



Instituto de Matemática e Estatística

USP

Model of Credit Loss and its use in Spread Decisions

University of São Paulo

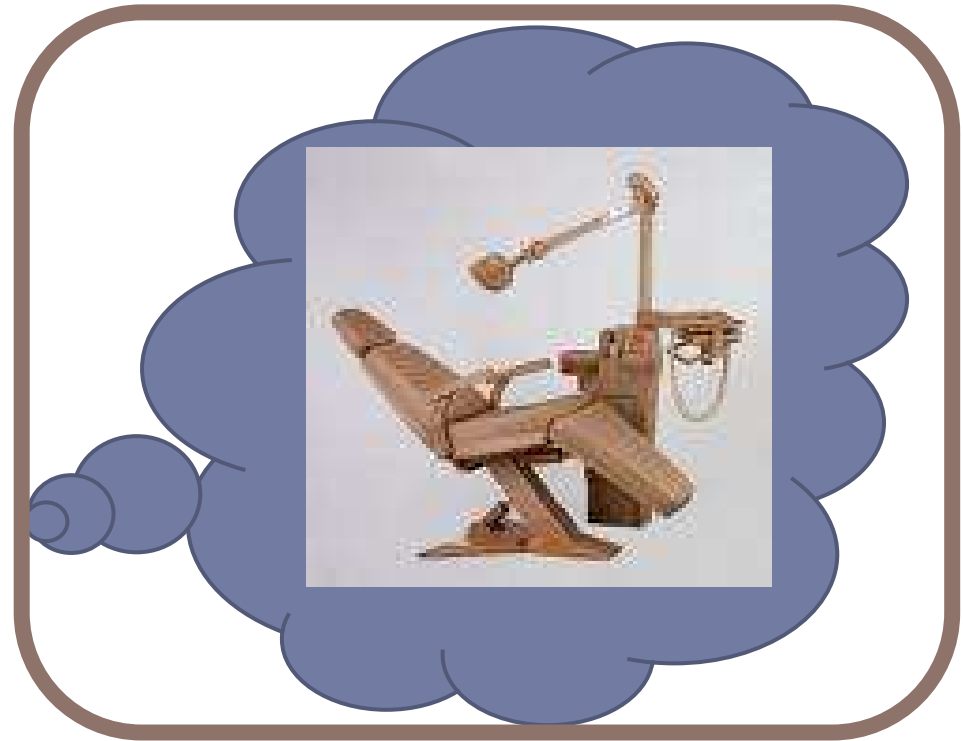
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Brief Introduction



What a credit loan is?



Mr. Magritte needs money to start his business.



What a credit loan is?



... And borrows it from a bank...



What a credit loan is?



With some customers of his own,
he expects paying the loan back.



What a credit loan is?



But things might not run so well...



What a credit loan is?



And Mr. Magritte might never pay back his debt.



Financial outcome of a portfolio

Amount lent

=



Target outcome

=



Revenues

=



Credit Losses

=

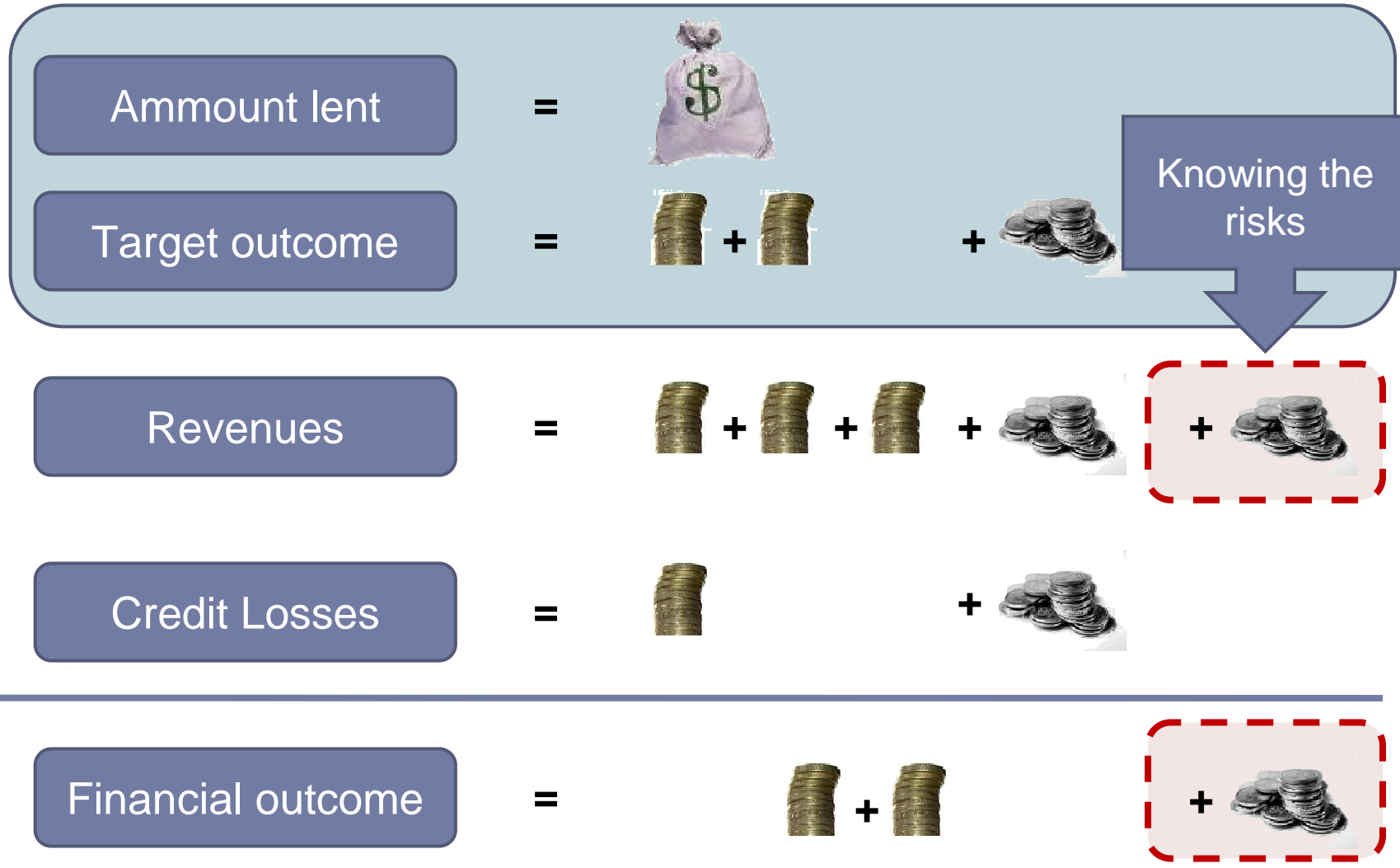


Financial outcome

=



Financial outcome of a portfolio



Measuring credit risk

Finished operations can be classified as:



Loans paid without default
Group 1

Paying after credit recovery efforts
Group 2

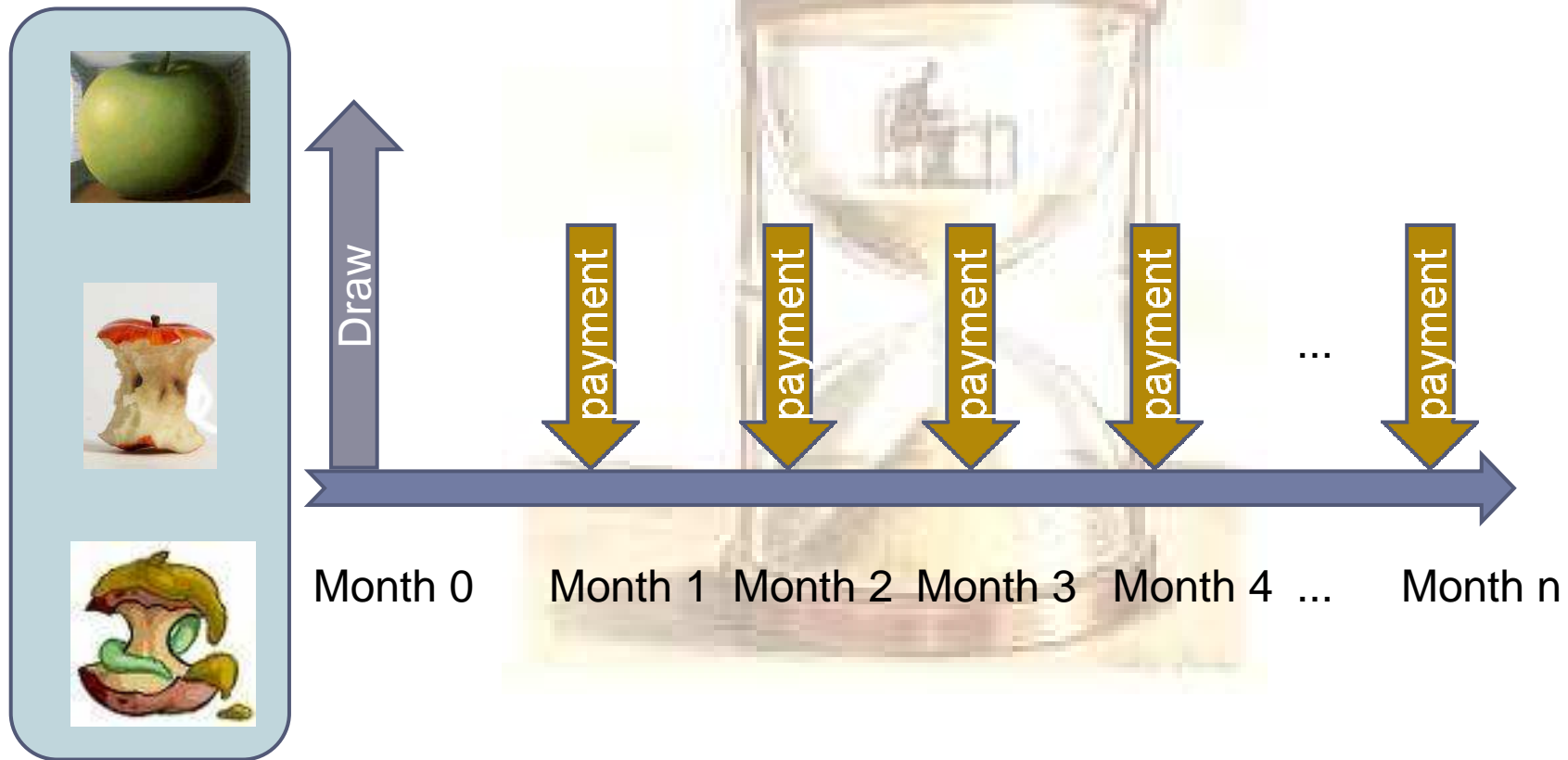


Finished as write off
Group 3



Measuring operations outcome

The outcome will also take into account the payments.



