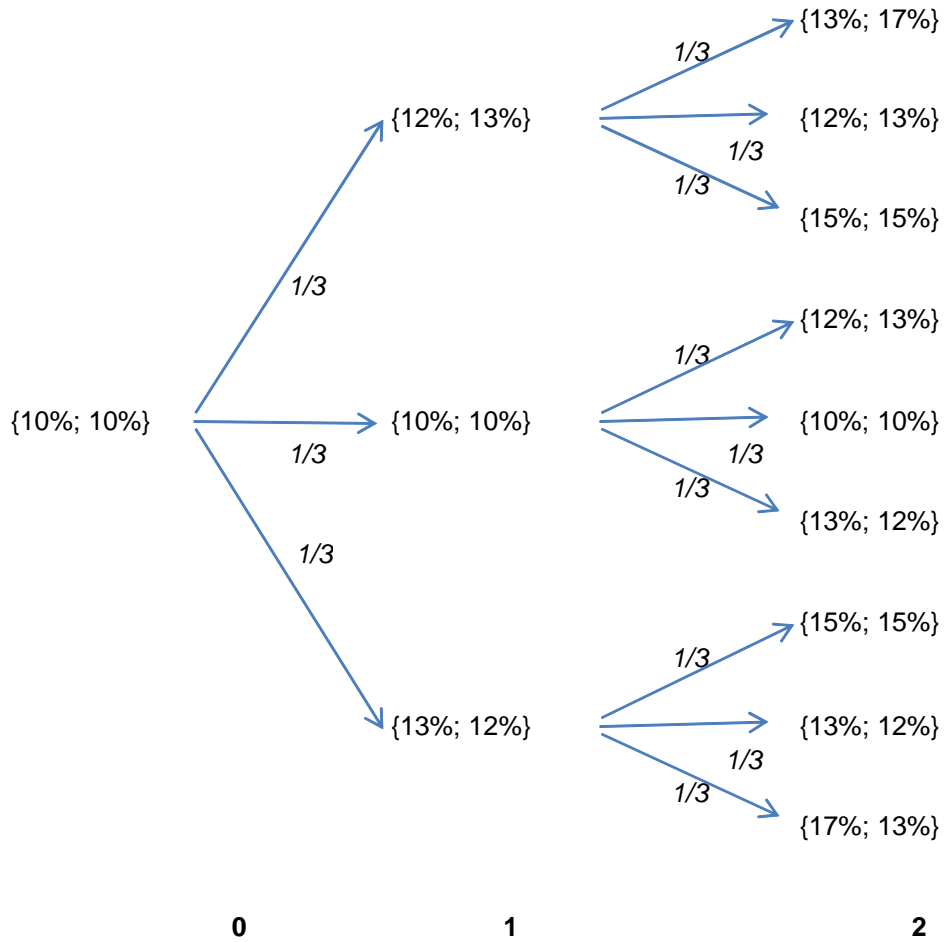


Figure 2. Trinomial tree for correlated errors.

With this tree we can calculate the result for each pair of errors at each period and node. In figure 3 we show the resulting profit and loss (P&L) for each realization of the trinomial tree from figure 2.

