Investment Decision in a New Credit Score System

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Credit Scoring and Credit Control XI
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- The **Credit Scoring System (CSS)** a **fundamental tool** to decrease the cost of credit concession and certainly responsible for the huge **credit concession** provided by banks.
- The model is not perfect and its **quality** is gauged by their **errors**.
  - Shifting economic conditions, population changes, model flaws and several others,
  - Banks and any user of CSSs are constantly measuring and reevaluating their systems to obtain an acceptable total error,
  - lower total error is always desired but it comes with a cost.

- The **investment decision** in a new CSS depend on:
  - the total error of the current system,
  - the error of a new system,
  - its cost.

- A Bank has always the **option to postpone** the decision of buying a new CSS.
  - **Thomas et al. (2002)** page 161, this is **an open question** and there is no framework to decide upon the timing to purchase a new CSS.
- This article provides a framework to decide upon the **timing to buy a new CSS**.
  - The investment **is irreversible**,
  - A CSS user has the option to acquire a new CSS at a later time, profiting for the possibility of new information,
  - decision financially similar to a **call option**
    - the investment is the exercise price and the option premium is the value to postpone the investment.

- We assume:
  - that **CSS errors grow** (the system decays) with time
  - at any time the errors can be bigger or smaller than expected.
    - **system errors are a random variables** and the main source of **uncertainty.**

– We define
  - **Stochastic process** for this random variable
  - calculate the option value of postponing the purchase of a new CSS.

➔**real options technique** to provide a framework for this problem.

- **NPV rule is not adequate** because it does not measure the value of the option to acquire the CSS at a future time.

- Real option approach has been applied to the problem of **investment decision** on new **software** by Sullivan et al. (1999), which is a very similar problem.

- The real option approach is extensively described in Dixit and Pindyck (1994).

Credit Decision

- For simplicity the **Bank maximizes its profit**.
  - the bank has to decide on the amount *v* to lend.
  - the bank has a total of *d* to lend out
  - pays *r* on the total money available *d*, and
  - charges from each client *r*+*s*.

- A new CSS cost *I* and can generate **two types of errors**.
  - It can forecast as bad an applicant which in fact is a good payer as α.
  - we call the population percentage of the bad payers that are misclassified as β.
  - Therefore the total system errors are:

$$[\alpha p + \beta(1-p)]$$

- The **credit concession problem** can have a different formulation (see Baesens and Van Gestel 2009).

- At the current market spread (*s*) the **demand for loans** from individuals considered good is **bigger** than the amount **available *d*:**

$$[(1-\alpha_t)p + \beta_t(1-p)]x_t > d$$

- The credit decision is:

$$\underset{\{v_t\}}{Max}\ [(1+r+s)(1-\alpha_t)p - \beta_t(1-p)]v_t - d(1+r) - I$$

$$subject\ to\ \ [(1-\alpha_t)p + \beta_t(1-p)]v_t \leq d. \qquad (2)$$

  - At each period *t* the bank should choose the **optimum credit conceded v_t**.