Financial Outcome

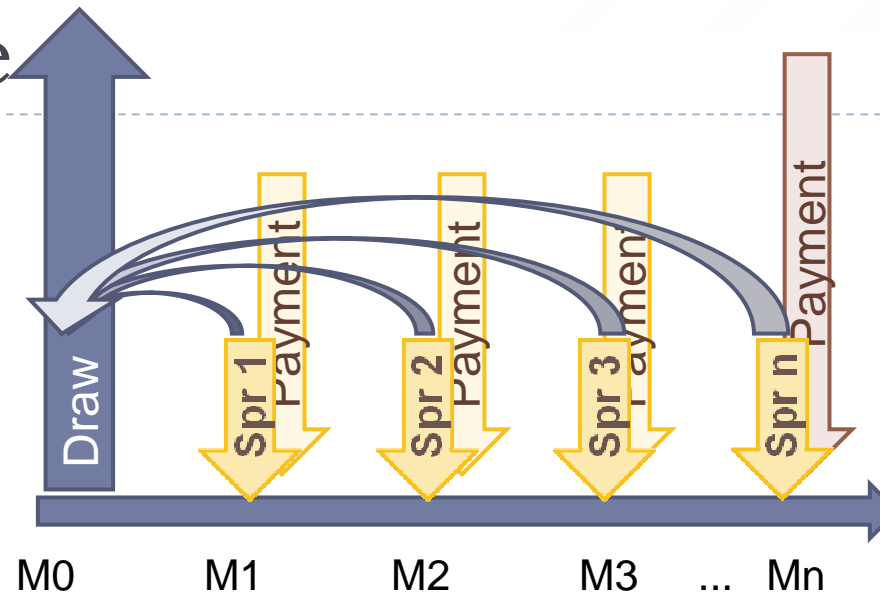
Present Value (VP)	100.000,00
Instalments (NI)	6
Interest Fee (i)	1,9%
Future Value (VF)	106.754,28
Payment (PMT)	17.792,38
Caption Cost (c)	1,0000%
Spread (spr)	0,8911%

The operation result depends on how the operation is finished and how many instalments were paid.

instalment	Balance	Corrected balance	Balance quit	Interest fee	Caption cost	Nominal spread
0	100.000,00	101.000,00	-	-	-	-
1	84.107,62	84.948,70	15.892,38	1.900,00	1.000,00	900,00
2	67.913,28	68.592,42	16.194,34	1.598,04	841,08	756,97
3	51.411,26	51.925,37	16.502,03	1.290,35	679,13	611,22
4	34.595,69	34.941,65	16.815,57	976,81	514,11	462,70
5	17.460,63	17.635,23	17.135,06	657,32	345,96	311,36
6-	0,00 -	0,00	17.460,63	331,75	174,61	157,15

Financial outcome

Present Value (VP)	100.000,00
Instalments (NI)	6
Interest Fee (i)	1,9%
Future Value (VF)	106.754,28
Payment (PMT)	17.792,38
Caption Cost (c)	1,0000%
Spread (spr)	0,8911%

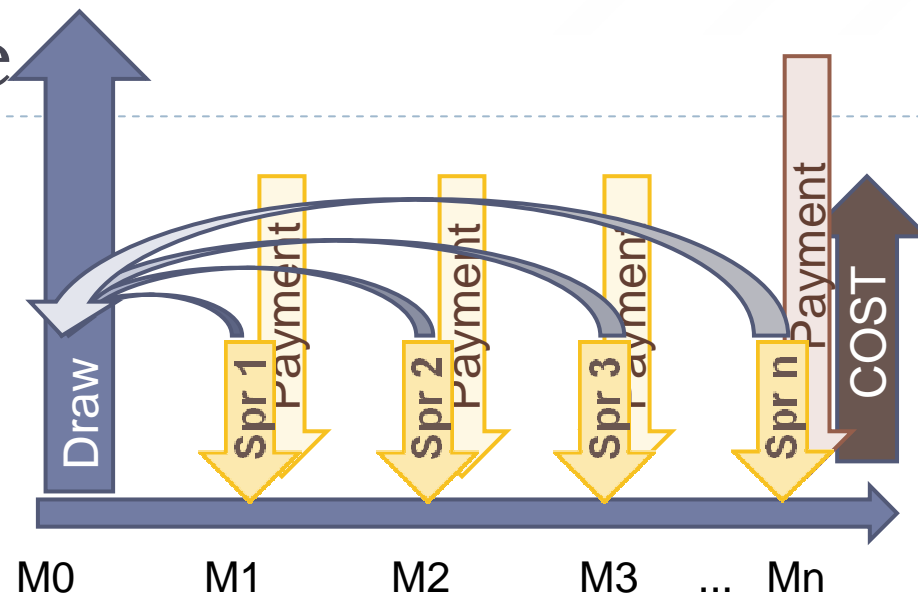


Instalment	Nominal Spread	PV Spread	Cumulated Spread	Operation Result
0	-	-	-	-
1	900,00	891,09	891,09	891,09
2	756,97	742,05	1.633,14	1.633,14
3	611,22	593,24	2.226,39	2.226,39
4	462,70	444,65	2.671,03	2.671,03
5	311,36	296,25	2.967,28	2.967,28
6	157,15	148,04	3.115,32	3.115,32

Group 1:
The financial outcome of a loan paid without default is the sum of the spreads paid at each instalment.

Financial outcome

Present Value (VP)	100.000,00
Instalments (NI)	6
Interest Fee (i)	1,9%
Future Value (VF)	106.754,28
Payment (PMT)	17.792,38
Caption Cost (c)	1,0000%
Spread (spr)	0,8911%

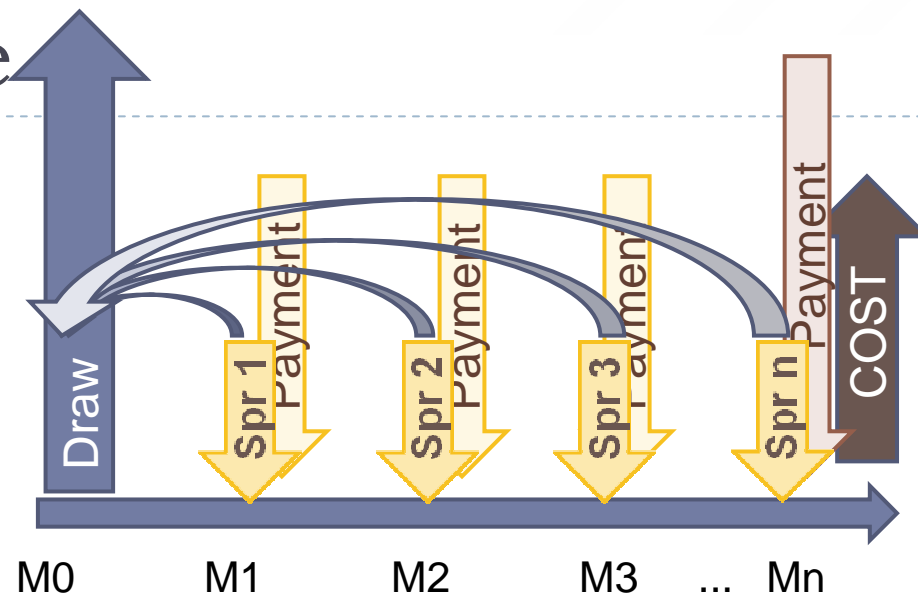


Instalment	Nominal Spread	PV Spread	Cumulated Spread	Operation Result
0	-	-	-	-
1	900,00	891,09	891,09	891,09
2	756,97	742,05	1.633,14	1.633,14
3	611,22	593,24	2.226,39	2.226,39
4	462,70	444,65	2.671,03	2.671,03
5	311,36	296,25	2.967,28	2.967,28
6	157,15	148,04	3.115,32	3.115,32

Group 2:
 Paying the loan under credit recovery efforts would produce an additional cost

Financial outcome

Present Value (VP)	100.000,00
Instalments (NI)	6
Interest Fee (i)	1,9%
Future Value (VF)	106.754,28
Payment (PMT)	17.792,38
Caption Cost (c)	1,0000%
Spread (spr)	0,8911%



Instalment	Nominal Spread	PV Spread	Cumulated Spread	Operation Result
0	-	-	-	-
1	900,00	891,09	891,09	891,09
2	756,97	742,05	1.633,14	1.633,14
3	611,22	593,24	2.226,39	2.226,39
4	462,70	444,65	2.671,03	2.671,03
5	311,36	296,25	2.967,28	2.967,28
6	157,15	148,04	3.115,32	3.115,32

Group 3:
A written off loan never has its final payment.

Expected financial outcome

Expected financial outcome:

$$E(O_i | \mathbf{x}_i) = \sum_{j=1}^3 E(O_i | Y_{ij} = 1; \mathbf{x}_i) \Pr(Y_{ij} = 1 | \mathbf{x}_i)$$

$$E(O_i | \mathbf{x}_i) = \sum_{j=1}^3 \sum_{t=1}^{t_i} E(O_i | Y_{ij} = 1; T_i = t; \mathbf{x}_i) \Pr(T_i = t | \mathbf{x}_i; Y_{ij} = 1) \Pr(Y_{ij} = 1 | \mathbf{x}_i)$$

$E(O_i | Y_{ij} = 1; T_i = t; \mathbf{x}_i)$ → Not random

$\Pr(T_i = t | \mathbf{x}_i; Y_{ij} = 1) = ?$ → Estimation needed.

$\Pr(Y_{ij} = 1 | \mathbf{x}_i) = ?$ → Estimation needed.

Expected financial outcome

$$E(O_i | \mathbf{x}_i) = \sum_{j=1}^3 \sum_{t=1}^{t_i} E(O_i | Y_{ij} = 1; T_i = t; \mathbf{x}_i) \Pr(T_i = t | \mathbf{x}_i; Y_{ij} = 1) \Pr(Y_{ij} = 1 | \mathbf{x}_i)$$

