Corporate loan PD modelling using external data

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Abstract

Basel II IRB framework requires the Probability of Default (PD) rating system, and its reaction under a stressed scenario, to be supported by a long history of data. Irrespective of the calibration methodology, the underlying data is clearly the major factor in ensuring a robust approach is adopted. Addressing the issue from a corporate lending bank's perspective, this paper demonstrates in practice how an external dataset can be used to complement the internal information in three main areas: estimation of forward looking PD, forecasting of PD in a stressed scenario, and modelling of the rating transition dynamics.

1 Introduction

Financial institutions primarily employ PD models to satisfy regulatory requirements with additional uses including measurement of obligor creditworthiness and risk based pricing. Traditionally PD modelling in corporate lending adopts a "fundamental" approach whereby the probability of default is modelled on the obligor's performance reflected in the financial statements. Ratios are constructed from financial statements to capture different aspects of an obligor's creditworthiness and the model output will be a credit score. The credit score is then transformed into a PD through a calibration process giving a more understandable rating that can be interpreted by business users.

The credit score can be calibrated to Point-in-Time (PiT) or Through-the-Cycle (TTC) PD ratings but in either case the long run average default rate (LRDR) of the portfolio needs to be estimated. Most financial institutions do not have adequate data covering the full economic cycle required to estimate LRDR and therefore need to rely on external data. There are a number of conceptual challenges in using external data to infer historical internal portfolio performance. Several adjustments are required to reconcile external data to the internal portfolio in order to estimate the historical portfolio composition and subsequently the LRDR. This paper presents a practical example of the steps that may be taken to achieve this reconciliation as well as the modelling techniques to forecast the PD and rating transition dynamics. The methodology can be augmented to suit other corporate portfolios to satisfy business use and regulatory requirements.

The paper is structured as follows. The data section first defines the data sources and assumptions and then proposes a PD rating system to be used throughout this paper. The methodology section discusses the approach to derive the internal

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portfolio's dynamics based on external data, and describes the PD forecasting model and rating transition model. The results of the proposed methodology are presented and discussed in the last section.

2 Data

2.1 Data sources

Two principal data types, financial and macroeconomic, are employed in this study as described below.

Financial Data

Financial filings and default information comes from the Jordan's Companies House database that contains financial information for 3,044,577 companies, with a total 11,837,130 annual observations. We approximate defaults by insolvency related events, which are captured by the filing types as summarised in Appendix 1. This is an assumption in relation to default recognition and would need to be accounted for as described in the following section.

A cohort based approach is adopted for default modelling. We only include companies in a given cohort (either semi-annual or annual) if they have not filed for any of the insolvency types (Appendix 1) in any previous cohort, and if they have financial information for up to 2 years prior to the cohort's starting date. The 2 years period allows for any possible changes in the fiscal year reporting dates or any late submission that is not yet updated in the database. This excludes companies that miss reporting deadlines regularly, or report incomplete accounts, and therefore can be classified as having less reliable data. A default is recorded in the first instance when a company files for any of the insolvency types in Appendix 1, given that it is live at the start of the given cohort.

For the purpose of this study, a subset of obligors from Companies House is also used as the development sample to parameterise the PD model.

Macroeconomic Data

Macroeconomic data is mainly sourced from DataStream, or alternatively from public sources such as the Office of National Statistics (ONS).

2.2 Portfolio Definition

External Portfolio

To illustrate the concept of using an external dataset to forecast and stress test the performance of a bank's historically limited corporate portfolio (thereafter the *internal portfolio*), we restrict the scope to medium to large UK companies. To exclude the small SMEs from the Companies House dataset, we use deflated £50m turnover cut-off point as a proxy of size classification. This subset of the full database defines the *external dataset*.

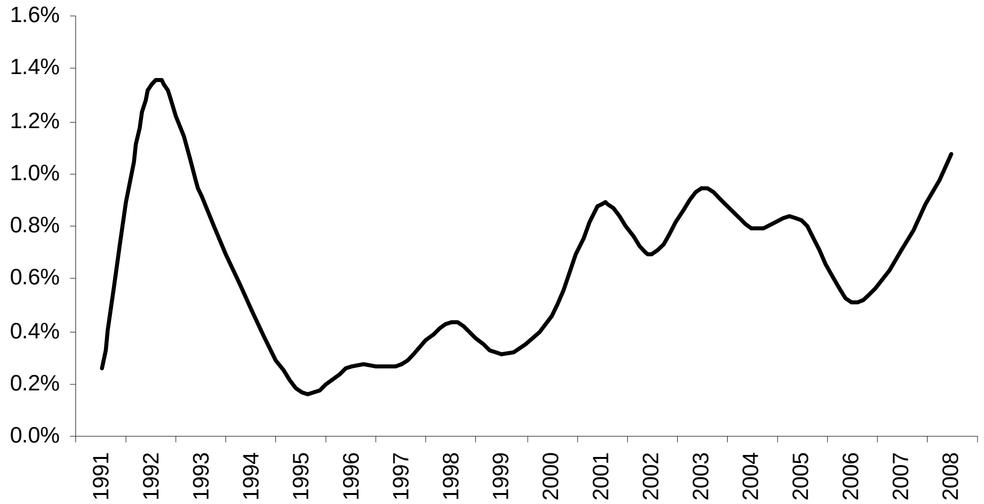
The following data assumptions are made to ensure that the external dataset can be used to model the internal portfolio:

- Data Assumption 1: *The external portfolio covers both contracting and booming economy periods and therefore is suitable to derive a long run average default rate (CEBS 383).* Under Basel II framework banks are required to use a long history of default data to build PD models. When an external dataset is used, it should contain the peak and trough of an economic cycle in order to fulfil regulatory modelling requirements.

- <u>Data Assumption 2:</u> *The internal portfolio is a subset of the external "market" portfolio, both in terms of companies' size and domicile.* An external dataset can only be used if it has similar characteristics to the internal dataset. If the underlying companies' characteristics are different across the two datasets, another external source should be sought. This assumption implies that the internal portfolio should contain UK based companies with turnover of more than £50m.
- <u>Data Assumption 3:</u> *Insolvency related events are reliable proxies for Basel II definition of default.* The external dataset contains only insolvency related filings which does not necessarily match the Basel II definition of default. If the default definition is materially different across the two datasets the default rates will be either under or over-stated. For simplicity we assume the insolvency filing types approximate the Basel II definition of default, although this might not be true in reality and additional adjustments are required to reconcile any difference in the definition of default.

Figure 1 shows the external default rates for annual cohorts formed between January 1991 and December 2008. The default rates follow a pattern reflecting the early 1990s recession, the deterioration in credit quality starting late in 2000, and the gradual movement to a downturn in 2008. It broadly supports Data Assumption 1 with a mix of booming and contracting years in the data history.