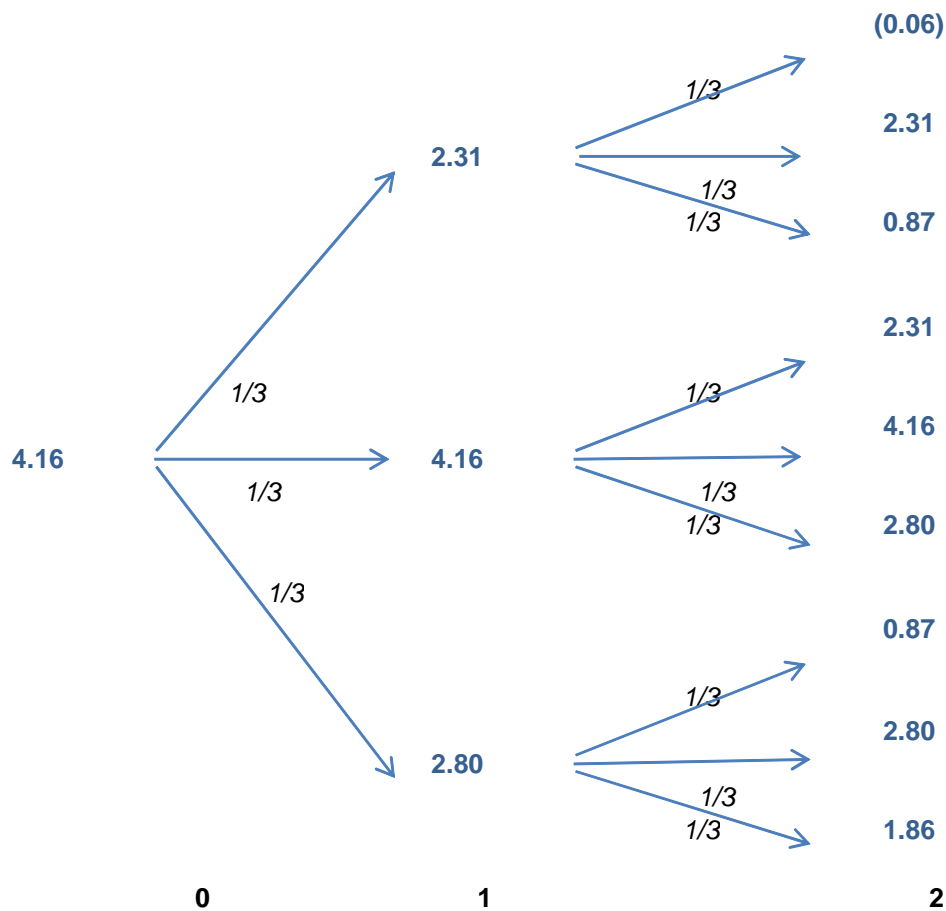


Figure 3. The resulting P&L for each realization for the process from figure 2.

Now we analyze three investment alternatives for the bank: (1) no change in the CSS; (2) buy a new CSS at period 0 and (3) buy a new CSS only at period 1. The purchase of a new CSS should be paid in the same period of the decision, but its effects are perceived only in the next period. For case (1) figure 3 shows the complete cash flow until period 2, the expected value of this decision is 8.85. In case (2), there should be a payment of 2 in period 0, and the cash flow at period 1 will change based on the new CSS. The cash flow for Period 1 will be constant and equal to period 0 in the former case, and the cash flow of period 2 will be equal to period 1 in the former case, figure 4 shows this complete flow. The expected value for this case is 8.84.