$$\Pr(Y_{i1} = 1 | \mathbf{x}_i) = \pi_{i1}$$

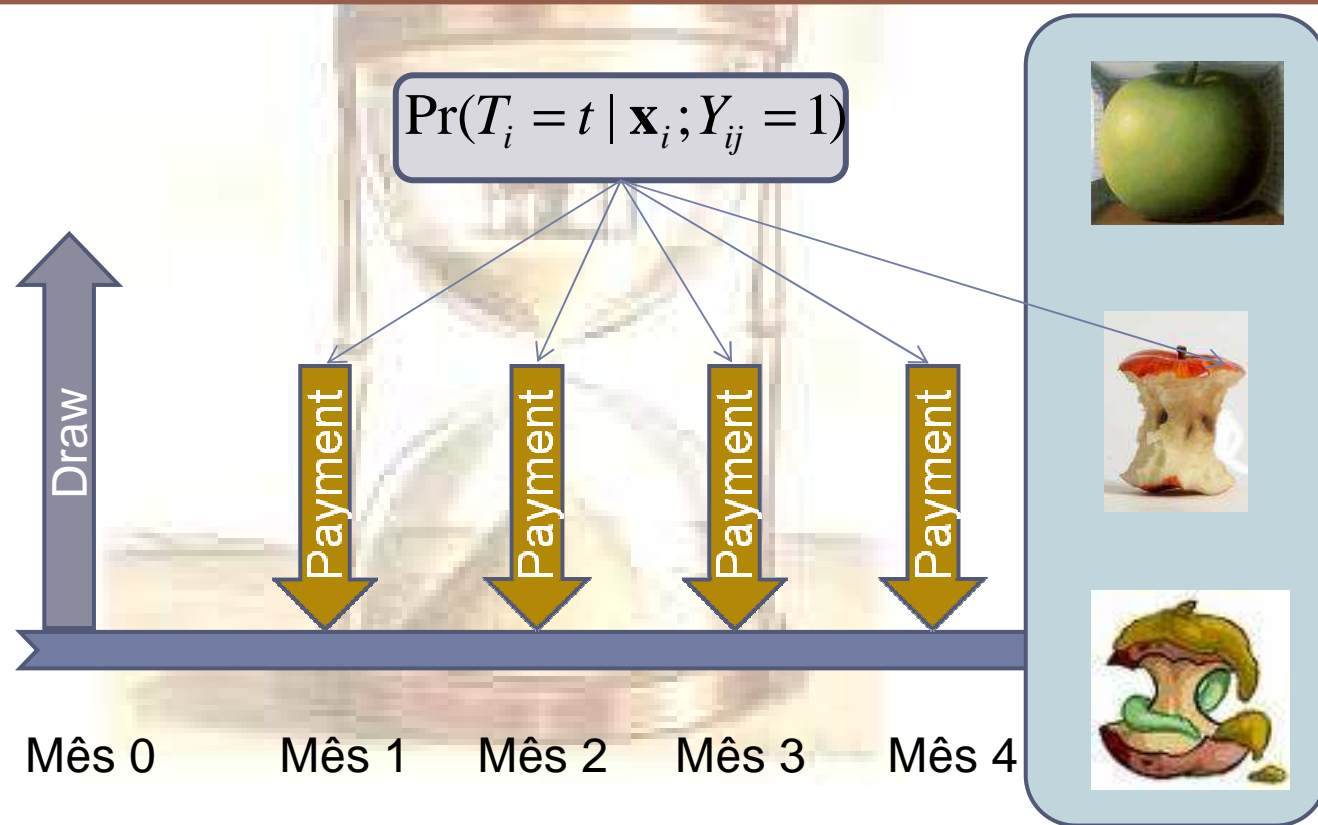
$$\Pr(Y_{i2} = 1 | \mathbf{x}_i) = \pi_{i2}$$

$$\Pr(Y_{i3} = 1 | \mathbf{x}_i) = \pi_{i3}$$



Expected financial outcome

$$E(O_i | \mathbf{x}_i) = \sum_{j=1}^3 \sum_{t=1}^{t_i} E(O_i | Y_{ij} = 1; T_i = t; \mathbf{x}_i) \Pr(T_i = t | \mathbf{x}_i; Y_{ij} = 1) \Pr(Y_{ij} = 1 | \mathbf{x}_i)$$



Group Classification

$$E(O_i | \mathbf{x}_i) = \sum_{j=1}^3 \sum_{t=1}^{t_i} E(O_i | Y_{ij} = 1; T_i = t; \mathbf{x}_i) \Pr(T_i = t | \mathbf{x}_i; Y_{ij} = 1) \Pr(Y_{ij} = 1 | \mathbf{x}_i)$$