Figure 1 External Annual Default Rates[#] 1991-2008



[#]Smoothed line of bi-annual 12-month rolling default rate. Minimum turnover cut off point at £50m

<u>Internal Portfolio Definition</u>

For illustrative purposes, the internal portfolio is selected from the external dataset, thus fulfilling Data Assumption 2. To address Data Assumption 3, we use the same definition of default as the external portfolio (i.e. insolvency events). To define the selection process for the internal portfolio, we require a number of conditions to be met regarding the portfolio characteristics:

- *<u>Portfolio Characteristic 1:</u> The internal portfolio has constant number obligors per year.* We choose to maintain equal portfolio size over the period to negate any portfolio size effects.
- *<u>Portfolio Characteristic 2:</u> The majority of companies are retained over consequent years.* This implies that the portfolio is not recycled each year.
- *<u>Portfolio Characteristic 3:</u> The portfolio risk profile over consequent years does not exhibit extreme movements.* This is consistent with a bank that does not change its strategy and risk appetite radically on a year-to-year basis.
- *<u>Portfolio Characteristic 4:</u> The internal portfolio is riskier than the external "market" portfolio, which is an indication of a bank seeking a higher return than the market.* The analysis can be performed in portfolios both less and more risky than the market. We have chosen a riskier portfolio in this paper for illustration purpose.

To create the internal portfolio, we use a semi-randomly selected sample of 5000 obligors from the external dataset, covering the period from 2000 to 2004. To satisfy Portfolio Characteristic 1, the obligors are equally spanned over the 5 years period, resulting in a 1000-obligor cohort per year.

We impose some restrictions to the random nature of the selection process to avoid extreme changes in risk profile on a year on year basis: after sorting companies by their Registration Number in an increasing order, companies ranked between 4501 and 5500 (the average total number of companies per year is approximately 6475) are selected. The Registration Number is an indicator of the Date of Incorporation, with higher Registration Numbers corresponding to more recently formed companies. This selection process may appear to be arbitrary but in reality it adheres to portfolio characteristics 2-4 for the following reasons:

- The majority of the companies will be retained and the risk profile will not be changed significantly over subsequent years. Some companies will drop out from the portfolio in each year due to either default or lack of financial data but this is expected to constitute only a small proportion of the dataset.
- Age of business is usually considered as a good discriminatory factor of credit risk although not perfectly correlated with defaults. Newly formed companies normally have higher leverage (higher proportion of fixed costs to total costs poses considerable challenges to operate above break even point), price setting limitations and higher barriers to funding. On the other hand, we expect well established companies to be more resilient to default risk. The chosen internal portfolio corresponds to companies that were incorporated between 1993 and 2000, and can therefore can be considered as newly formed. Consequently, the hypothetical internal portfolio will be riskier than the market portfolio riskier than the market average.

Table 1 contains a high level summary of both internal and external portfolios for the period 2000-2004, covering Good/Bad frequencies and incorporation date ranges.

Table 1 Summary of internal and external portfolios

Cohort	Internal Portfolio				External Portfolio			
	Goods	Bads	Default Rate	Incorporation Dates	Goods	Bads	Default Rate	Incorporation Dates
01/01/2000-31/12/2000	991	9	0.9 %	07/1993-03/1998	6091	28	0.46 %	12/1863-09/1999
01/01/2001-31/12/2001	982	18	1.8 %	02/1994-06/1998	6273	56	0.88 %	12/1863-12/2000
01/01/2002-31/12/2002	984	16	1.6 %	12/2002-01/1999	6415	45	0.70 %	12/1863-11/2001
01/01/2003-31/12/2003	978	22	2.2 %	04/1995-06/1999	6596	63	0.95 %	12/1863-12/2002
01/01/2004-31/12/2004	985	15	1.5 %	10/1995-12/1999	6755	54	0.79 %	12/1863-06/2004
Overall	4920	80	1.6 %	07/1993-12/1999	32130	246	0.76 %	12/1863-06/2004

2.3 PD Rating System

An illustrative PD rating system, which will serve as proxy for a typical corporate bank PD model, is built on the hypothetical internal portfolio. A typical corporate PD model is likely to incorporate the quantitative aspects but also qualitative questions and possibly expert judgement-based overrides. We do not intend to replicate a full corporate PD model and therefore only a small set of accounting variables are use for model build. The full set of variables considered for model development, along with their derivations, is shown in Table 2.

Table 2 Accounting variables for model development

Accounting Variable	Derivation
Current Ratio	Current Assets / Current Liabilities
Quick Ratio	(Current Assets – Stock & Work In Progress) / Current Liabilities
Cash Ratio	Cash & Bank Deposits / Current Liabilities
EBIT To Turnover	Profit Before Interest Paid / Turnover
Debt Ratio	Total Liabilities / Total Assets
Interest Cover	Profit Before Interest Paid / Interest Paid
Debt To EBIT	Total Liabilities / Profit Before Interest Paid
Return On Assets	Turnover / Average Total Assets
Return on Equity	Turnover / Average (Total Assets – Total Liabilities)
Net Worth	Total Assets-Total Liabilities
Z-Score Variable 1	(Current Assets – Current Liabilities) / Total Assets
Z-Score Variable 2	Retained Profits / Total Assets
Z-Score Variable 3	Profit Before Interest Paid / Total Assets
Z-Score Variable 4	(Total Assets – Total Liabilities) / Total Liabilities
Z-Score Variable 5	Turnover / Total Assets

All nominal amounts are deflated using the Retail Price Index (RPI) sourced from ONS. All variables were winsorized to 1^{st} and 99^{th} percentiles, except for *Debt To EBIT* (winsorized at values 1 and 9), *Interest Cover (*winsorized at values -1 and 10), and *Net Worth (*winsorized at values 0 and 25). Monotonic transformations, such as logarithms, square roots and squares were also tested for all variables.

The accounting variables are regressed against annual defaults using a simple logistic regression. The model's likelihood is maximized by Fisher Scoring. The model can be summarized as follows:

$$D_i \mid X_i \sim Bernoulli(PD_i),$$

$$Log\left(\frac{PD_i}{1-PD_i}\right) = Z_i = \mathbf{b} \cdot X_i,$$

where D_i is the vector of default indicators, X_i is the matrix of explanatory variables, $\mathbf{b}$ is the vector of the coefficients to be estimated, and Z_i is the vector of the scores (log odds). Matrix X_i is augmented to include a column of ones for the intercept.

To choose the best model we use a mix of stepwise selection and manual exclusion of variables where coefficient signs are counter-intuitive. Four explanatory variables emerged from the selection, namely *Log Cash Ratio (CR)*, which is a measure of liquidity, *Interest Cover (IC)*, which measures the ability to meet the financial obligations, *Log Net Worth (NW)*, which is a proxy for size and *EBIT To Turnover (ET)*, which is a measure of profitability. The final model is shown below with standard errors reported in brackets:

$$Z_i = \underset{(0.5264)}{1.4735} - \underset{(0.0679)}{0.3154}\,CR - \underset{(0.0137)}{0.0451}\,IC - \underset{(0.0281)}{0.1992}\,NW - \underset{(0.4261)}{2.5292}\,ET$$

The discriminatory power of the model, as measured by Somers' D, is 35.2%. Once the model is estimated we calibrate it to 2004 default experience by rerunning a logistic regression with the score Z_i as the independent variable. The calibration parameters are:

$$Z_i^{cal} = -\underset{(0.2356)}{2.0501} + \underset{(0.1413)}{0.5468}\,Z_i$$

The rating scale is built by considering the calibrated rating distribution of the internal portfolio and external portfolios in order to determine the range and grade boundaries. Summary statistics for the scores from both datasets are shown in Table 3.