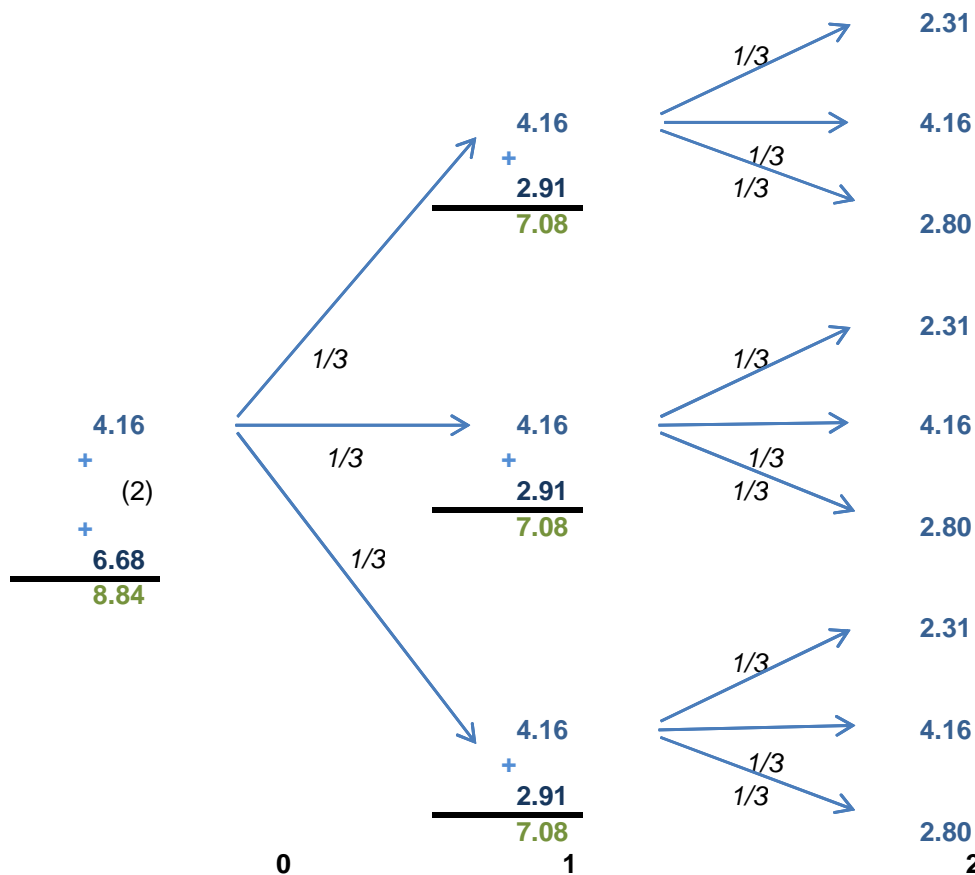


Figure 4. The resulting P&L for each realization for the process with a new CSS purchased at period 0.

In the last case the purchase of a new CSS occurs at period 1, implying a cost of 2 in this period. The cash flow at period 2 should also change to incorporate the effect of the new CSS. In figure 5 we show the flow and the expected value is 8.89.

Comparing the three alternatives, the most valuable one is (3), waiting until period 1 to purchase a new CSS. The traditional investment analysis based on NPV would lead to the decision (2) because its NPV is positive, but postponing this decision has greater value, and the bank does have this option. If we wait one period to see what happens to $\{\alpha_t, \beta_t\}$, the NPV can be even bigger depending on the realized tree node.

If the investment were reversible (could be sold next period) then as long as NPV is positive it should be done, depending on what happens to the next period error the system can be sold. Another similar issue arises from the fact that the bank cannot wait to make the investment, for instance, in the case of a higher competition that decreases the demand for loans ($d < v_t$).