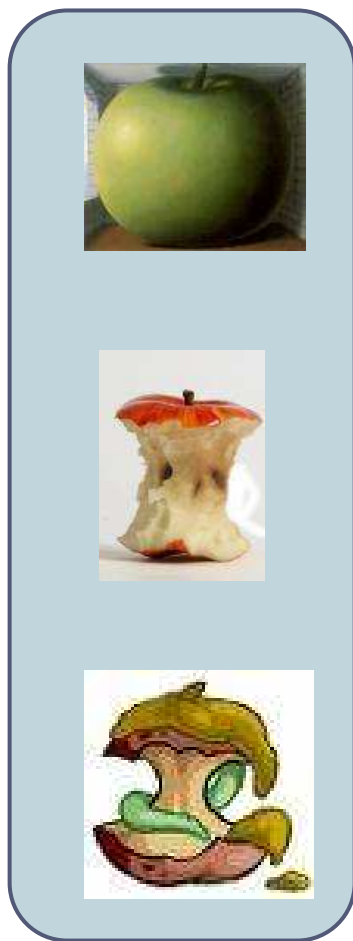
Loan ended without default
Group1

Loan ended needing credit recovery efforts
Group2



Loan ended by write off
Group3

Group Classification



Y_1

Y_2

Y_3

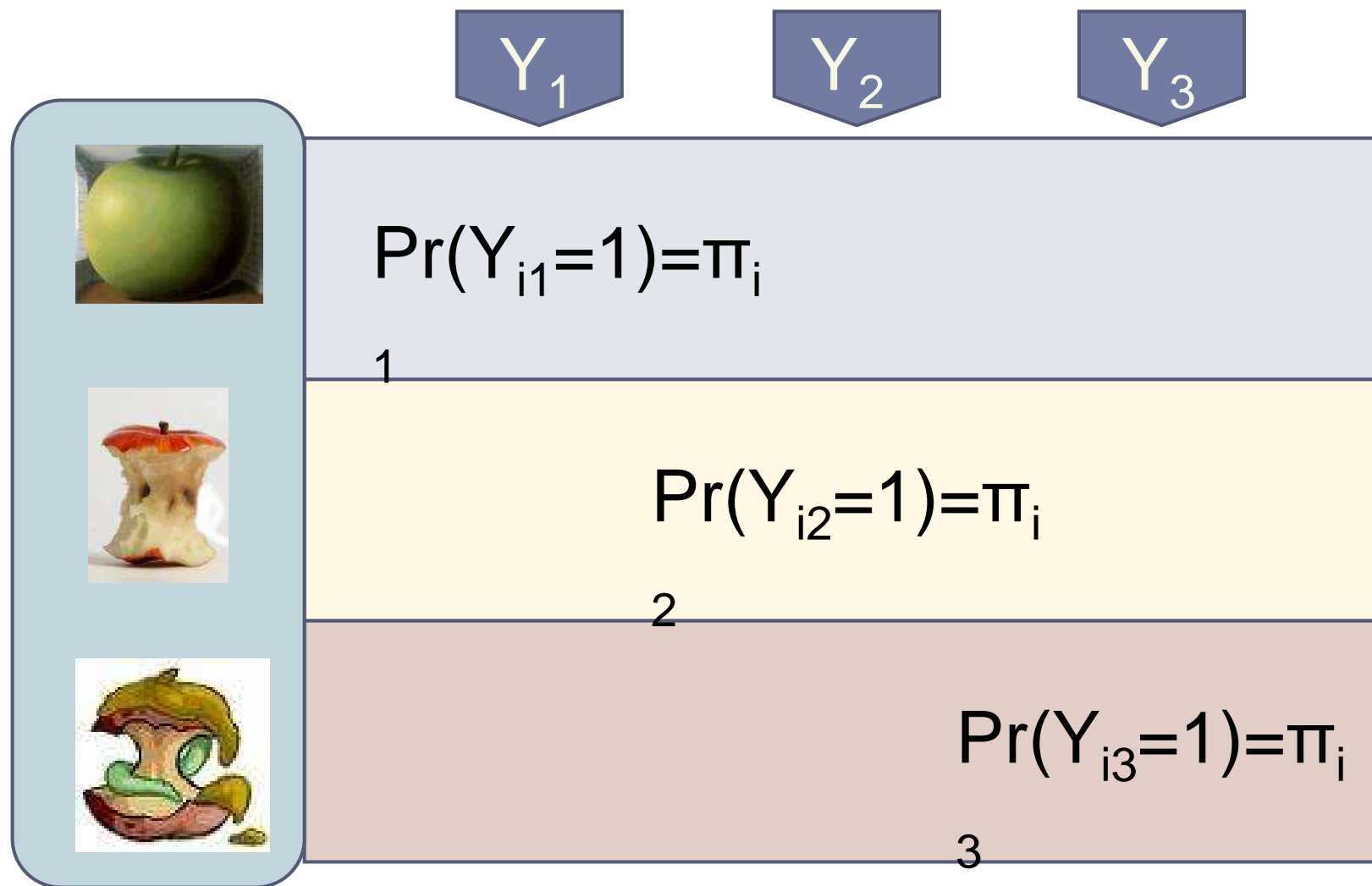
1 0 0

0 1 0

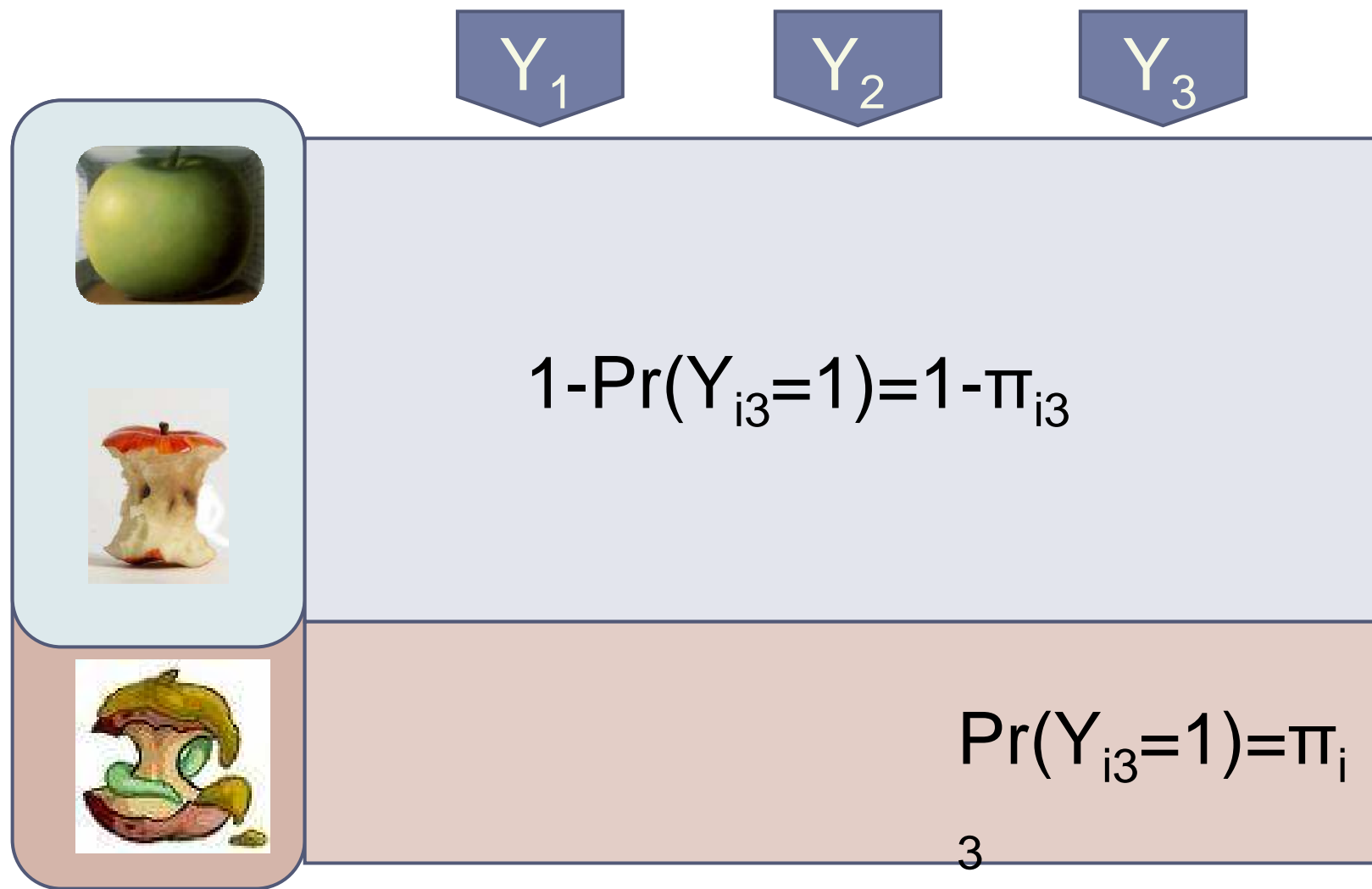
0 0 1



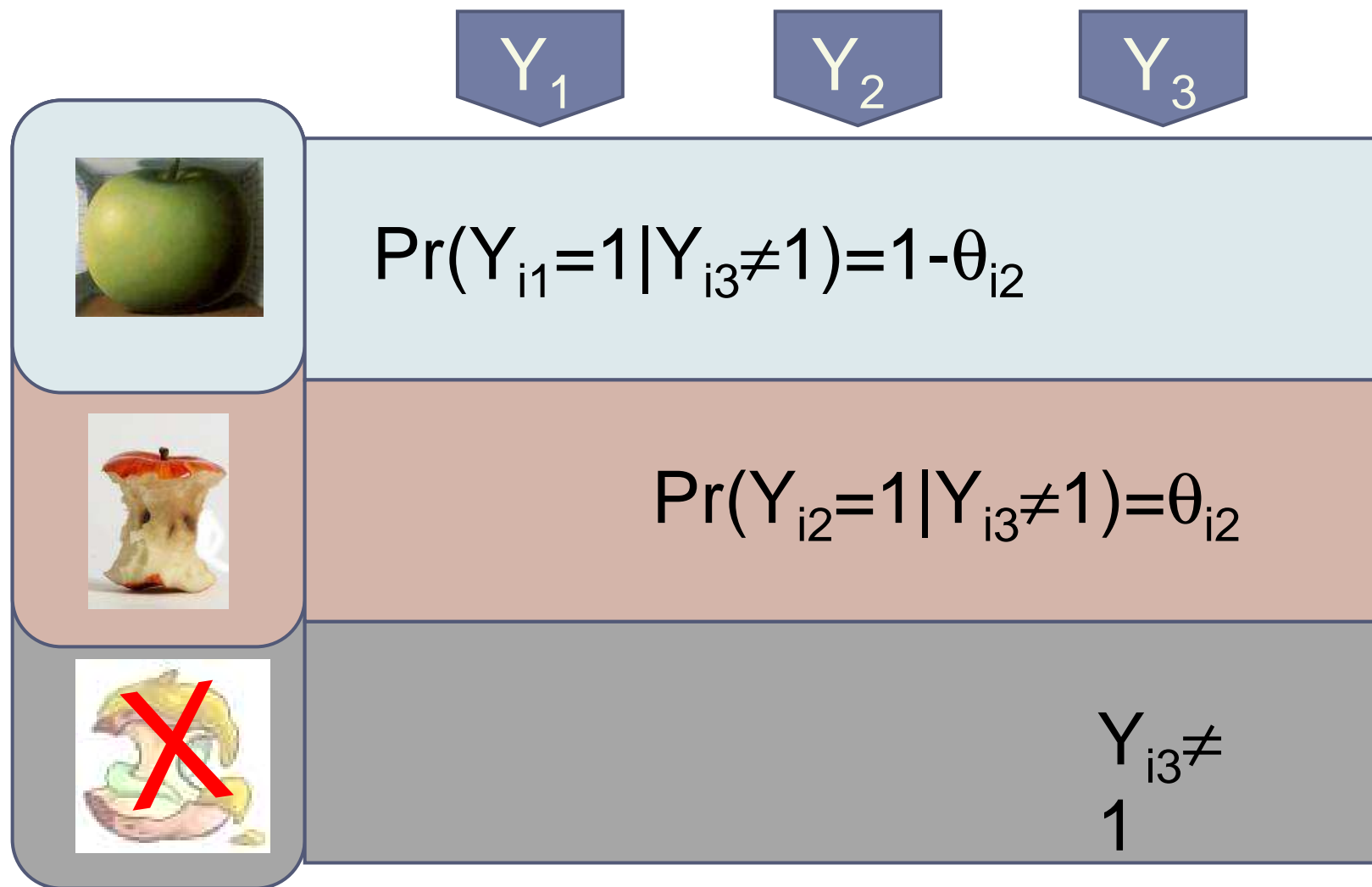
Group Classification



Group classification

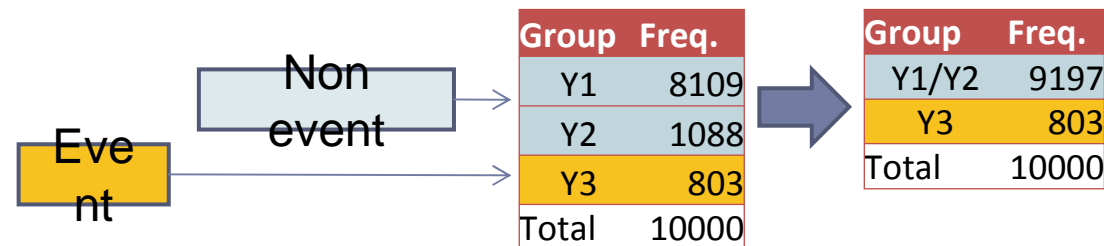


Group classification



Predictive model for π_{i3}

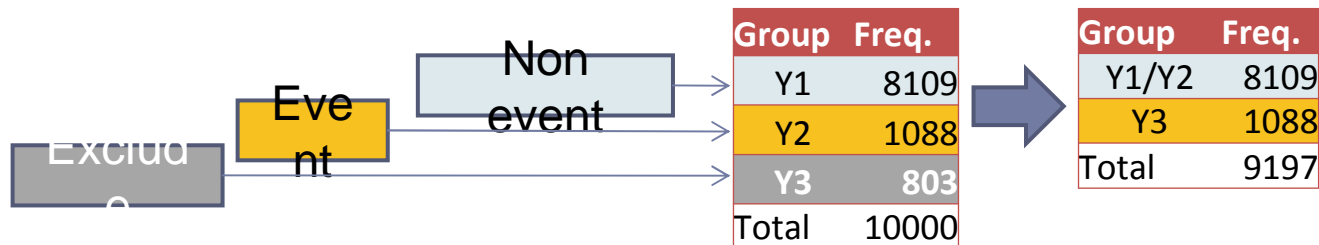
$$\ln\left(\frac{\pi_{i3}}{1-\pi_{i3}}\right) = \alpha_1 + \beta_{11}x_{i1} + \beta_{12}x_{i2} + \dots + \beta_{1p}x_{ip}$$



Parameter	Estimate	Standard Error	Chi Square	P-value
Term=6	-2,523	0,159	251,2	<0,0001
Term=12	-2,265	0,152	223,0	<0,0001
Term=18	-1,567	0,151	107,4	<0,0001
Term=24	-0,465	0,147	9,979	0,0020
Positive history	-1,334	0,083	260,2	<0,0001
Entity's age	-0,084	0,005	292,4	<0,0001
Borrowing	0,457	0,218	4,399	0,0360
Negative History	1,565	0,064	598,0	<0,0001

Predictive model for θ_{i2}

$$\ln\left(\frac{\theta_{i2}}{1-\theta_{i2}}\right) = \alpha_2 + \beta_{21}x_{i1} + \beta_{22}x_{i2} + \dots + \beta_{2p}x_{ip}$$



Parameter	Estimate	Standard Error	Chi Square	P-value
Term=6	-1.964	0.1151	291.1	<.0001
Term=12	-1.604	0.1073	223.6	<.0001
Term=18	-1.215	0.1088	124.7	<.0001
Term=24	-0.550	0.1104	24.9	<.0001
Positive History	-0.725	0.0468	239.3	<.0001
Entity's age	-0.030	0.0040	53.8	<.0001
Entity's age * Neg. History	0.001	0.0037	6.9	0.0085
Negative History	0.558	0.0870	41.1	<.0001

Estimators for π_{i1} , π_{i2} and π_{i3}

Linear Model

No linear model necessary

$$\hat{\theta}_{i2} = \frac{1}{1 + e^{-(\hat{\alpha}_2 + \hat{\beta}_{21}x_{i1} + \hat{\beta}_{22}x_{i2} + \dots + \hat{\beta}_{2p}x_{ip})}}$$

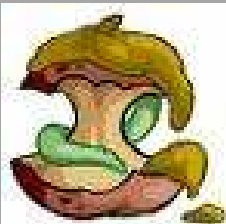
$$\hat{\pi}_{i3} = \frac{1}{1 + e^{-(\hat{\alpha}_1 + \hat{\beta}_{11}x_{i1} + \hat{\beta}_{12}x_{i2} + \dots + \hat{\beta}_{1p}x_{ip})}}$$