Table 3 Score Summary Statistics

Summary Statistics	Internal Portfolio	External Portfolio
Mean	-2.97460	-2.98947
Median	-3.02066	-3.05351
Mode	-3.40914	-4.37918
Std Deviation	0.50903	0.52861
Inter Quartile Range	0.62290	0.64337
Max	-0.32891	-0.32891
Min	-4.69247	-4.91905

We split scores from -4.82 (corresponding to a PD of 0.80%) to -0.8473 (corresponding to a PD of 30%) into 9 equidistant intervals, with scores falling either below (low risk) or above (high risk) this range to form two extra rating grades. The final numbers of 11 rating grades (RG) are large enough to provide sufficient diversification and small enough to give well populated grades. The complete rating system is shown in Table 4.

Table 4 Rating Grade Scale

Rating Grade	Scores			PDs		
	Low	Mid Point	High	Low	Mid Point	High
1	-Inf	-5.03	-4.82	0%	0.65%	0.80%
2	-4.82	-4.59	-4.37	0.80%	1.01%	1.25%
3	-4.37	-4.15	-3.93	1.25%	1.55%	1.93%
4	-3.93	-3.71	-3.49	1.93%	2.39%	2.96%
5	-3.49	-3.27	-3.05	2.96%	3.67%	4.53%
6	-3.05	-2.83	-2.61	4.53%	5.58%	6.86%
7	-2.61	-2.39	-2.17	6.86%	8.41%	10.27%
8	-2.17	-1.95	-1.73	10.27%	12.48%	15.09%
9	-1.73	-1.51	-1.29	15.09%	18.13%	21.63%
10	-1.29	-1.07	-0.85	21.63%	25.59%	30%
11	-0.85	-0.41	Inf	30%	40%	100%

3 Methodology

A rating system can be Through-The-Cycle (TTC), Point-in-Time (PiT) or fall somewhere in between and be referred to as hybrid. In TTC systems there is no migration between RGs and the state of the economy is captured by the volatility of default rates within each of the grades. On the other hand, the PiT system has stable default rates within each of the RG and the state of the economy is captured by the migration of obligors to a higher or lower grade. It is common practice to adopt a hybrid rating system given that in practice it is very difficult to maintain either of those extreme cases due to the nature of the modelling factors. We choose a hybrid rating system to illustrate the methodology and therefore expect both migrations and fluctuations in default rates between RGs.

For the purpose of this paper, we assume all inputs to the internal rating system are available for the external portfolio. In practice, practical limitations may restrict the extent to which the scoring model can be replicated in external portfolio. For instance, financial ratios might not be well populated or the PD model may include qualitative factors which are not available in external dataset.

The scoring system can be characterised by the default rates in each RGs as well as the migration between RGs. To reconcile the two portfolios we rely on the general assumption that the migration of the scoring system is the same between the internal and external portfolio, whilst the grade specific default rates can be different. The external dataset is used to proxy the dynamics of the internal portfolio and the reconciliation process involves the following steps:

Step 1: Define the dynamics for the transitions between RGs in the external dataset
Step 2: Use the transition dynamics to approximate the behaviour of the synthetic "historical" internal portfolio
Step 3: Adjust for the differences in the default rates between internal and external portfolios
Step 4: Model the approximate default rates for the internal portfolio.
Step 5: Build a rating transition model
Step 6: Use both the transition dynamics and modelled default rates for PD forecasting and stress testing

Semi-annual cohorts are chosen to increase the granularity of the historical default rate time series and improve the link with macroeconomic variables. The analysis can be also performed on annual or quarterly basis, depending on data availability.

Step 1: Transition Dynamics

The first step is to identify the transition dynamics of the rating system. We allow the transition matrix to evolve both forwards and backwards as a 1st order Markov Chain. The forward process is used to forecast while the backward process to weight the external portfolio, as shown below. The 1st order Markov process is widely used in the finance industry to describe rating transition matrices and simplifies many of the calculations. Nevertheless, the methodology can be also applied to higher order processes if necessary. We also make the assumption that companies belonging to the same RG form a homogeneous group in respect to score and that the transitions between RGs are conditionally independent.

The transition matrix can be decomposed into a transient and an absorbing part. The transient part captures the migrations from non default states to non default states (good book), while the absorbing part refers to defaults. The transition matrix decomposition can be written as:

$$P_{trans} = \begin{bmatrix} & TR & ABS \\ TR & \begin{bmatrix} P_{1,1} & ..... & P_{1,11} \\ .... & .... & .... \\ P_{11,1} & ..... & P_{11,11} \end{bmatrix} & \begin{bmatrix} PD_1 \\ .... \\ PD_{11} \end{bmatrix} \\ & ABS & ABS \\ ABS & [0 \quad .... \quad 0] & 1 \end{bmatrix}$$

In practice there is another absorbing state for companies that are included in the cohort but are lost during the course of the cohort period. We assume the run off is distributed evenly across the other states, thus not changing their relative proportion.

The transitions from RG $S_t = i$ at time t to RG $S_{t+1} = j$ at time $t+1$, given the conditional independence assumption, can be regarded as realizations from a binomial distribution:

$$T_{ij,t+1} \mid S_t = i, I_{ij,t} \sim Binom\left(P_{ij,t}^{trans}, N_{i,t}\right)$$

where $T_{ij,t}$ are the transition from $S_t = i$ to $S_{t+1} = j$, $P_{ij,t}^{trans} = P\left(S_{t+1} = i \mid S_t = j, I_{ij,t}\right)$ is the conditional migration probability, $N_{i,t}$ is the number of companies at time t, and $I_{ij,t}$ is the information set for any particular transition. Similarly the reverse chain transitions can be written as:

$$T_{ij,t} \mid S_{t+1} = i, I_{ij,t+1} \sim Binom\left(P_{ij,t+1}^{trans}, N_{i,t+1}\right)$$

The information set for the forward and backward chains can differ. Its inclusion makes both chains time inhomogeneous.

Step 2: Historical rating distribution of internal portfolio

The second step is to form the synthetic historical internal portfolio. Data Assumption 2 states that the internal portfolio is a subset of the external portfolio, and given that the internal portfolio is large enough to represent a significant proportion of the market, we make the additional assumption that the forward and backward migrations between grades are proportionally the same between internal and external portfolios. In other words, $P(S_{t+1} | S_t, I_t)$ and $P(S_t | S_{t+1}, I_{t+1})$ are assumed to be the same for both internal and external portfolios.