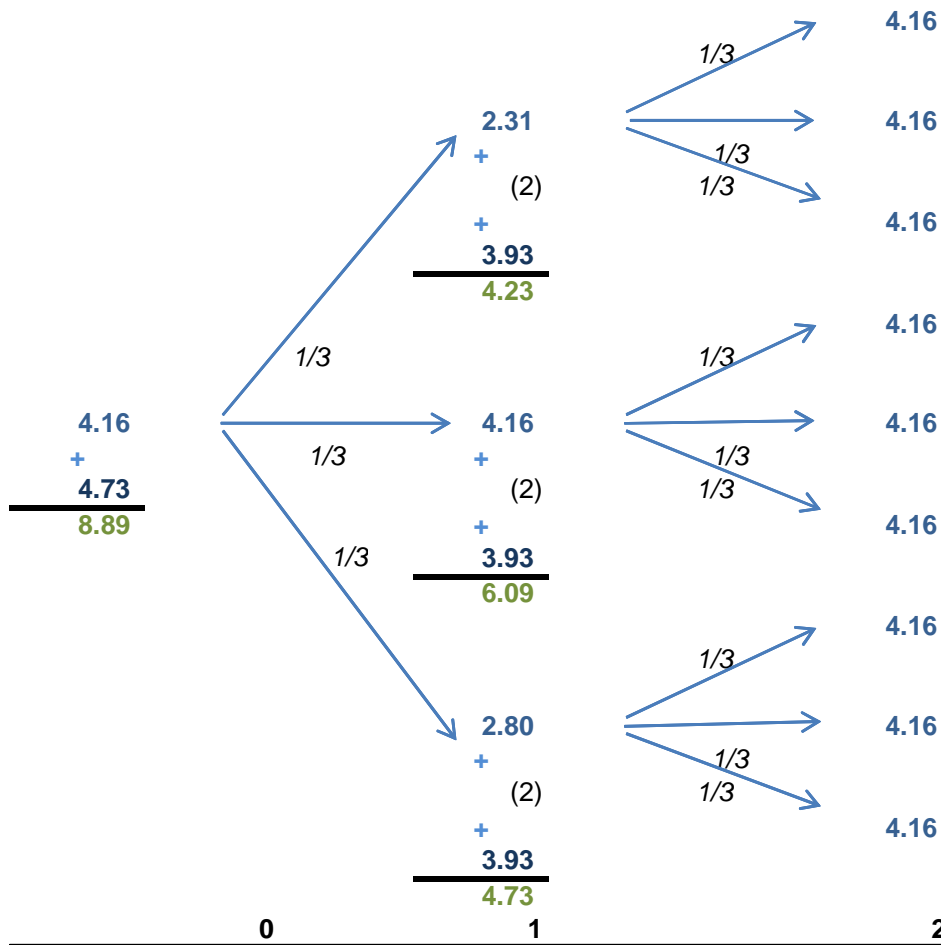


Figure 5. The resulting P&L for each realization for the process with a new CSS purchased at period 1.

Based on this framework we now will investigate the impact of each variable on the timing decision.

4. Sensitivity Analysis

In our credit concession model and investment timing decision, several variables compete for the expected P&L, they are: CSS cost (I), average error decay, error standard deviation, spread, interest rates, and the proportion of good payers in the population (p). We take the former example as a basic case to investigate the effect of each of the above variables on the timing of the decision.

In the case of the CSS cost, we calculate the value of each three investment decision for CSS costs varying from 0 to 10. In figure 6 we show this value for each investment decision alternative. For a CSS cost below 1.5 the change should be done in $t=0$, if it is above 1.5 the change should be postponed and maybe done in the next period, depending on what happens to the system error.

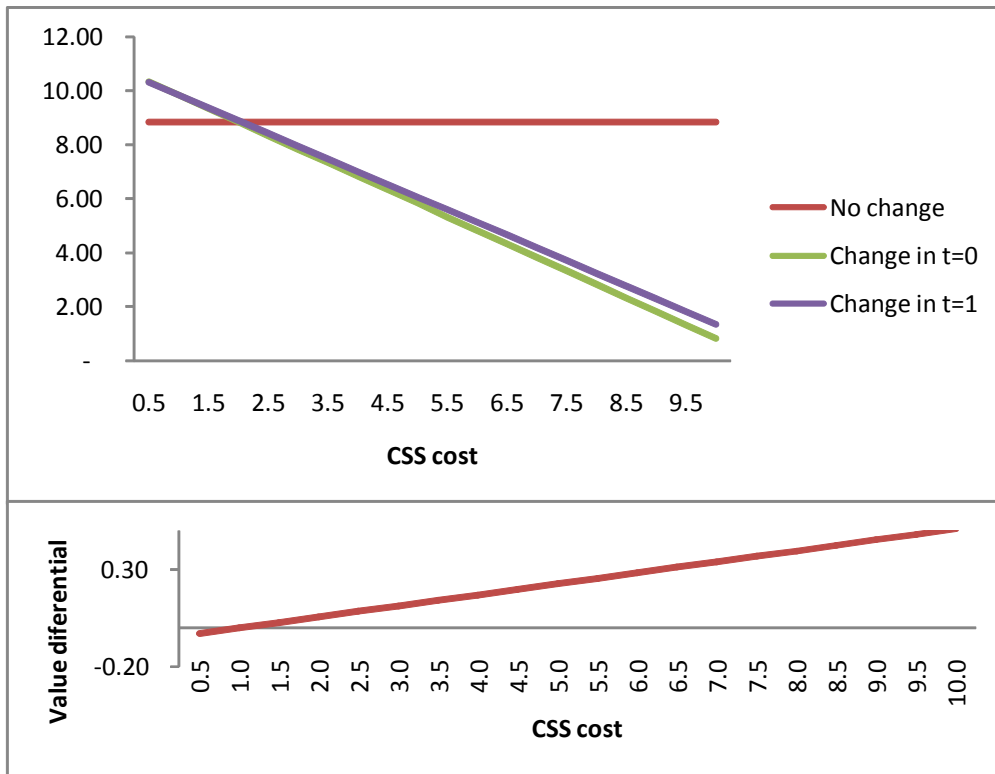


Figure 6. In the first graph there is the value of each investment decision for varying CSS costs. In the second graph only the difference between the value of changing in $t=1$ and the value of changing in $t=0$.