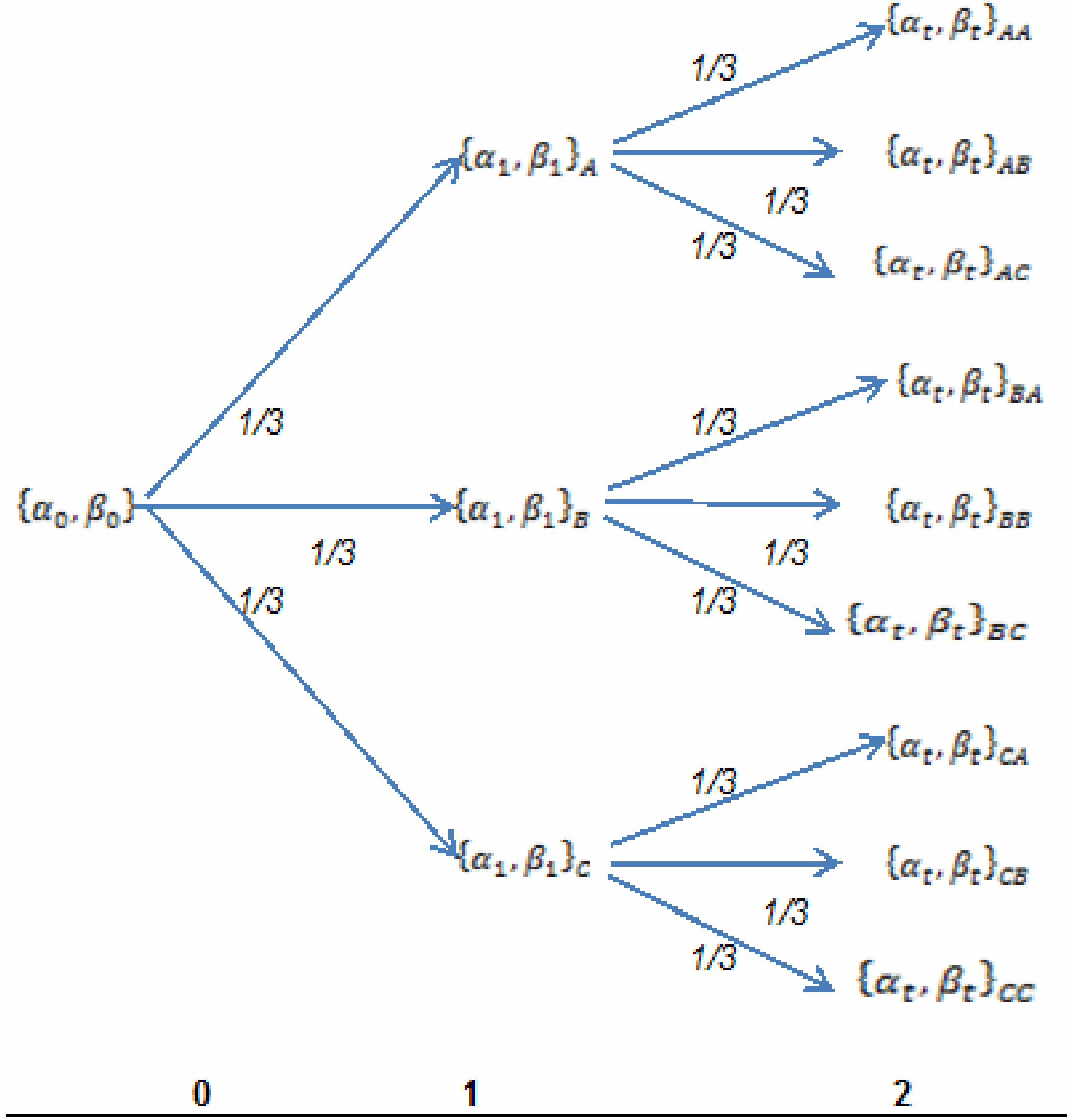
- As long as the expected gain is positive, the **optimum** is *d*, otherwise it is zero.
- The **sequence** $\{\alpha_t, \beta_t\}_{t \geq 1.}$ **defines** the CSS quality or their total errors.
- A **CSS without forecasting power**, one would expect both errors been 0.5, this is the error of a pure random model.
  - borrowers can **gamble** the system and end up with a CSS with errors below 0.5.

CSS Investment Decision

- The investment decision faced is whether to **purchase** a new CSS **or not:**
  - depends on the CSS errors.

- Any **new CSS** is defined by:
  – a flat cost of *I*,
  – initial errors of $\{\alpha_0, \beta_0\}$,
  – a sequence of errors $\{\alpha_t, \beta_t\}_{t \geq 1.}$.

- The **CSS decay** is characterized by:
  - both errors growing at a rate $\mu>0$,
  - volatility σ per period,
  - The errors are unrelated to any other variable from the credit decision,
  - There is an upper limit at (1,1), the maximum error and a lower limit at (0,0) which is the minimum error,
  - We assume a lognormal diffusion process for both errors.
    - avoids negative values for the errors.
  - These two lognormal processes are approximated by a trinomial tree according to Amin (2001).
    - The jumps (upward and downward) and their probabilities (q) are chosen to match the growth rate, volatility of the errors and their boundaries.