The historical backwards migration probability is first derived from the external portfolio, shown as the dotted line moving backwards in Figure 2. The implied historical composition ($S_{i,t-1} \dots S_{i,t-5}$) for the internal portfolio is derived by recursively applying the observed backward transition probabilities from the external portfolio to the initial state, $S_{e,t}$, of the internal portfolio, as shown in Figure 3. The historical forward migration probability is calculated based on the same principle using $S_{e,t-5}$ as the initial state of the Markov Chain.

Figure 2 Backward and forward migration of external portfolio

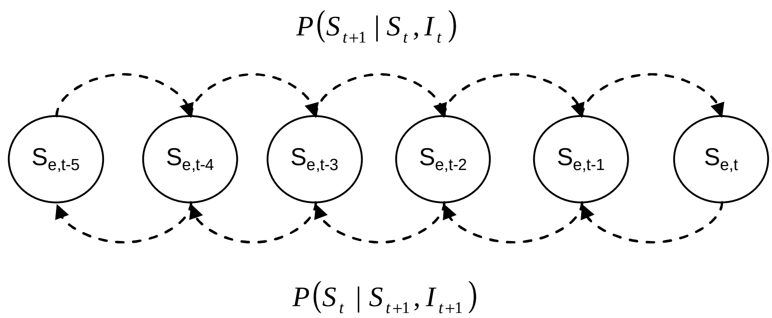
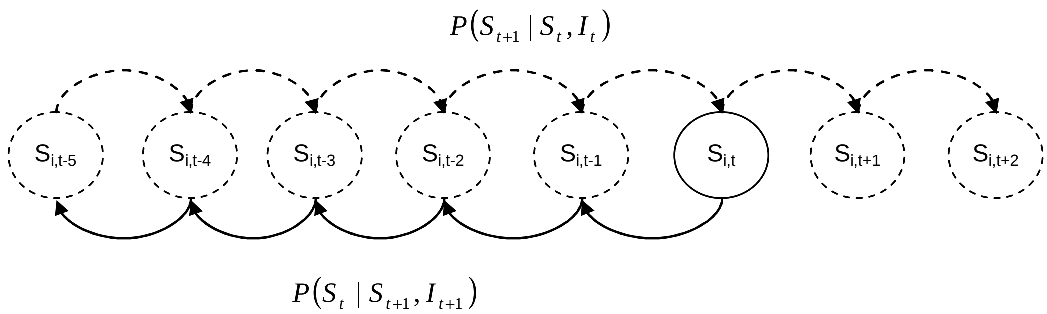


Figure 3 Backward and forward migration of internal portfolio

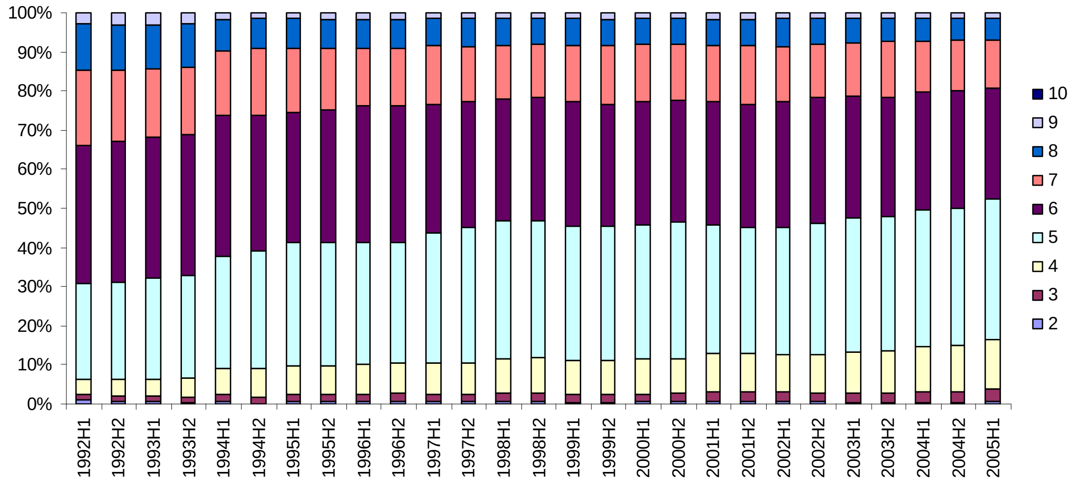


The initial states for the backward chains are observable for both the internal and external portfolio. However, the initial state for the internal portfolio is derived by first running the backward chain although it is directly observable in the external portfolio.

The historical internal portfolio composition $S_{i,t}$, as well as the forward migration probability $P(S_{t+1} | S_t, I_t)$, is used in later sections to estimate the PD forecasting model and rating migration model respectively. See Step 4 and 5.

The calculated historical internal portfolio composition is shown in Figure 4. The PD risk rating resembles a bell shape distribution with majority of the obligors fall within risk rating 5 and 6 and relatively stable throughout the observation period. The lack of volatility suggests that the PD rating system leans towards TTC in the TTC-PiT approach.

Figure 4 Historical internal portfolio rating distribution



Step 3: Default Rate Level Adjustment

If the model has the same discriminatory power internally and externally, under the RG homogeneity and conditionally default occurrence assumptions, we expect to see the same default rate for the two portfolios. However, given that the scoring model is built on the internal portfolio, we expect it to better discriminate credit risk for companies that belong to this portfolio. Therefore, the homogeneity in respect to scores does not imply a consistent homogeneity in default risk across the two portfolios and we expect to see differences in default rates.

We use the overlapping period of 5 years (2000 – 2004) to align the default rates in the two portfolios. For each of the RGs with non-zero default rate, we assume a linear relationship in log odds space between internal and external portfolio. Default rate is aligned by adjusting both the intercept and the slope of the log odds ratio.

Step 4: PD Forecasting Model

We derive the time series of theoretical default counts (split by RG) by adjusting the difference in default rate in each RG of the synthetic historical internal portfolio. This long history of defaults forms the basis for building a predictive model to forecast the grade specific and aggregate portfolio PD level.

Default is a complicated process that may not have a straightforward link with the real, monetary or financial markets economic variables. Therefore, a latent factor is included to capture movements in default rates that are not reflected in the observable variables. Latent factors are widely used in default modelling (Tasche (2006), McNeil and Wendin (2006), Koopman and Lucas (2008), Duffie et al (2007)). The Generalized Linear Model (GLM) model is extended to include latent variables and becomes known as Generalized Linear Mixed Model (GLMM). Furthermore, we allow the unobservable factor to follow a 1st order stationary autoregressive process to capture any autocorrelation effect. The model can be summarized as follows:

$$D_{k,t} \mid I_t^d \sim Binomial\left(PD_{k,t}, N_{k,t}\right)$$

$$Log\left(\frac{PD_{k,t}}{1-PD_{k,t}}\right) = a_k + \mathbf{b}_k \cdot X_t + u_t$$

$$u_t = \varphi \cdot u_{t-1} + e_t,\ \ e_t \sim N\left(0, \sigma^2\right)$$

where $D_{k,t}$ is the number of defaults for RG k at time t, $PD_{k,t}$ is the grade and time specific probability of default, $N_{k,t}$ is the number of companies in grade k at time t, a_k is grade specific average default intensity, X_t is the vector of observable macroeconomic variables at time t, $\mathbf{b}_k$ is the sensitivity of the probability of default for grade k to the macroeconomic variables, u_t is the unobservable factor, $|\varphi| < 1$ measures the mean reversion rate of the latent factor and e_t are iid normally distributed disturbances with zero mean and σ^2 variance.