The same analysis is done for other variables. In figure 7 we show the same results for the mean and the standard deviation of the errors alpha and beta. For mean below 15% the CSS should not be changed, and only for mean above 27% the change should be done at $t=0$. For any standard deviation the change should be postponed.

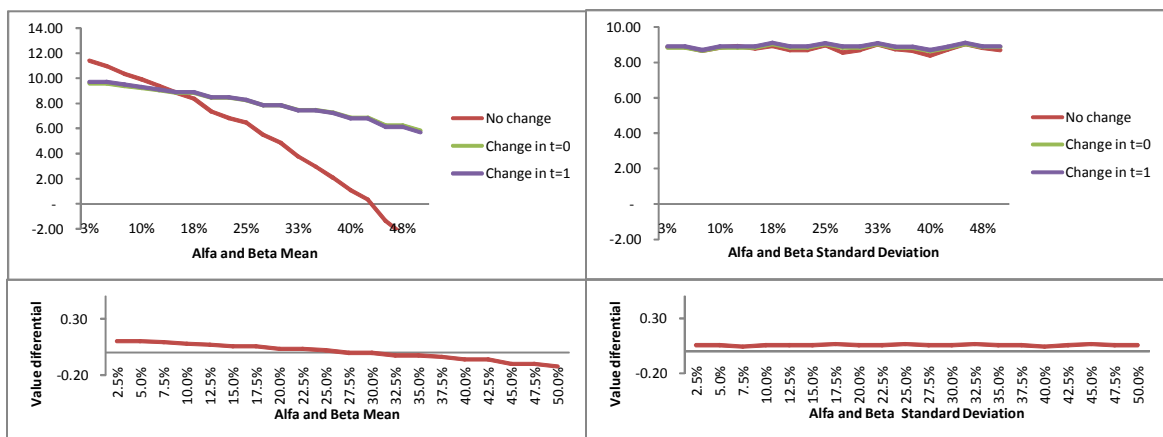


Figure 7. The first graph on the left shows the value of each investment decision for varying mean of errors. In the second graph on the left only the difference between the value of changing in $t=1$ and the value of changing in $t=0$. On both graphs on the right side the same analysis is done for the errors standard deviation.

In figure 8 we show the same results for the correlation of the errors alpha and beta and the spread charged. Whatever the correlation and the spread no change should be done at $t=0$.