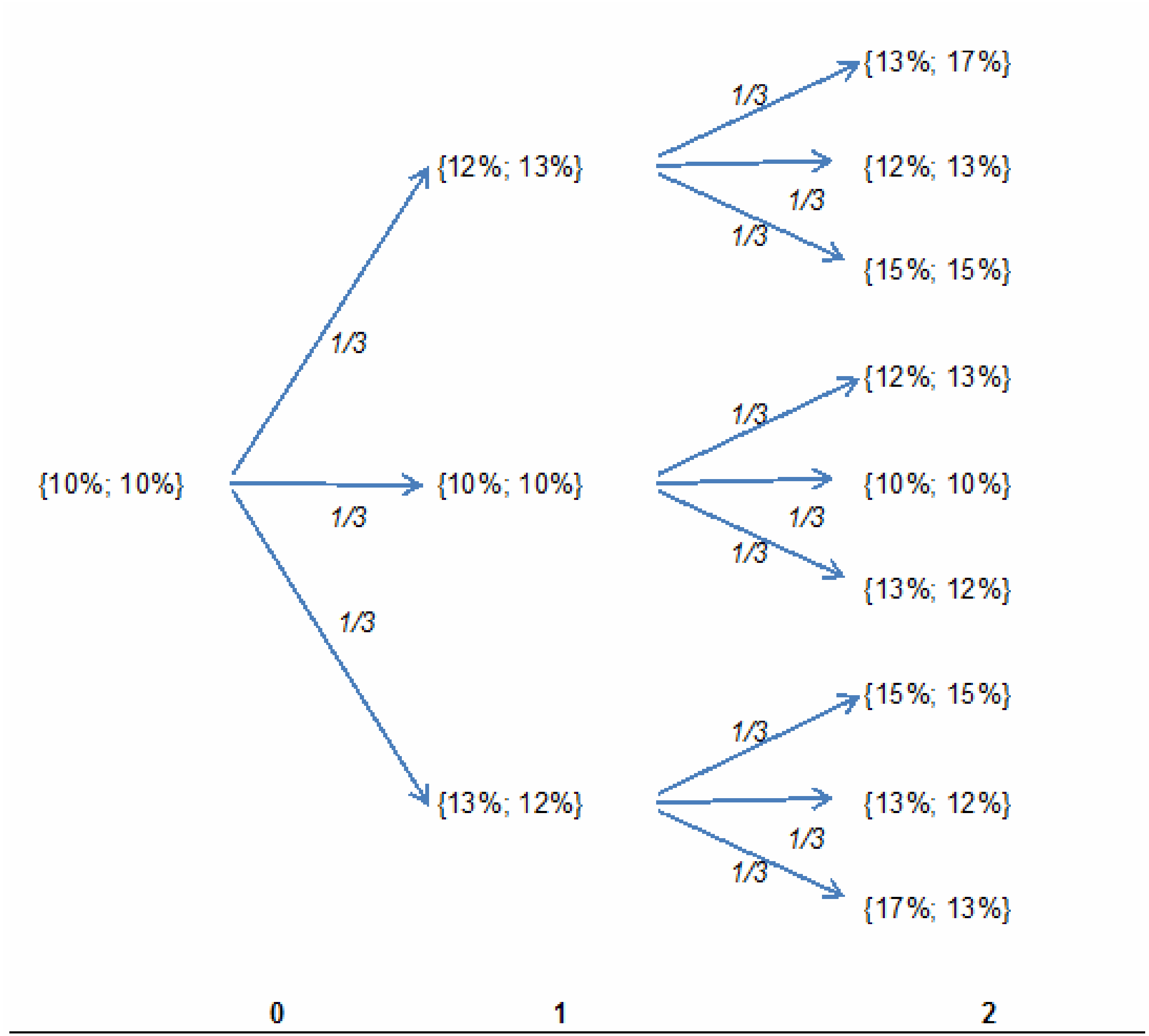
{α_0, β_0}
1/3
1/3
1/3
{α_1, β_1}_A
{α_1, β_1}_B
{α_1, β_1}_C
1/3
1/3
1/3
{α_t, β_t}_AA
{α_t, β_t}_AB
{α_t, β_t}_AC
{α_t, β_t}_BA
{α_t, β_t}_BB
{α_t, β_t}_BC
{α_t, β_t}_CA
{α_t, β_t}_CB
{α_t, β_t}_CC
0
1
2

- **Timing** the acquisition of a new CSS.
  - A new CSS will on average diminish the errors,
  - There is a cost to do it,
  - the decision can be postponed.
- The **value** of **the option to postpone** this decision.
  - The best decision is the one that is most valuable,
  - The value of waiting to invest must be compared to the current loss of not having a new CSS.
    - A bank may maintain a poor performance CSS due to a possible improvement in the future.
    - The errors should increase persistently in order to trigger the purchase of a new CSS.