A number of observable macroeconomic variables are also used to predict defaults and these can be broadly classified into 4 groups: business cycle, interest rate, financial markets, and corporate market conditions. All variables are measured on a quarterly basis; however, if a variable is observed more frequently then the average across the quarter is taken. The variables are summarised in Table 5:

Table 5 Macroeconomic covariates

Variable	Type	Description
GDP Growth	Business Cycle	Log Real GDP Change 0-2 Lags, Trailing Log Real GDP Change (TrGDP) 0-2 Lags
Inflation	Business Cycle	Log GDP Deflator Change 0-2 Lags
Unemployment rate (ILO)	Business Cycle	0-2 Lags Log Change 0-2 Lags
Capital Utilisation Gap, % GDP	Business Cycle	0-2 Lags
3M LIBOR	Interest Rate	0-2 Lags
2Y Swap Rate	Interest Rate	0-2 Lags
10Y Swap Rate	Interest Rate	0-2 Lags
Short-Long Spread	Interest Rate	3M-10Y Difference 0-2 Lags
Short-Medium Spread	Interest Rate	3M-2Y Difference 0-2 Lags
Commodities Index	Financial Markets	Log Change 0-2 Lags Trailing Log Change 0-2 Lags
FTSE 100 Index	Financial Markets	Log Change 0-2 Lags Trailing Log Change 0-2 Lags
Dow Jones Index	Financial Markets	Log Change 0-2 Lags Trailing Log Change 0-2 Lags
Nikkei Index	Financial Markets	Log Change 0-2 Lags Trailing Log Change 0-2 Lags
S&P 500 Index	Financial Markets	Log Change 0-2 Lags Trailing Log Change 0-2 Lags
DAX Index	Financial Markets	Log Change 0-2 Lags Trailing Log Change 0-2 Lags
Relative Return On Investment	Corporate Market Conditions	0-2 Lags
Corporate Gearing	Corporate Market Conditions	0-2 Lags
Real Corporate Profits	Corporate Market Conditions	Log Change 0-2 Lags Trailing 0-2 Lags

The selection process involves 3 steps. First, single factor regression is employed to check whether the lag or the quarterly change of each variable is significant against defaults. The combinations of variables that provide the best fit to the data are then chosen using stepwise regression. The last step involves checking the coefficient signs for counter-intuitive results. The procedures yield the following model:

$$\begin{bmatrix} PD_{4,t} \\ PD_{5,t} \\ PD_{6,t} \\ PD_{7,t} \\ PD_{8,t} \\ PD_{9,t} \end{bmatrix} = F\left(\begin{bmatrix} \underset{(0.4906)}{-5.8163} \\ \underset{(0.2135)}{-4.2848} \\ \underset{(0.2130)}{-4.5254} \\ \underset{(0.1974)}{-4.2434} \\ \underset{(0.2006)}{-4.0643} \\ \underset{(0.2861)}{-3.9467} \end{bmatrix} + \begin{bmatrix} \underset{(22.8838)}{-81.4683} & \underset{(26.1877)}{-99.4897} \\ \underset{(4.4073)}{-47.7586} & \underset{(8.2913)}{42.1413} \\ \underset{(3.9124)}{-18.6253} & \underset{(6.7903)}{15.3668} \\ 0 & \underset{(5.8277)}{10.1551} \\ 0 & \underset{(5.8277)}{10.1551} \\ \underset{(9.5717)}{-33.5506} & \underset{(5.8277)}{10.1551} \end{bmatrix} \cdot \begin{bmatrix} \text{TrGDP Growth}_{\text{t-1}} \\ \text{SLSpread}_{\text{t-1}} \end{bmatrix} + u_t \right)$$

$$u_t = \underset{(0.1758)}{0.7351} \cdot u_{t-1} + e_t,\ e_t \sim N\left(0, \underset{(0.1071)}{0.1708}\right)$$

where $F(x) = \dfrac{1}{1+\exp^{-(x)}}$ is the logistic function, TrGDP is 12 months trailing GDP and SLSpread is Short minus Long Spread. RG 1 and 2 do not have any defaults historically where PD cannot be derived empirically and therefore is assumed to be 0 in this study. RG 3 has only two observation periods with default and an average default rate of 0.128% for the whole period. For simplicity, we retain 0.128% as a constant for RG 3 in subsequent analysis, although more sophisticated treatment of low default portfolio are also widely discussed in other papers (Pluto and Tasche (2006), Forrest (2005), Benjamin, Cathcart and Ryan (2006)).

Step 5: Rating Migration Model

We focus on the transition model for the forward chain given that the concept is the similar for backward chain transition model. The conditional probabilities $P_{ij,t}^{trans} = P\left(S_{t+1} = i \mid S_t = j, I_{ij,t}\right)$ are modelled by a logistic function similar to the logistic regression specification employed in the scoring model. There is an option to model each of the pairs i, j separately but we choose to build a generic model with all rating transitions tied to two factors: a downgrade and an upgrade factor. Given that the information set only consists of the downgrade or upgrade factor f_t, the conditional probabilities can be rewritten as $P_{ij,t}^{trans} = P\left(S_{t+1} = i \mid S_t = j, f_t\right)$. The model can be expressed as:

$$Log\left(\frac{P_{ij,t}^{trans}}{1 - P_{ij,t}^{trans}}\right) = \lambda_{ij} + \beta_{ij} \cdot f_t, \text{ for } i \neq j$$

$$f_t = \begin{cases} f_t^{down}, \text{if } i < j \\ f_t^{up}, \text{if } i > j \end{cases}$$

where λ_{ij} is the event specific average migration intensity and β_{ij} is the event specific sensitivity to the upgrade or downgrade factor f_t. In order to get a transition matrix, each row of P^{trans} should sum to one, and therefore $P_{ii,t}^{trans} = 1 - \sum_{i<j} P_{ij,t}^{trans} - \sum_{i>j} P_{ij,t}^{trans} - \sum_{i} PD_{i,t}$ for all i.