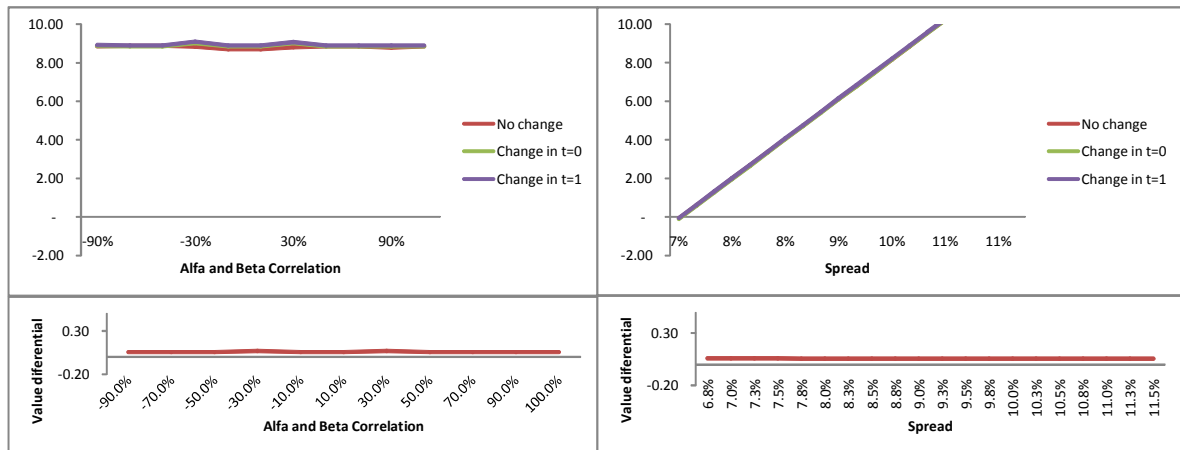


Figure 8. The first graph on the left shows the value of each investment decision for varying correlations between the two types of errors. In the second graph on the left only the difference between the value of changing in $t=1$ and the value of changing in $t=0$. On both graphs on the right side the same analysis is done for the spread.

In figure 9 we show the results for the interest rates and the proportion of goods. Whatever the level of interest rate and the proportion of goods, changing in $t=0$ is not the best decision.

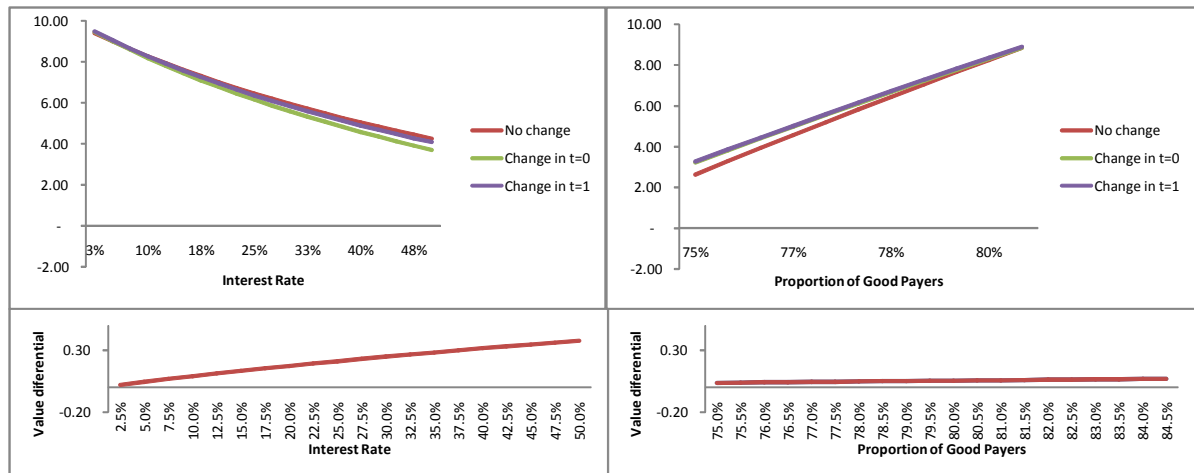


Figure 9. The first graph on the left shows the value of each investment decision for varying interest rates. In the second graph on the left only the difference between the value of changing in $t=1$ and the value of changing in $t=0$. On both graphs on the right side the same analysis is done for the proportion of good payers.

This is a local analysis that departs from the numbers used in the example from section 2. Obviously, if the starting points differ the impact of each variable should also be different than above. In summary, the results above rarely show that changing the CSS at $t=0$ is the best alternative, although in general it does have a positive NPV. In fact only for a very high error decay mean or very low CSS cost would purchasing a new CSS at $t=0$ be the most valuable decision. The other variables had no impact in the timing to change the CSS.

5. Conclusion

We have provided a framework for the timing of the purchased decision of a new CSS, which has not been done so far. This framework is very flexible to allow many different credit environments and should be adapted to each specific company.

Using a very basic case we simulated the effect of several variables in the timing of the purchase of a new CSS. The results have suggested that only the cost of the new CSS and its average decay affect the timing of the decision.

The model can be extended to include different credit concession rules or different stochastic processes for the errors. Also the competition effect may change the credit concession model with varying ν and spread may have impact on the results. More competition shall affect demand for loans and may decrease the value of the option to postpone the investment. The model can also be modified for other sources of uncertainty such as proportion of good payers, spread or demand for loans.

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