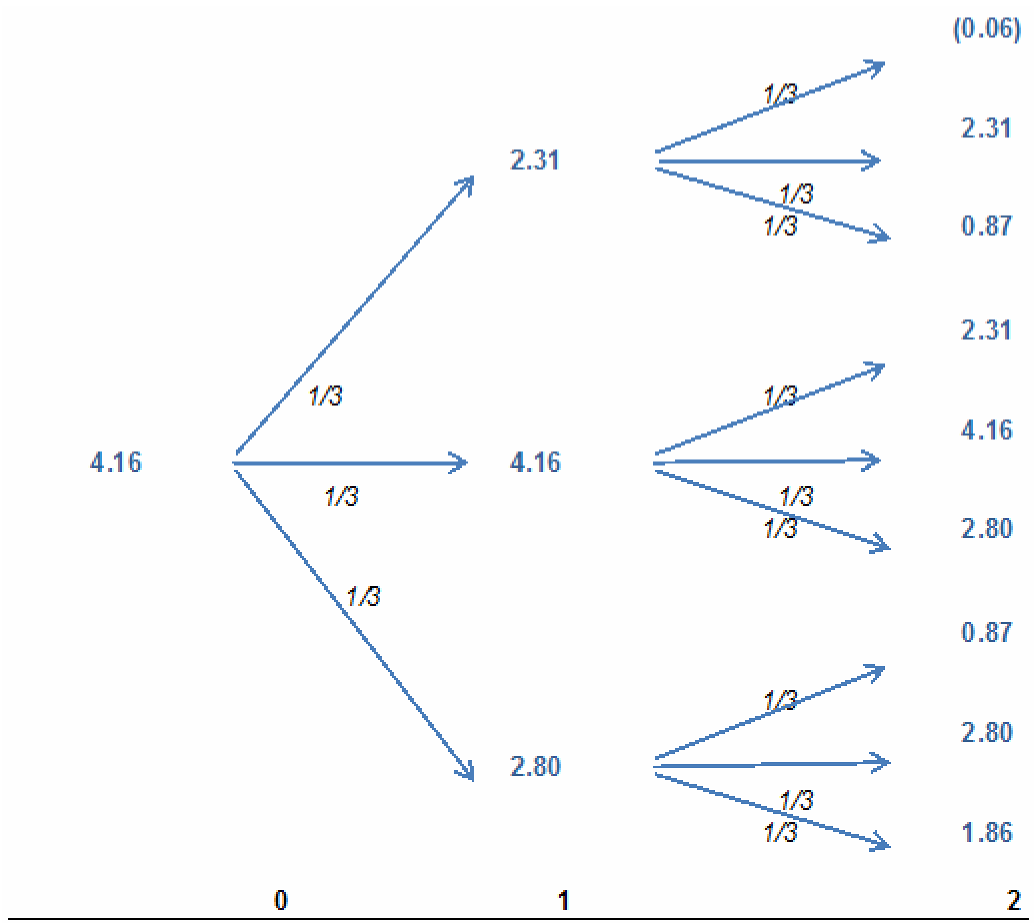
- At any period, the bank can purchase a new CSS paying *I*.
- The decision is also path dependent:
  - a higher error in any moment may trigger the decision to buy a new CSS if the error in past periods also increased.

Example

- A bank has *d*= 100 available to lend,
- A new CSS costs 2,
- the spread is 10%,
- the interest rate is 6%,
- the proportion of good payers 80%,
- A new CSS has initial errors of (10%,10%), average decay per period is 15%, its standard deviation 10% and its correlation 50%.


{10%; 10%}
1/3
{12%; 13%}
{10%; 10%}
{13%; 12%}
{13%; 17%}
{12%; 13%}
{15%; 15%}
{12%; 13%}
{10%; 10%}
{13%; 12%}
{15%; 15%}
{13%; 12%}
{17%; 13%}
0
1
2

Resulting profit and loss (P&L)



- Three **investment alternatives** for the bank:

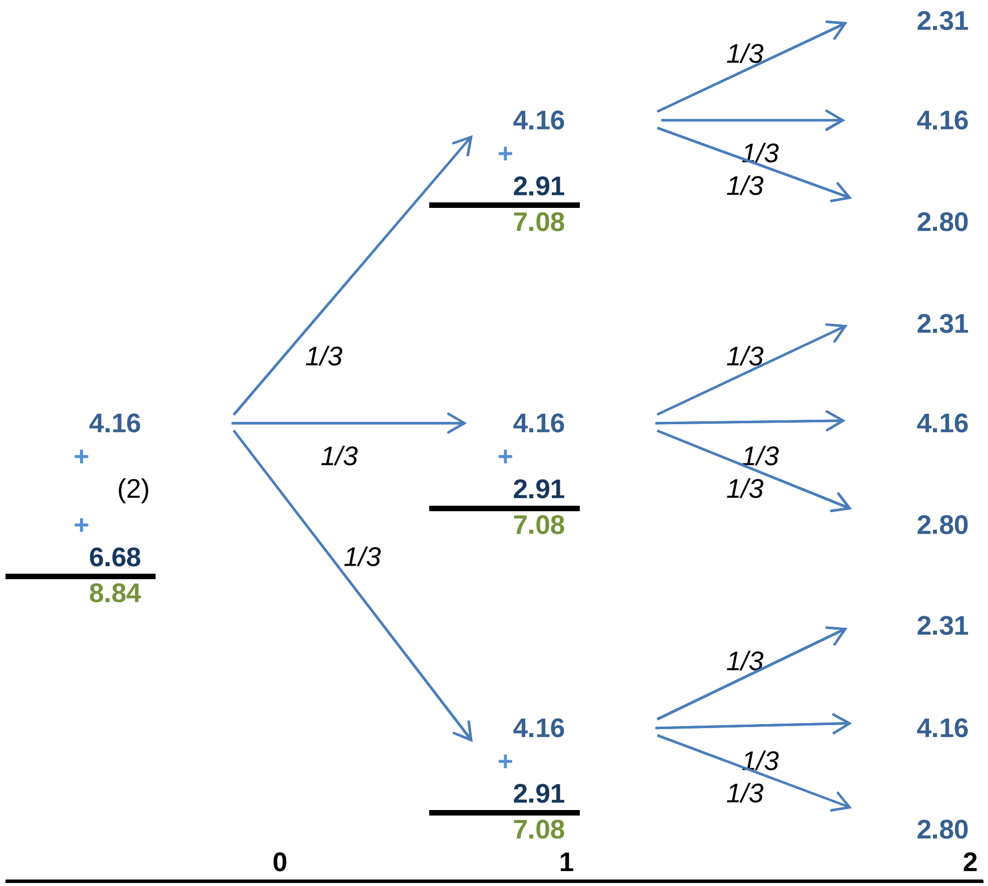
  (1) no change in the CSS,

  (2) buy a new CSS at period 0,

  (3) buy a new CSS only at period 1.

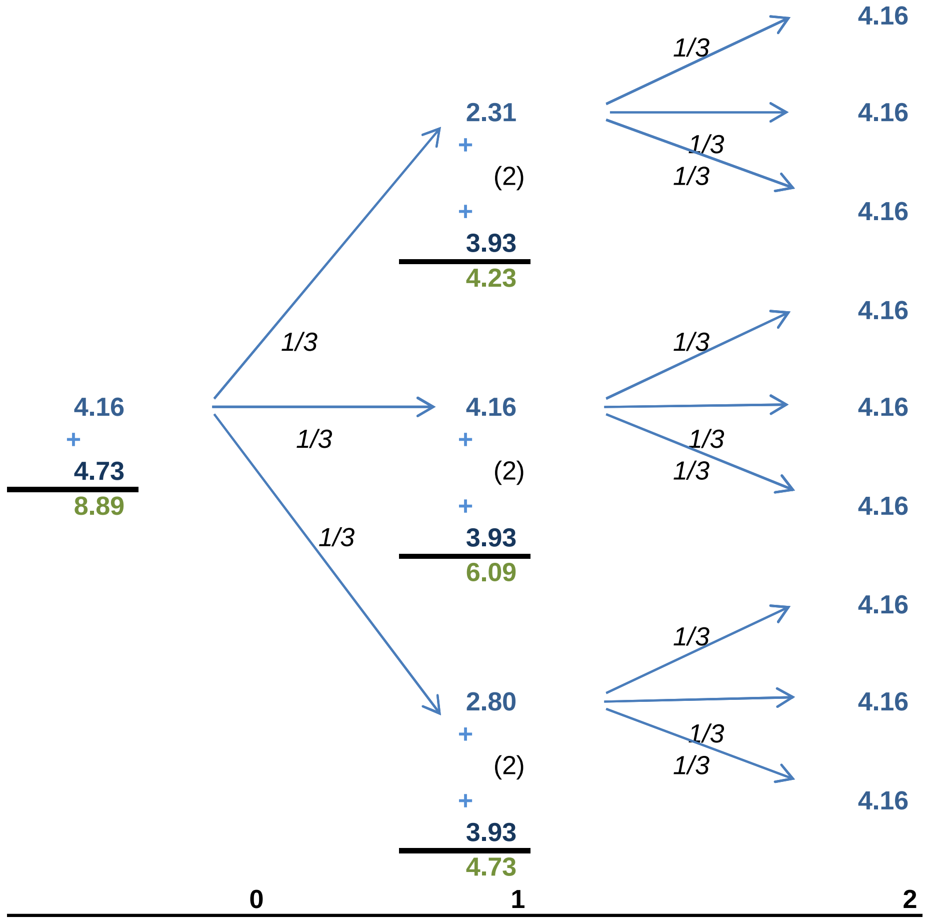
- A **new CSS** should be paid in the same period of the decision, but its **effects** are perceived in the **next period**.

Case 2



4.16
+
(2)
+
6.68
8.84
1/3
1/3
1/3
4.16
+
2.91
7.08
4.16
+
2.91
7.08
4.16
+
2.91
7.08
1/3
1/3
1/3
2.31
4.16
2.80
2.31
4.16
2.80
2.31
4.16
2.80
0
1
2

Case 3


4.16
+
4.73
8.89
1/3
1/3
1/3
2.31
+
(2)
+
3.93
4.23
4.16
+
(2)
+
3.93
6.09
2.80
+
(2)
+
3.93
4.73
1/3
1/3
1/3
4.16
4.16
4.16
4.16
4.16
4.16
4.16
4.16
4.16
0
1
2

case (1) the expected value is 8.85.
case (2) the expected value is 8.84.
case (3) the expected value is 8.89.

- The **most valuable** is (3), waiting until period 1 to purchase a new CSS.

- The **NPV analysis** would lead to the decision (2) because its NPV is positive, but postponing this decision has greater.