To derive the factor f_t, we aggregate downgrades and upgrades into two sets of binary events modelled separately. The probabilities for downgrade, $P_t^{down} = P\left(S_{t+1} > S_t \mid S_t, X_t^{down}\right)$, and upgrade, $P_t^{up} = P\left(S_{t+1} < S_t \mid S_t, X_t^{down}\right)$, are modelled respectively as follows:

$$T_t^{down} \mid S_t, X_t^{down} \sim Binom\left(P_t^{down}, N_t\right)$$
$$Log\left(\frac{P_t^{down}}{1-P_t^{down}}\right) = f_t^{down} = \lambda_{down} + X^{down}\beta_{down}, \text{ and}$$

$$T_t^{up} \mid S_t, X_t^{up} \sim Binom\left(P_t^{up}, N_t\right)$$
$$Log\left(\frac{P_t^{up}}{1-P_t^{up}}\right) = f_t^{up} = \lambda_{up} + X^{up}\beta_{up},$$

where T_t^{down}, T_t^{up} are the number of downgrades and upgrades respectively, $\lambda_{down}, \lambda_{up}$ are the average downgrade and upgrade intensities, X^{down}, X^{up} are the matrices of explanatory variables (which can be different for downgrades and upgrades), and β_{down}, β_{up} are the coefficients vectors.

The semi-annual transition data exhibit a high degree of seasonality. We choose to include the seasonal dummies $season_t$ (taking value of 1 if the cohort corresponds to the second half of each year) to account for seasonality in the estimation.

The information set $I_{ij,t}$ at time t is usually taken to be either observable (macroeconomic) or unobservable (latent) variables. The use of macroeconomic variables is intuitive and provides some clear advantages in forecasting and stress testing (see Nickell et al (2000), and Bangia et al (2002)). Latent variables are proved to capture movements that are not related to the macroeconomic environment and in some cases the effect is greater than that of observable variables (Gagliardini and Gourieroux (2005), and Koopman et al (2009)). The proposed rating system does not exhibit any significant level of autocorrelation to justify the use of complex latent factor structures. Therefore both f_t^{down} and f_t^{up} are modelled as functions of observable macroeconomic variables only (X^{down} and X^{up} respectively). Furthermore, to simplify forecasting and stress testing it is assumed that the macroeconomic variables that drive transitions are the same with the factors that drive defaults. This would imply: $X^{down} = X^{up} = X^{trans} = X^{default}$.

Both aggregate factors and individual transitions are estimated by maximum likelihood using Fisher scoring. The estimated models for the downgrade and upgrade factor are:

$$f_t^{down} = -\underset{(0.03609)}{3.3949} + \underset{(0.02229)}{0.9256} \cdot 1_{July-December} + \underset{(1.0441)}{14.9021} TrGDP_{t-1}$$
$$f_t^{up} = -\underset{(0.03516)}{3.28} + \underset{(0.02217)}{0.9274} \cdot 1_{July-December} + \underset{(1.0263)}{9.9325} TrGDP_{t-1} - \underset{(0.9622)}{8.5121} SLSpread_{t-1}$$

The interest rate spread is dropped from the downgrade model due to non-significant coefficient. It should be noted that the trailing GDP growth has directionally the same effect for both downgrades and upgrades. This indicates that an increase in GDP growth introduces more migrations within each RG. The estimated model for the individual transition probabilities $P_{ij,t}^{trans} = P(S_{t+1} = i \mid S_t = j, f_t)$ can be found in Appendix 2.

Step 6: Forecasting and Stress Testing

To properly calibrate the rating model, we require the 1 year forward looking grade and portfolio level PD. The PD model has two sources of forecasting power. Firstly, coming from lagged values of observable variables and secondly from the stationary latent factor. Forecasting macroeconomic variables is outside the scope of this paper. Therefore, we use 2004 as the starting point to forecast defaults based on the actual trailing GDP growth, short-long interest rate spread time series for the period 2005-2008 and the MSE expected value for the latent factor, $u_t = 0.7351 \cdot u_{t-1}$

For stress testing purpose, the bank would be likely to introduce a stress scenario in the middle of the 5 years horizon in 2004. Business planning is usually focused on horizons shorter than 5 years given the difficulty to predict the market conditions far in the future. To form a synthetic stress scenario we use the autoregressive properties of the two time series that affect default and migrations, trailing GDP growth and short-long interest rate spread. For both series a simple AR(1) was fitted by least squares and the estimates are shown below:

$$TrGDP_t = \underset{(0.05543)}{0.979} \cdot TrGDP_{t-1} + \varepsilon_t^{GDP},\ \varepsilon_t^{GDP} \sim N\left(0, 0.0161^2\right)$$
$$Spread_t = \underset{(0.14611)}{0.65563} \cdot Spread_{t-1} + \varepsilon_t^{Spread},\ \varepsilon_t^{Spread} \sim N\left(0, 0.012899^2\right)$$

The downturn conditions are defined in terms of PD increase and therefore trailing GDP growth percentiles correspond to the lower end of the distribution (due to negative sign in PD forecasting model), while short-long spread percentiles correspond to the upper end of the distribution (due to positive sign in PD forecasting model). The stress percentiles are chosen as follows to simulate a stress scenario with the worst year in the middle.

Table 6 Stress scenario as the percentile of AR(1) distribution

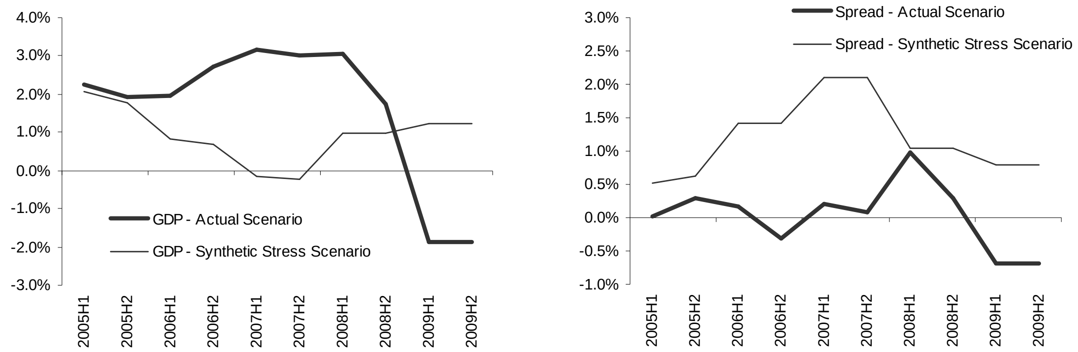
Percentile	2005	2006	2007	2008	2009
TrGDP	20%	5%	1%	10%	15%
SLSpread	80%	95%	99%	90%	85%

Under a real world stress scenario correlations between GDP, interest rates and other macroeconomic variables should be explored and stress forecasts of those variables are likely to be derived from structural or reduced form VARs. To illustrate the concept, a simplified "factor-push" approach to stress scenario is employed in this paper.

4 Results

Based on the data observation period 2005-2008 we have an actual 5-year economic scenario leading to 2009, and an economic shock showing TrGDP dropping by around 1.85% in the last 6 months of 2008. The synthetic stress scenario, according to the AR(1) model and confidence interval approach, shows the worst year in the middle of the 5-year outlook and gradually reverts to mean level towards the end of 5-year period.