Estimative

$$\hat{\pi}_{i1} = (1 - \hat{\pi}_{i3}) \times (1 - \hat{\theta}_{i2})$$

$$\hat{\pi}_{i2} = (1 - \hat{\pi}_{i3}) \times \hat{\theta}_{i2}$$

$$\hat{\pi}_{i3} = \hat{\pi}_{i3}$$

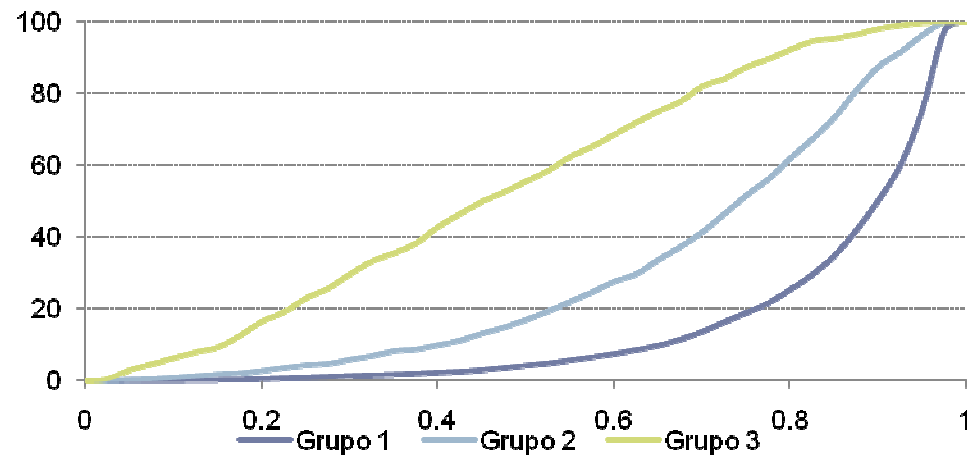


Model assessment

Homogeneity test

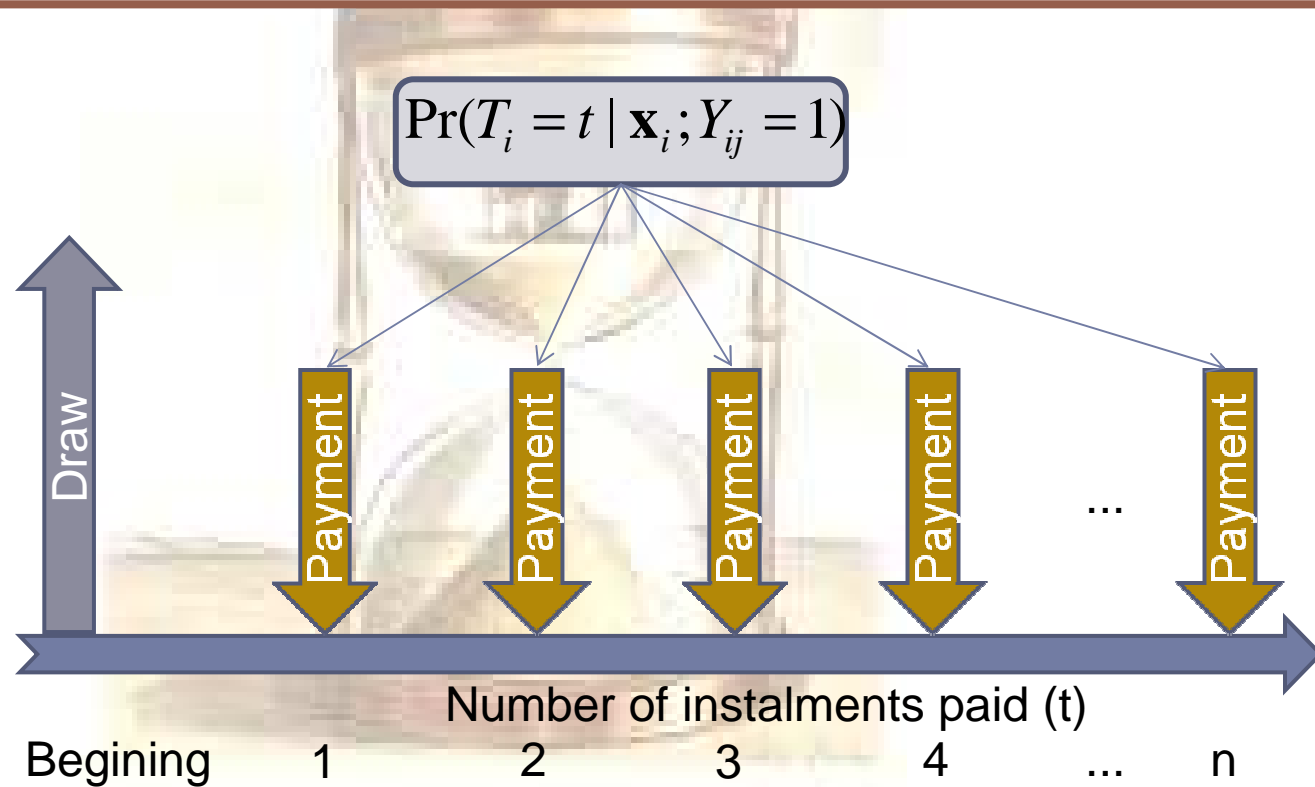
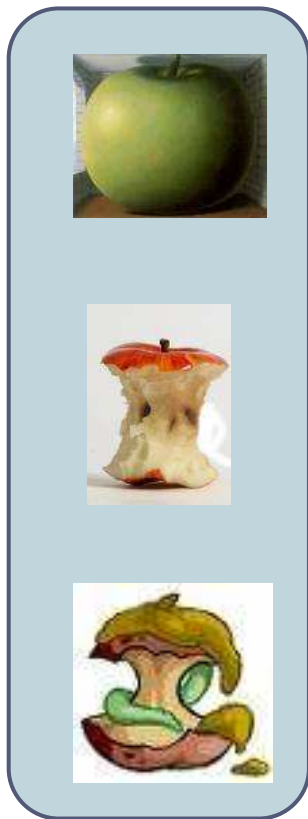
Faixa P1	p1	p2	p3	Y1	Y2	Y3
low-10,520	34%	21%	45%	35%	19%	46%
0,520-10,691	62%	22%	17%	60%	22%	17%
0,691-10,782	74%	18%	9%	73%	18%	9%
0,782-10,843	81%	14%	4%	80%	16%	4%
0,843-10,885	87%	11%	3%	87%	12%	1%
0,885-10,919	90%	8%	1%	89%	9%	2%
0,919-10,945	93%	6%	1%	95%	5%	1%
0,945-10,963	95%	4%	0%	96%	4%	0%
0,963-10,975	97%	3%	0%	97%	3%	0%
0,975-high	98%	2%	0%	99%	1%	0%

CDF of estimates



Number of instalments paid

$$E(O_i | \mathbf{x}_i) = \sum_{j=1}^3 \sum_{t=1}^{t_i} E(O_i | Y_{ij} = 1; T_i = t; \mathbf{x}_i) \Pr(T_i = t | \mathbf{x}_i; Y_{ij} = 1) \Pr(Y_{ij} = 1 | \mathbf{x}_i)$$



Number of instalments paid

$$E(O_i | \mathbf{x}_i) = \sum_{j=1}^3 \sum_{t=1}^{t_i} E(O_i | Y_{ij} = 1; T_i = t; \mathbf{x}_i) \Pr(T_i = t | \mathbf{x}_i; Y_{ij} = 1) \Pr(Y_{ij} = 1 | \mathbf{x}_i)$$

The hazard function:

$$h(t | Y_{ij} = 1) = \Pr(T_i = t | T_i > t; \mathbf{x}_i; Y_{ij} = 1)$$

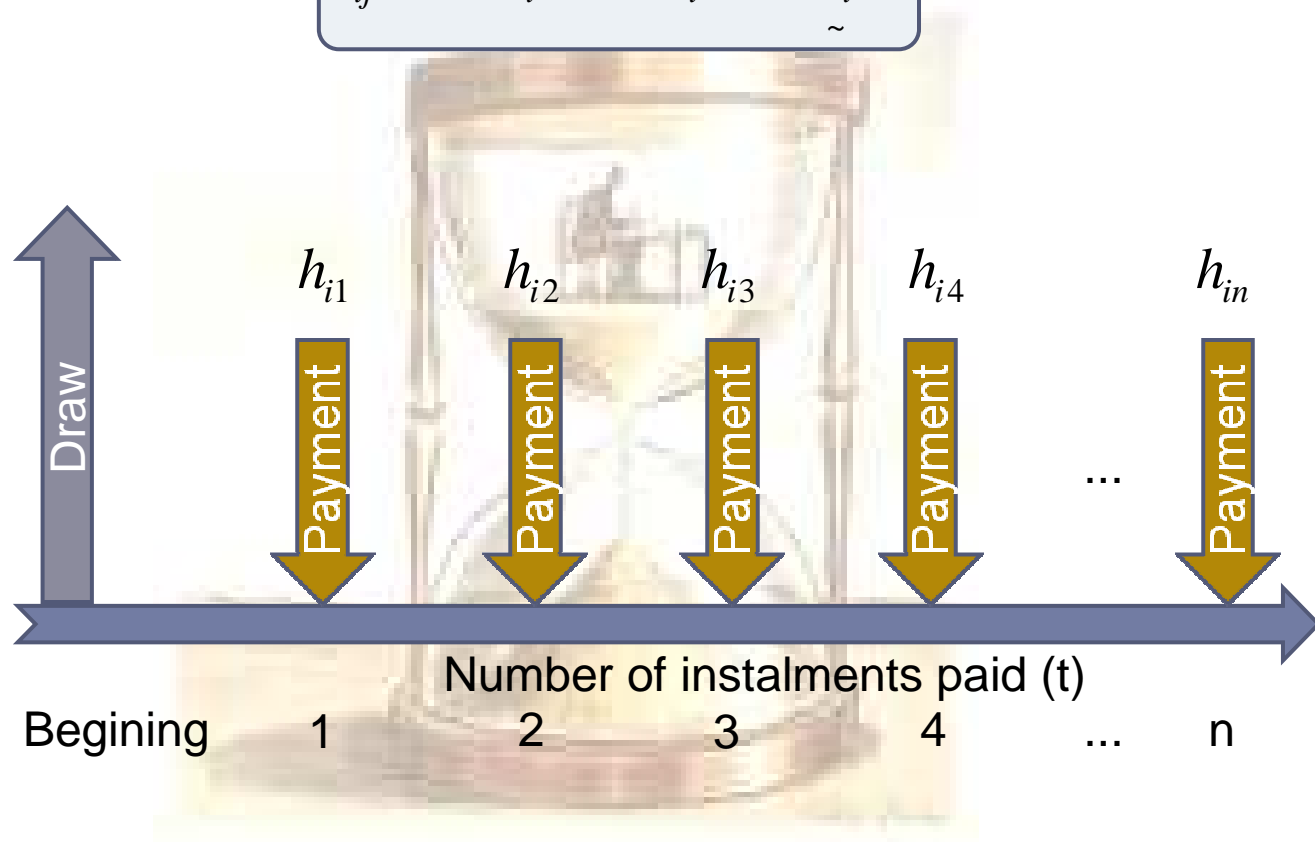
Number of instalments:

$$\Pr(T_i = t | \mathbf{x}_i; Y_{ij} = 1) = h(t) \prod_{k=1}^{t-1} (1 - h(k))$$

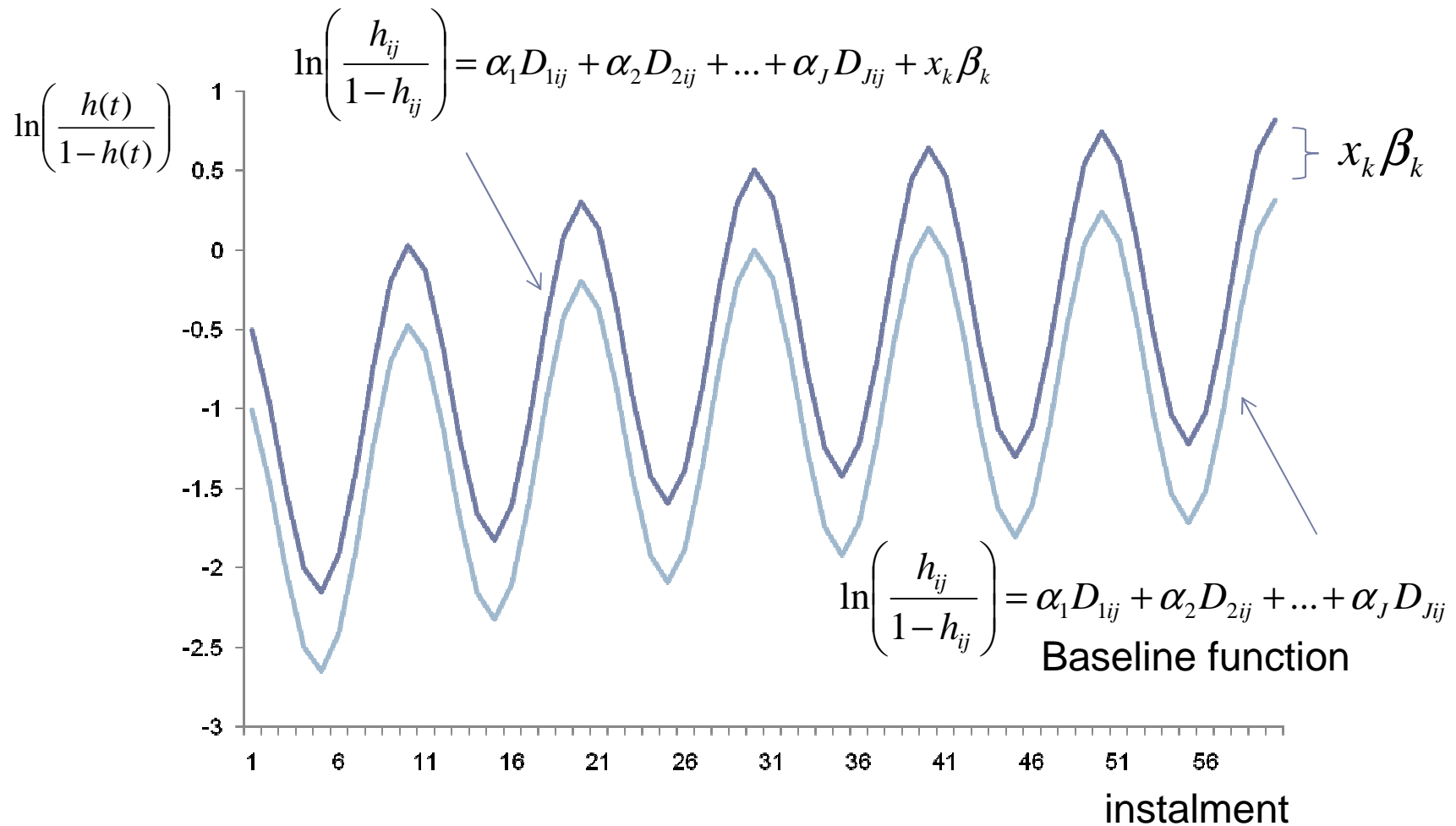


Number of instalments

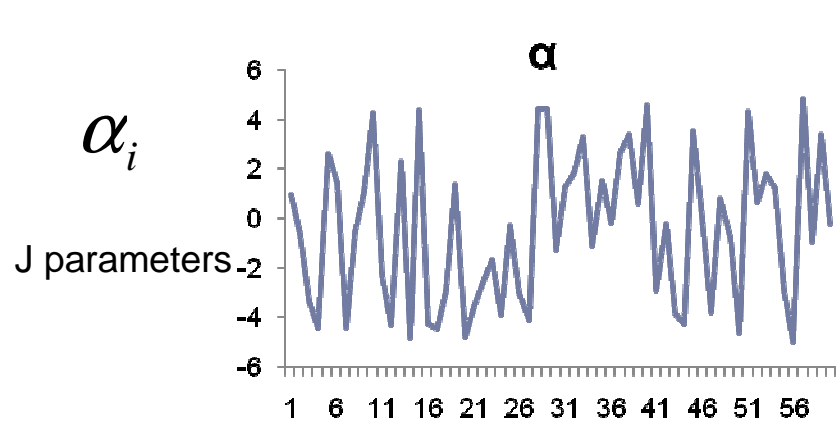
$$h_{ij} = P(T_i = j | T_i \geq j; x_i)$$



Logit Additive risks

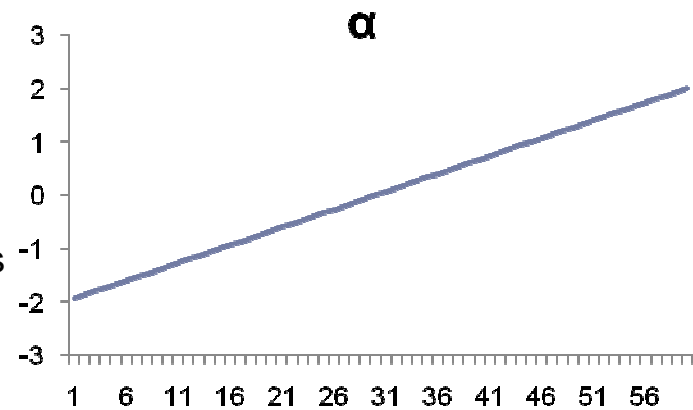


Baseline hazard risk function



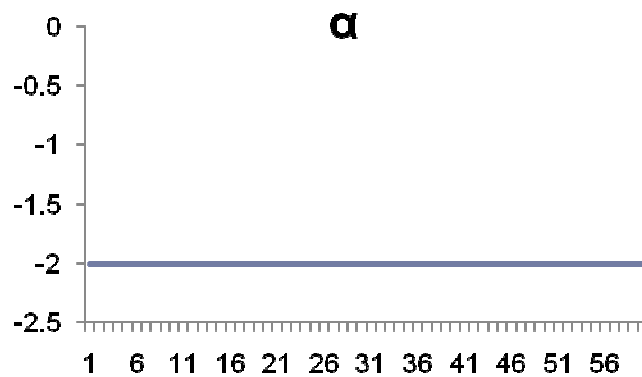
$$\alpha_i = \alpha + \beta_\alpha t$$

2 parameters

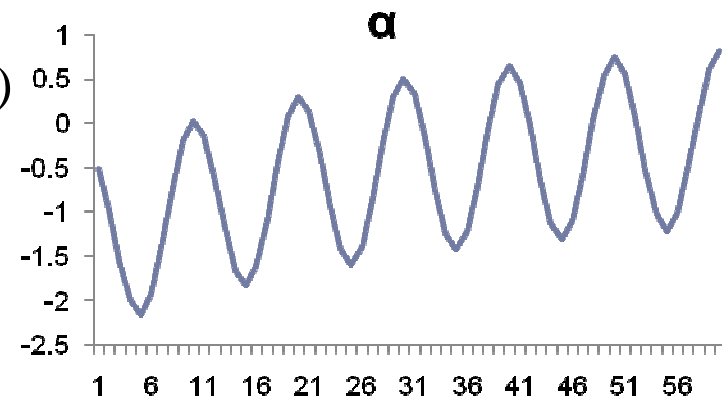


$$\alpha_i = \alpha$$

1 parameters



$$\alpha_i = g(t)$$



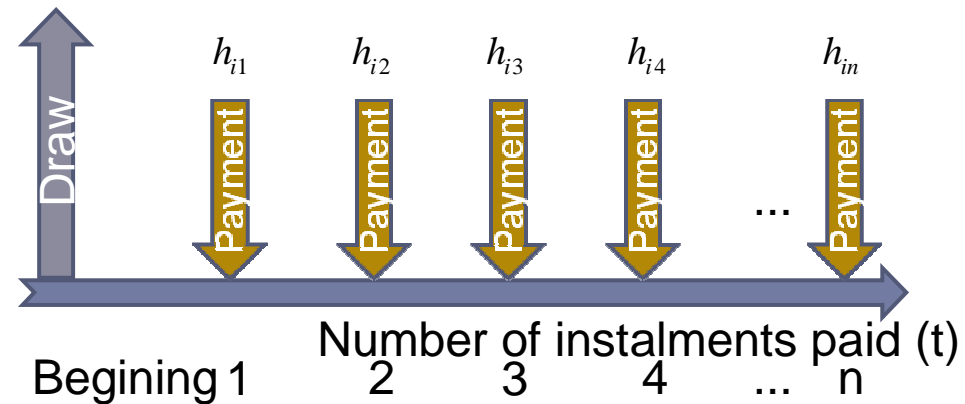
Data preparation example



Loan	Value	Term	Group	t
1	100	12	1	3
2	150	6	2	2
3	120	36	3	5

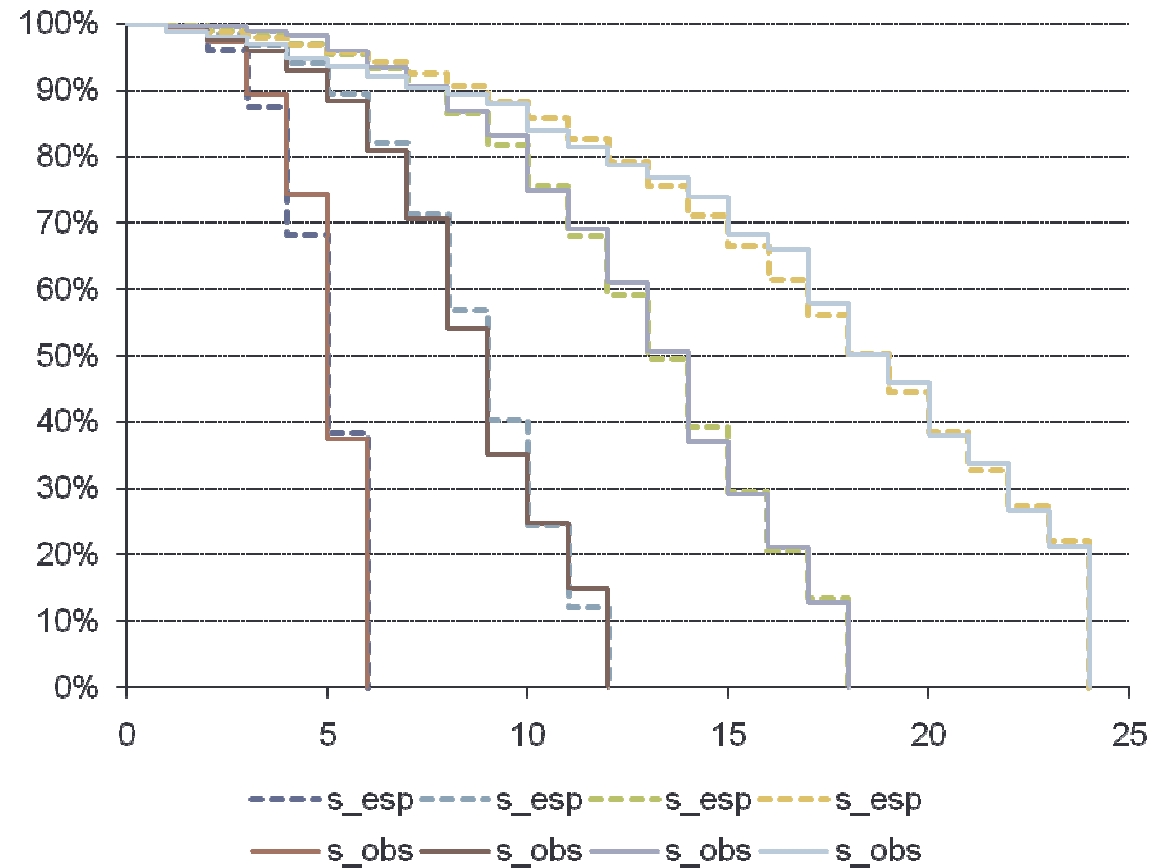
Loan	Value	Term	Group	t	D1	D2	D3	D4	D5	D6	i	Y1	y2	y3
1	100	12	1	3	1	0	0	0	0	0	1	0	0	0
1	100	12	1	3	0	1	0	0	0	0	2	0	0	0
1	100	12	1	3	0	0	1	0	0	0	3	1	0	0
2	150	6	2	2	1	0	0	0	0	0	1	0	0	0
2	150	6	2	2	0	1	0	0	0	0	2	0	1	0
3	120	36	3	5	1	0	0	0	0	0	1	0	0	0
3	120	36	3	5	0	1	0	0	0	0	2	0	0	0
3	120	36	3	5	0	0	1	0	0	0	3	0	0	0
3	120	36	3	5	0	0	0	1	0	0	4	0	0	0
3	120	36	3	5	0	0	0	0	1	0	5	0	0	1