- If the **investment were reversible** (could be sold next period) then as long as NPV is positive it should be done

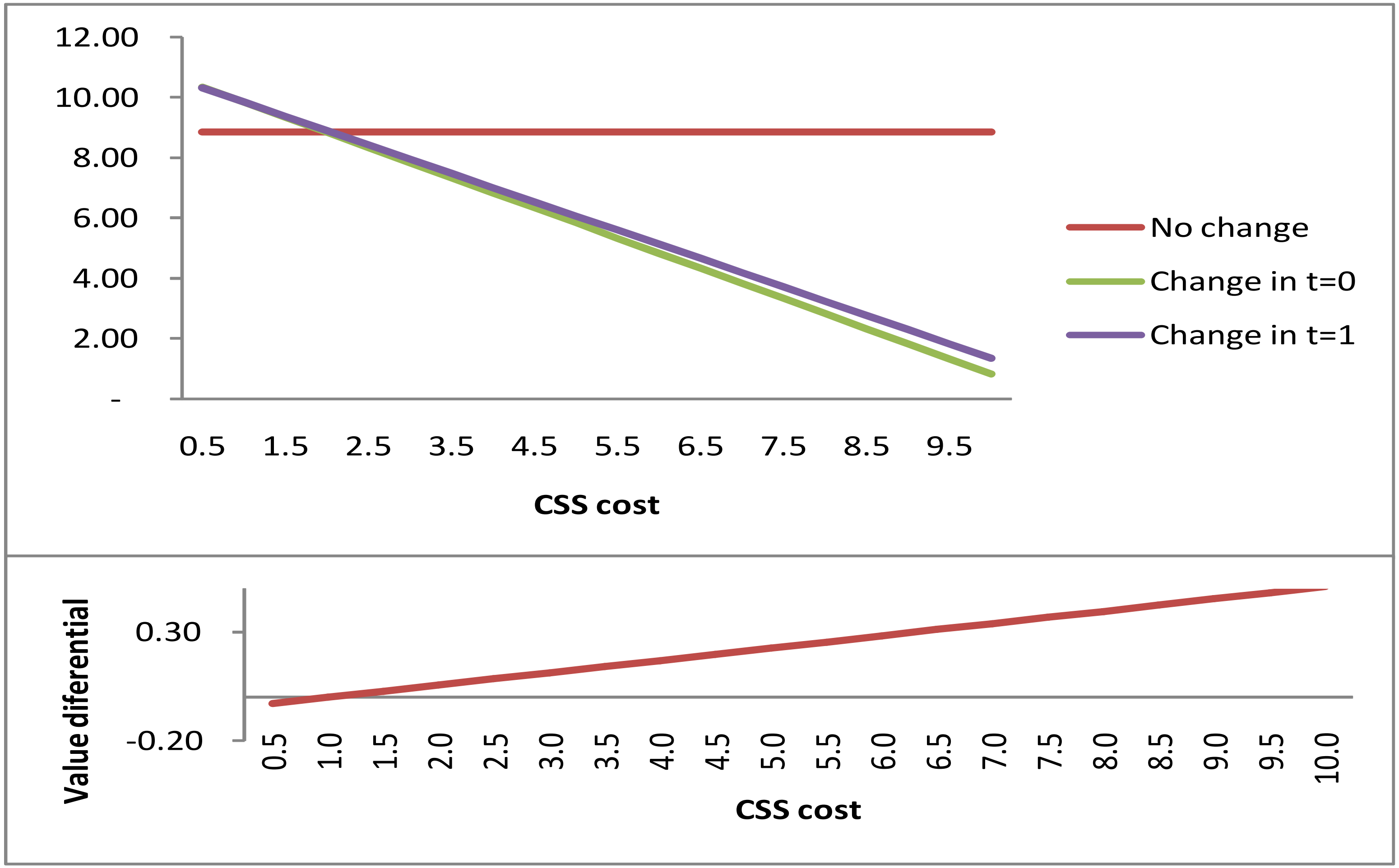
- Another issue arises from the fact that the bank cannot wait to make the investment, for instance, in the case of a higher **competition that decreases** the demand for loans .

Sensitivity Analysis

- Several **variables** compete for the expected P&L, they are:
  - CSS cost (*I*),
  - average error decay and error standard deviation,
  - spread,
  - interest rates,
  - proportion of good payers in the population (*p*).

- We take the former example as a **basic case**.
- The value of each three investment decision for **CSS costs varying** from 0 to 10.
  - For a CSS cost below 1.5 the change should be done in t=0,
  - if it is above 1.5 the change should be postponed and maybe done in the next period, depending on what happens to the system error.