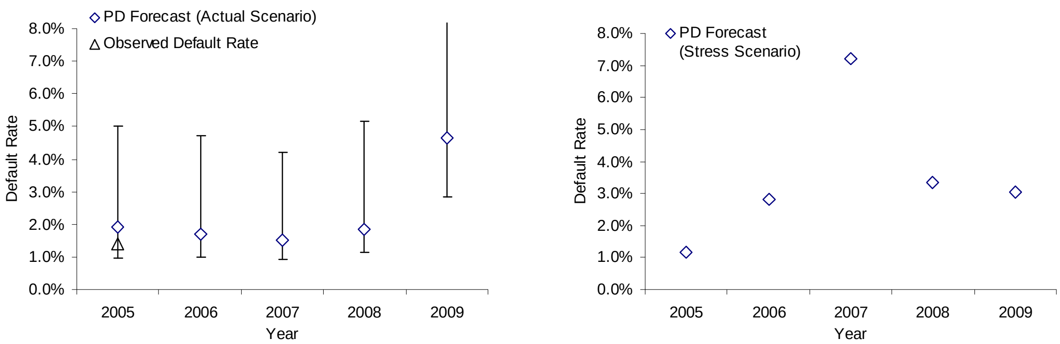
Figure 5 GDP and Spread# – actual and forecast from 2005 to 2009



the H22009 data point of actual scenario is replicated from 2009H1 data

Based on the actual scenario, the PD forecasting model yields stable PDs from 2005 to 2008, but a higher PD in 2009, as shown in Figure 6. The observed default rate in 2005 falls within the 90% confidence interval of the PD estimates thus validating the PD forecasting model. Given the macroeconomic forecast, the PD forecast can be extended beyond 2009 to simulate all possible outcomes. The PD forecast in the stress scenario is also available by feeding the stress factor forecast into the PD forecasting model.

Figure 6 PD forecast under actual# and stress scenarios



the error bar corresponds to 90% confidence interval

The risk rating distributions as modelled in accordance with the proposed approach under both the actual scenario and the synthetic stress scenario are presented below.

Figure 7 Transition for actual scenario

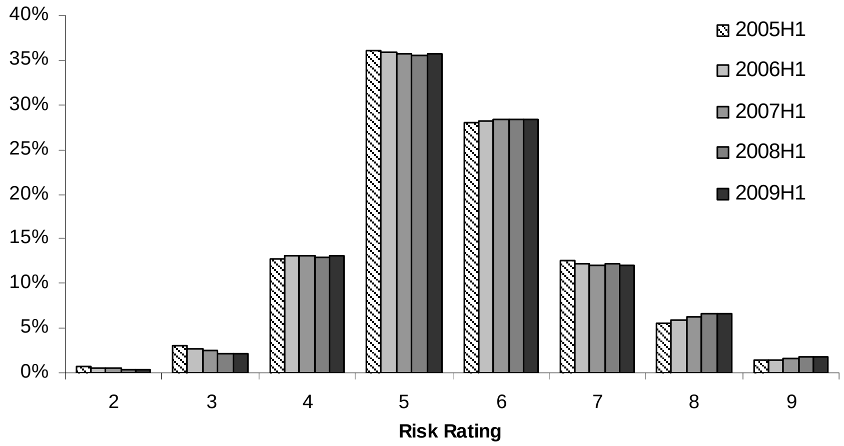
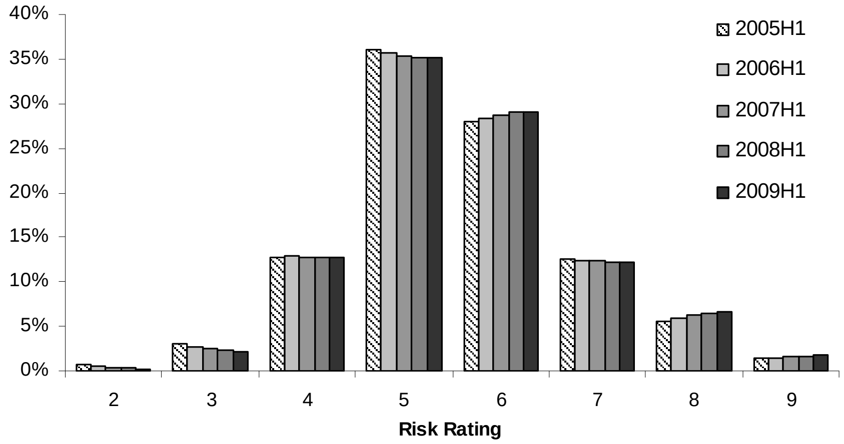
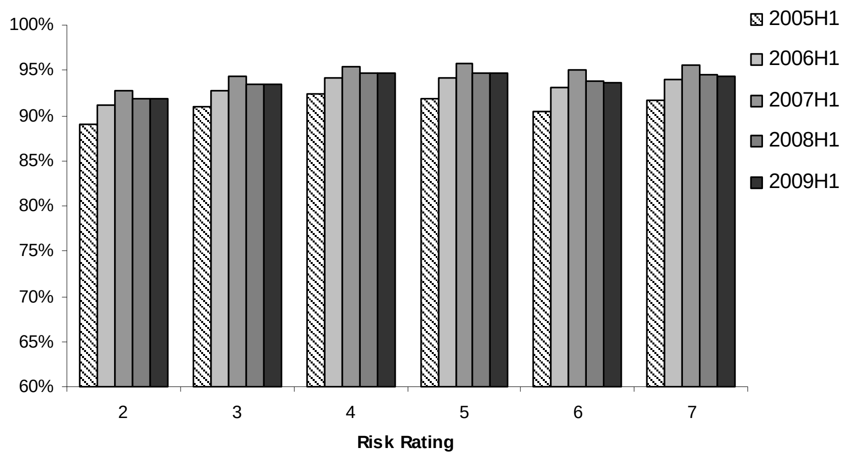


Figure 8 Transition for synthetic stressed scenario



The aggregated risk rating in Figure 7 and Figure 8 show a very stable distribution without any obvious migration throughout the 5-year horizon. The raw data in the stress scenario has shown circa 10% to 15% of the obligors either migrate upwards or downwards in each year, as shown in Figure 9. Although the individual risk rating shows a moderate migration outwards, the net effect is not apparent at portfolio level.

Figure 9 Percentages of Risk Ratings with no migration (stress scenario)



One may expect to see a greater degree of migrations and transitional dynamics in the outcome. The stable results are likely attributed to the following reasons:

- *Stable underlying risk rating system:* historically the risk rating has exhibited high stability throughout the data history, as shown in Figure 4. As a hybrid system, the risk rating is more akin to a TTC calibration than a PiT system. The lower cyclicality has ensured low level of migrations between risk ratings even in a stress scenario.
- *Same economic factors driving PD rating system and rating transition:* the rating transition model employs the same macroeconomic factors in the PD rating system for simplicity. In reality, the PD rating system and rating transition may be driven by different factors. The sensitivity of the rating migration model is reduced for not having bespoke factors specific to the model.
- *Data coverage:* this paper assumes that the external data covering 1992 to 2008 to be a full economy cycle. However given that the early 1990s UK downturn was considered to include 1990 and 1991, the lack of data in this period may cause estimation bias in the model and yield a model which is less responsive to downturn scenario.

Some of the issues are addressed by default if one is to apply the methodology in the real world such as adopting the actual PD rating system with moderate cyclicality. Several other issues need to be carefully managed during design stage, for instance having separate model factors in forecasting PD and rating distribution. It should also be noted that a real risk rating system is likely to be more complicated than what is presented in this paper which may pose challenges to adopt the proposed methodology.

5 Conclusion

This paper has shown, using a synthetic risk rating and dataset, the concept of utilising external data to inform the internal portfolio historical risk rating distribution, PD forecast, rating migration and stress testing. The major contribution of this paper is to demonstrate a practical solution in PD modelling given a lack of internal data if appropriate external proxy data and macroeconomic data are available. Once the PD forecasting model and rating migration model are built, the ability to forecast portfolio PD in any given macroeconomic scenario would benefit across the bank's credit risk management practices including PD calibration, business planning, capital and loss provisioning.

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Appendices

Appendix 1 Filing types to approximate default in Companies House dataset

Filing type	
Admin Abstracts & Payments	F 1.14 Ending of Moratorium
Admin Progress Report	Liq Statement of Rec/Payments
Admin Receivers Report 3.10	Meeting of Creditors (Gazette)
Administration Notice Gazette	Move from Admin to Dissolution
Administration Order	Move from Admin to Liquidation
Administrator App Gazette	Notice ceasing to act Receiver
Administrators Final Notice	Notice of Admin Order
App of Liquidator, Court Wind Up	Petitions Winding Up (Gazette)
App of Official Receiver (F14)	Petitions Winding Up Dismissed
App of Provisional Liquidator	Receivers Abstract of R&P 3.6
App of Receiver by a Court	Receivers Report
App/Liquidator Compulsory (Gazette)	Receivership Doc. Lodged
App/Liquidator Creditors (Gaz)	Receivership Notice Gazette
Appoint of Liquidator (Gazette)	Resolution Creditors Wind Up
Appointment of Administrator	Resolution to Wind Up
Completion of Winding Up	Statement of Admin Proposals
Constitution of Liq Committee	Statement of Admin Proposals
Court Order Notice of Wind Up	VA Revocation of Suspension
Dis. Winding Up	Variation of Admin Order
Discharge of Admin Order	Vol Arrangement Completion
End of Administration	Voluntary Arrangements
F 1.11 Start of Moratorium	Winding Up Orders (Gazette)
F 1.12 Extension of Moratorium	

Appendix 2 Transition probabilities estimates

The estimates and their standard errors for the individual transition model $Log\left(\frac{P_{ij,t}^{trans}}{1-P_{ij,t}^{trans}}\right)=\lambda_{ij}+\beta_{ij}\cdot f_t$, where $f_t=\begin{cases} f_t^{down}, \text{if } i<j \\ f_t^{up}, \text{if } i>j \end{cases}$ depending on whether the event is a downgrade or an upgrade, can be found in Table 7. Coefficients which are not significant at the 10% significance level are dropped.

Table 7: Transition probabilities estimates

Transition Event	**Downgrade**		**Transition Event**	**Upgrade**	
	λ_{ij}	β_{ij}		λ_{ij}	β_{ij}
2 => 3	-	0.7796 (0.07716)	**3 => 2**	-	1.605 (0.06775)
2 => 4	-2.7809 (0.2661)	-	**4 => 2**	-4.2015 (1.3725)	1.1745 (0.6107)
2 => 5	-3.3162 (0.3393)	-	**4 => 3**	-2.0280 (0.3146)	0.8401 (0.1352)
2 => 6	-	1.5931 (0.1871)	**5 => 2**	-	3.9765 (0.1663)

Transition Event	Downgrade		Transition Event	Upgrade	
	λ_{ij}	β_{ij}		λ_{ij}	β_{ij}
2 => 7	-4.8481 (0.7099)	-	**5 => 3**	-3.9650 (0.5698)	1.0082 (0.2489)
2 => 8	-	-	**5 => 4**	-1.2060 (0.1164)	0.7977 (0.0495)
2 => 9	-	-	**6 => 2**	-9.8251 (0.7071)	-
3 => 4	-	0.8283 (0.02949)	**6 => 3**	-4.7845 (0.9195)	0.9847 (0.3996)
3 => 5	-	1.4031 (0.05463)	**6 => 4**	-2.3547 (0.3043)	1.0682 (0.1331)
3 => 6	-4.1245 (0.1811)	-	**6 => 5**	0.2243 (0.0881)	1.0209 (0.0377)
3 => 7	-5.7797 (0.4089)	-	**7 => 2**	-	4.5839 (0.5261)
3 => 8	-	3.4793 (0.5069)	**7 => 3**	-	3.3559 (0.1558)
3 => 9	-	-	**7 => 4**	-2.5545 (0.6107)	1.1990 (0.2701)
4 => 5	-	0.8377 (0.01092)	**7 => 5**	-1.0780 (0.2128)	0.9082 (0.09099)
4 => 6	-1.6861 (0.3155)	0.9410 (0.1348)	**7 => 6**	0.8216 (0.1283)	1.1902 (0.05524)
4 => 7	-2.0136 (0.7865)	1.550 (0.3547)	**8 => 2**	-	-
4 => 8	-6.8720 (0.2583)	-	**8 => 3**	-	4.2774 (0.5080)
4 => 9	-	4.5023 (0.5249)	**8 => 4**	-3.9208 (1.1715)	0.9603 (0.5064)
5 => 6	-0.2193 (0.09069)	0.9313 (0.03831)	**8 => 5**	-1.0986 (0.3252)	1.0434 (0.1410)
5 => 7	-1.9412 (0.2191)	1.0082 (0.09423)	**8 => 6**	-	1.1490 (0.01932)
5 => 8	-3.1288 (0.3721)	0.9639 (0.1597)	**8 => 7**	-	1.0833 (0.0180)
5 => 9	-5.5163 (1.0043)	0.8040 (0.4248)	**9 => 2**	-	-
6 => 7	-	1.2190 (0.01026)	**9 => 3**	-	-
6 => 8	-1.4545 (0.2253)	1.1440 (0.09761)	**9 => 4**	-	-
6 => 9	-4.4387 (0.5348)	0.6462 (0.2217)	**9 => 5**	-2.1512 (1.0033)	0.9218 (0.430)
7 => 8	-0.7121 (0.2057)	0.9884 (0.08724)	**9 => 6**	-	1.3317 (0.04918)
7 => 9	-2.5179 (0.5333)	1.0617 (0.2293)	**9 => 7**	-1.620 (0.5866)	0.6759 (0.2449)
8 => 9	-1.3650 (0.3439)	0.9419 (0.1454)	**9 => 8**	-	0.9555 (0.03277)