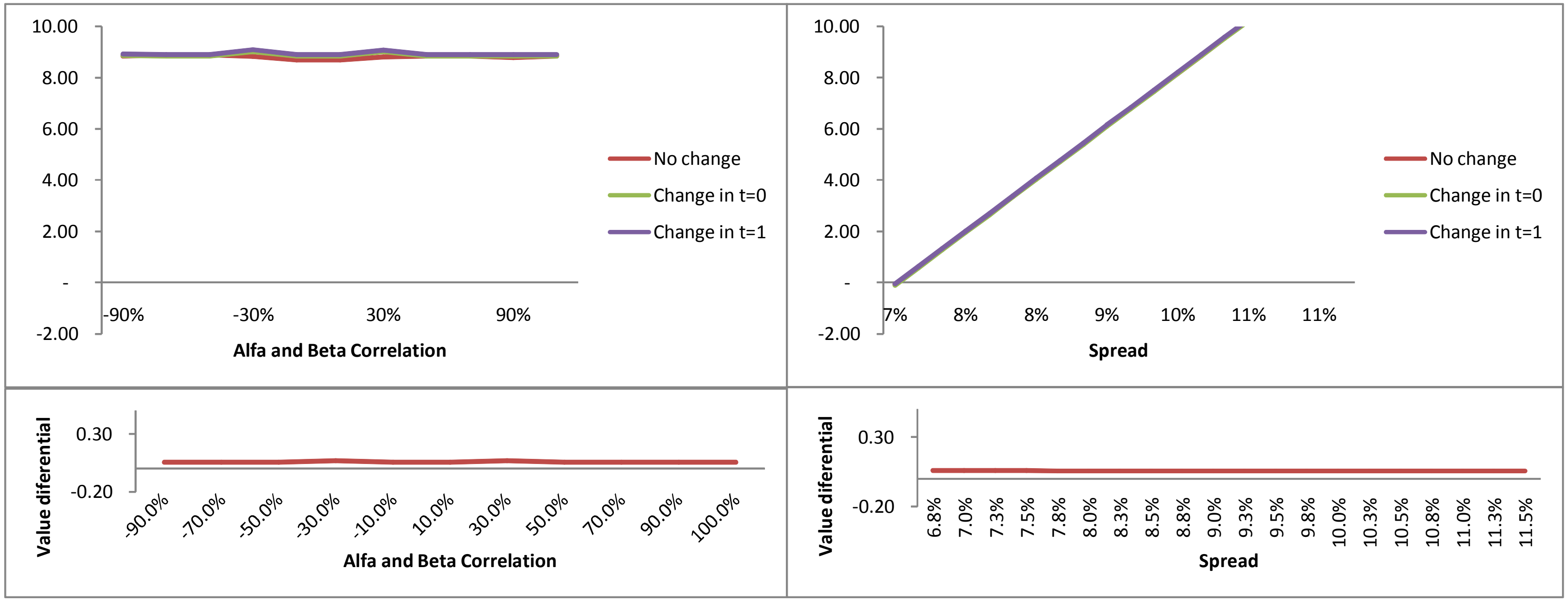
Cost effect


12.00
10.00
8.00
6.00
4.00
2.00
-
0.5 1.5 2.5 3.5 4.5 5.5 6.5 7.5 8.5 9.5
CSS cost
No change
Change in t=0
Change in t=1
Value diferential
0.30
-0.20
0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0 5.5 6.0 6.5 7.0 7.5 8.0 8.5 9.0 9.5 10.0
CSS cost

Alfa, Beta Mean and STD

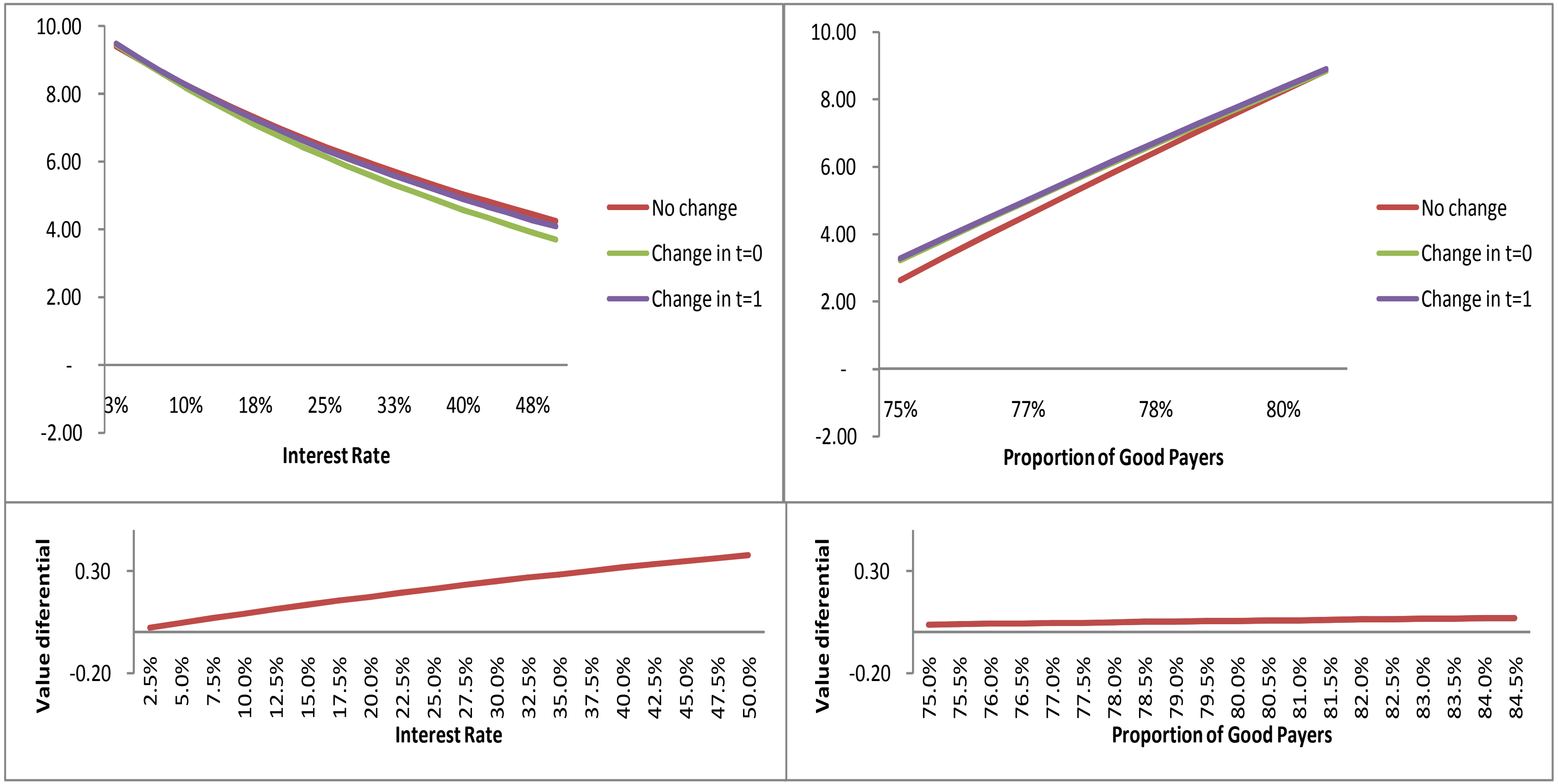
- The same results for the mean and the standard deviation of the **alpha and beta**.
  - For mean below 15% the CSS should not be changed,
  - only for mean above 27% the change should be done at *t*=0.
- For any standard deviation the change should be postponed.

Alfa, Beta correlation and spread



- Whatever the **correlation** and the **spread** no change should be done at t=0.

- Whatever the level of **interest rate** and the **proportion of goods**, changing in t=0 is not the best decision.



10.00
8.00
6.00
4.00
2.00
-
-2.00
3%
10%
18%
25%
33%
40%
48%
Interest Rate
No change
Change in t=0
Change in t=1
10.00
8.00
6.00
4.00
2.00
-
-2.00
75%
77%
78%
80%
Proportion of Good Payers
No change
Change in t=0
Change in t=1
Value diferential
0.30
-0.20
2.5%
5.0%
7.5%
10.0%
12.5%
15.0%
17.5%
20.0%
22.5%
25.0%
27.5%
30.0%
32.5%
35.0%
37.5%
40.0%
42.5%
45.0%
47.5%
50.0%
Interest Rate
Value diferential
0.30
-0.20
75.0%
75.5%
76.0%
76.5%
77.0%
77.5%
78.0%
78.5%
79.0%
79.5%
80.0%
80.5%
81.0%
81.5%
82.0%
82.5%
83.0%
83.5%
84.0%
84.5%
Proportion of Good Payers

- This is a **local analysis** that departs from the numbers used in the example from section 2.
  - if the starting points differ the impact of each variable should also be different.
- The results above rarely show that changing **the CSS at *t*=0** is the best alternative, although in general is does have a positive NPV.
  - only for a very high error decay mean or very low CSS cost would purchasing a new CSS at *t*=0 be the most valuable decision.
  - The other variables had no impact.

Conclusion

- We have a **framework** for the timing of the purchased of a new CSS, which has not been done so far.
- Very flexible framework to many **different credit environments**
- The results suggested that only **the cost of the new CSS** and its average decay affect the timing of the decision.
- Can be extended to include different **credit concession rules** or different stochastic processes for the errors.
- The **competition effect** may change the credit concession model with varying *v* and spread may have impact on the results.
- The model can also be modified for **other sources of uncertainty** such as proportion of good payers, spread or demand for loans.

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