Number of instalments | Group 1



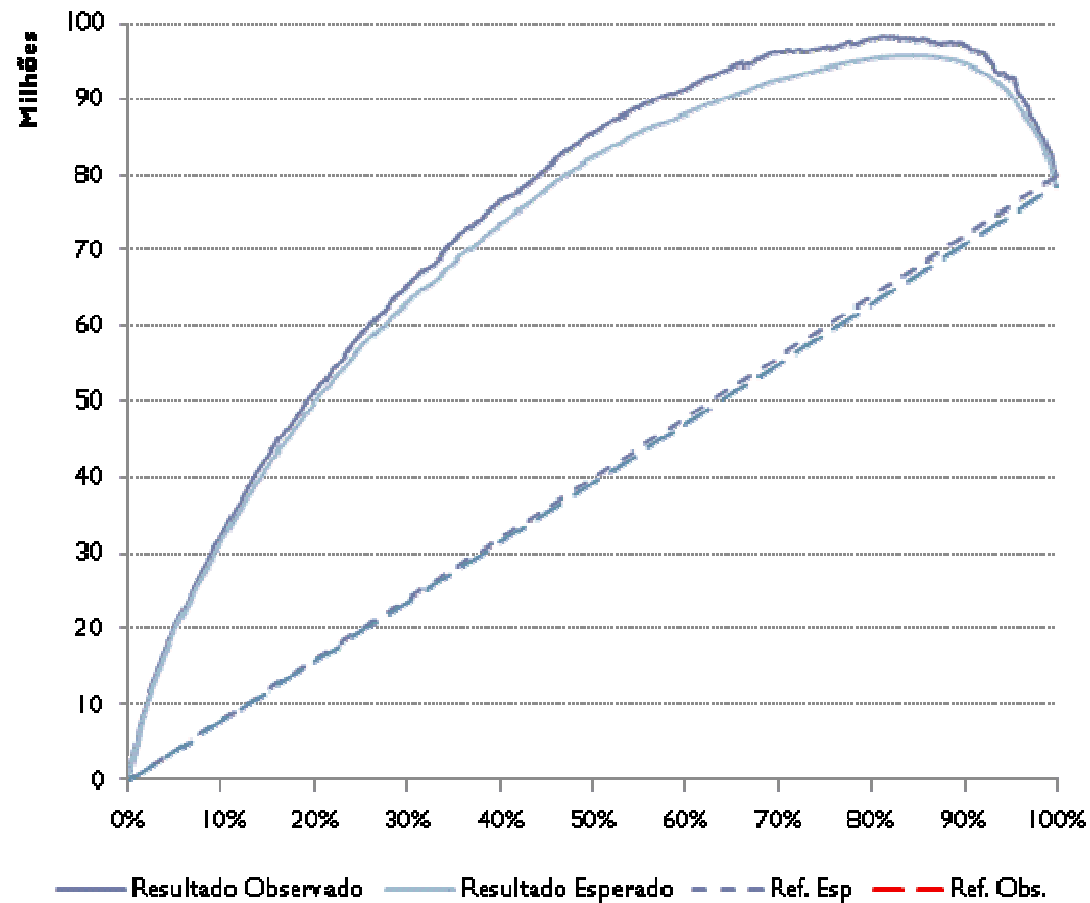
Parâmetro	GL	Estimativa	Erro Padrão	Estatística de Wald	Nível Descritivo
Intercepto	1	-22.694	0.0982	5.335.238	<.0001
pz12	1	-0.5943	0.1128	277.746	<.0001
pz18	1	-15.731	0.1207	1.697.546	<.0001
pz24	1	-1.972	0.1350	2.132.553	<.0001
t	1	0.3346	0.0261	1.640.278	<.0001
pz12*t	1	-0.1114	0.0277	161.801	<.0001
pz18*t	1	-0.0865	0.0269	103.279	0.0013
pz24*t	1	-0.1597	0.0268	356.038	<.0001
hist_pos	1	-0.6239	0.0201	9.629.424	<.0001
idade	1	-0.0495	0.00142	12.173.643	<.0001
endividamento	1	0.3071	0.072	182.021	<.0001
hist_neg	1	0.6478	0.0264	6.026.891	<.0001

Model fit



$$E(\hat{O}_i | \mathbf{x}_i) = \sum_{j=1}^3 \sum_{t=1}^{t_i} \hat{E}(O_i | Y_{ij} = 1; T_i = t; \mathbf{x}_i) \hat{\Pr}(T_i = t | \mathbf{x}_i; Y_{ij} = 1) \hat{\Pr}(Y_{ij} = 1 | \mathbf{x}_i)$$

Model quality



Spread definition

Dichotomy:

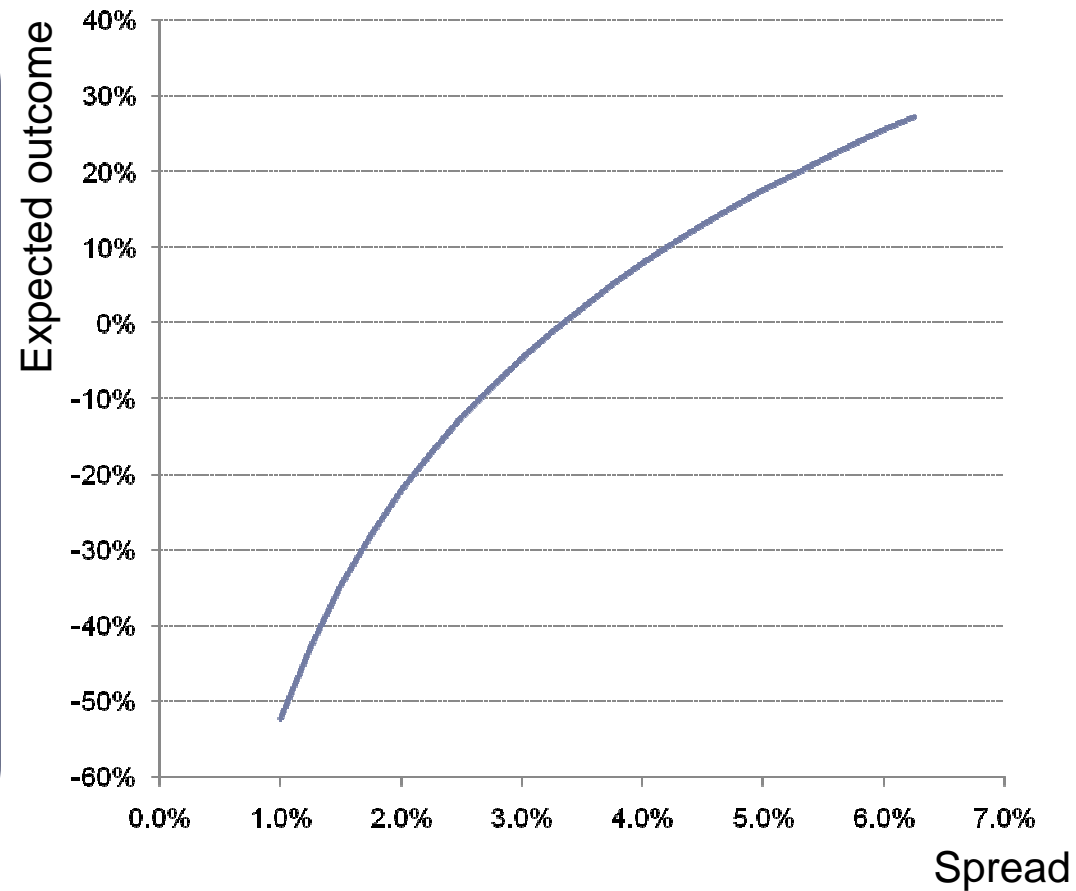
Guess:

A=1%

B=6%

$C=(1\%+6\%)/2$

Target: 10%



Spread definition

Dichotomy:

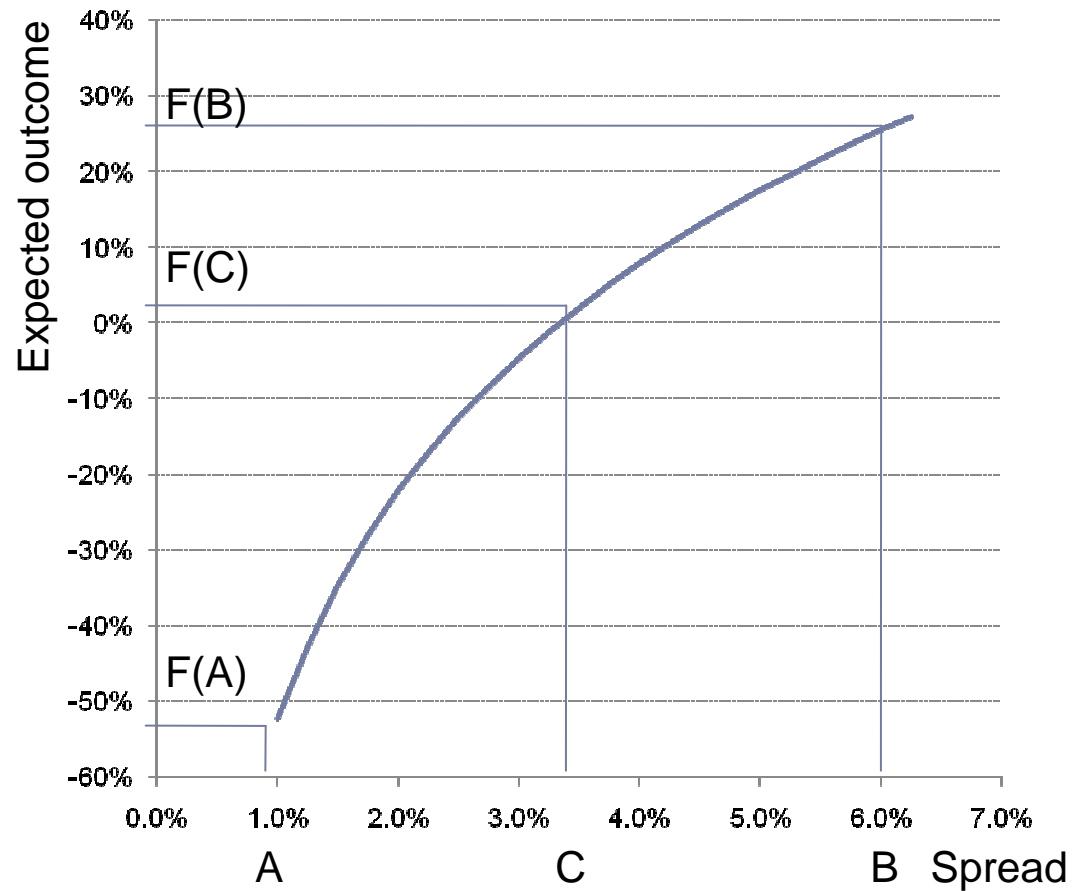
Guess:

A=1%

B=6%

$C=(1\%+6\%)/2$

Target: 10%



Spread definition

Dichotomy:

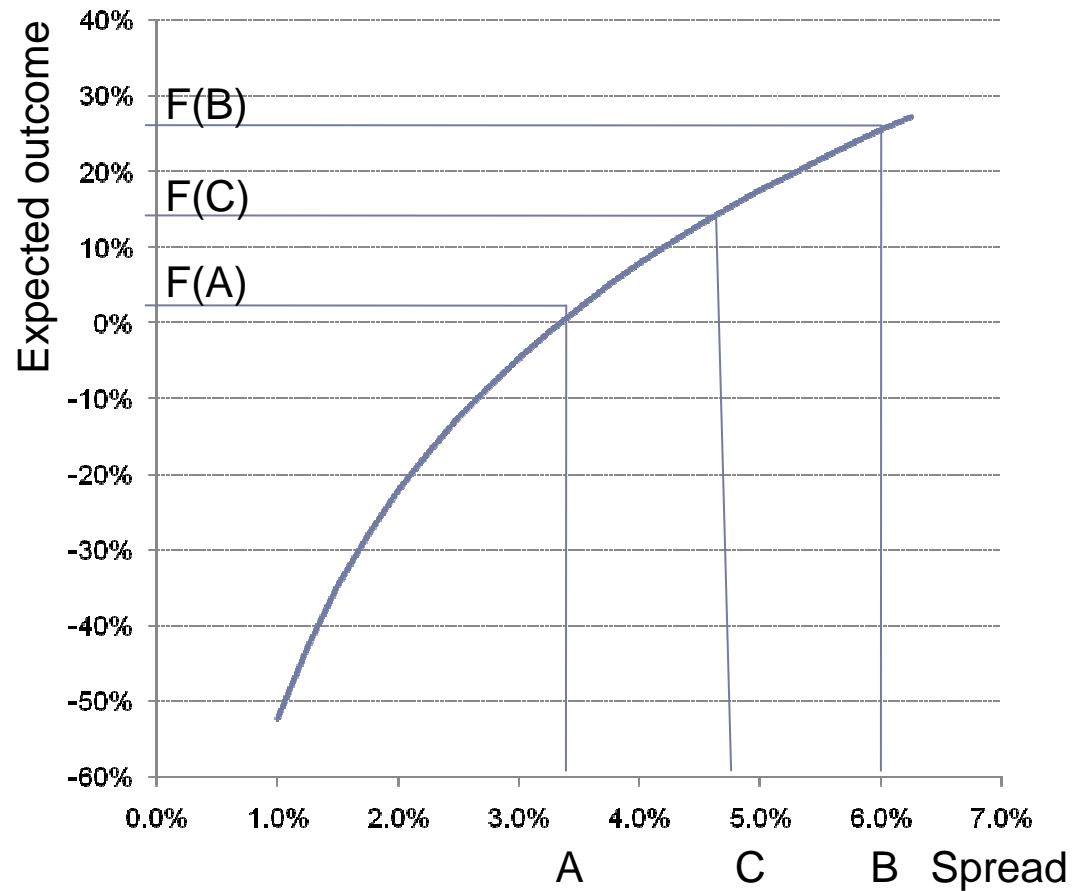
Guess:

A=1%

B=6%

$C=(1\%+6\%)/2$

Target: 10%



Spread definition

Dichotomy:

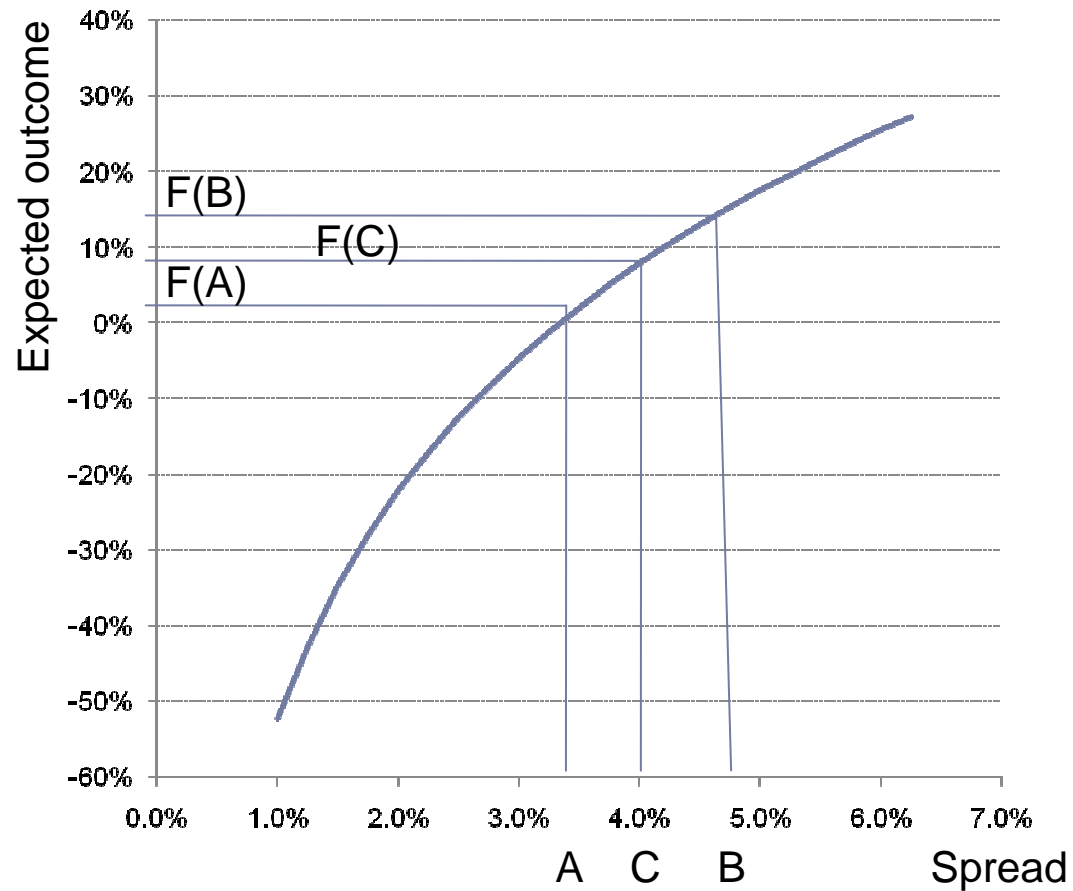
Guess:

A=1%

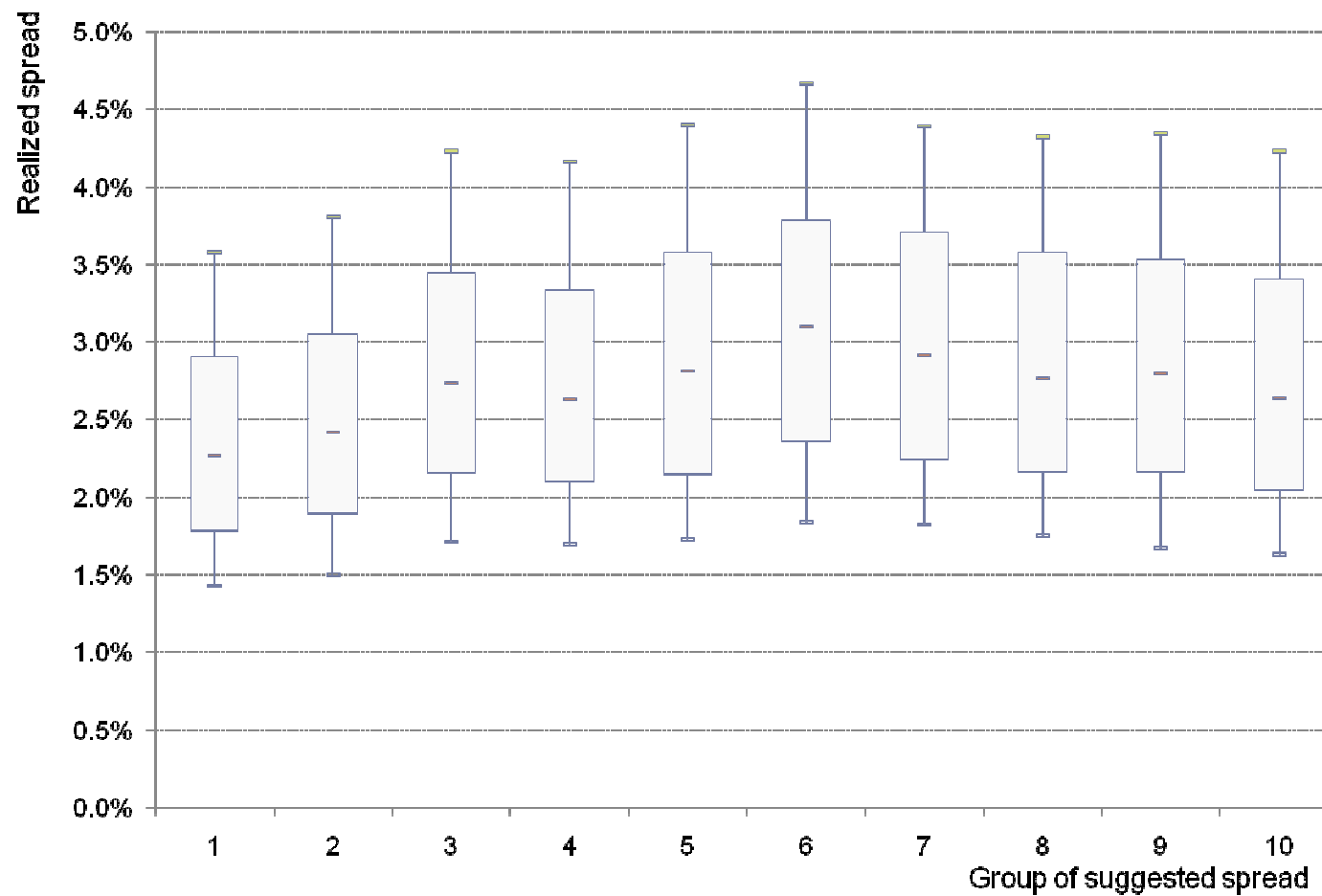
B=6%

$C=(1\%+6\%)/2$

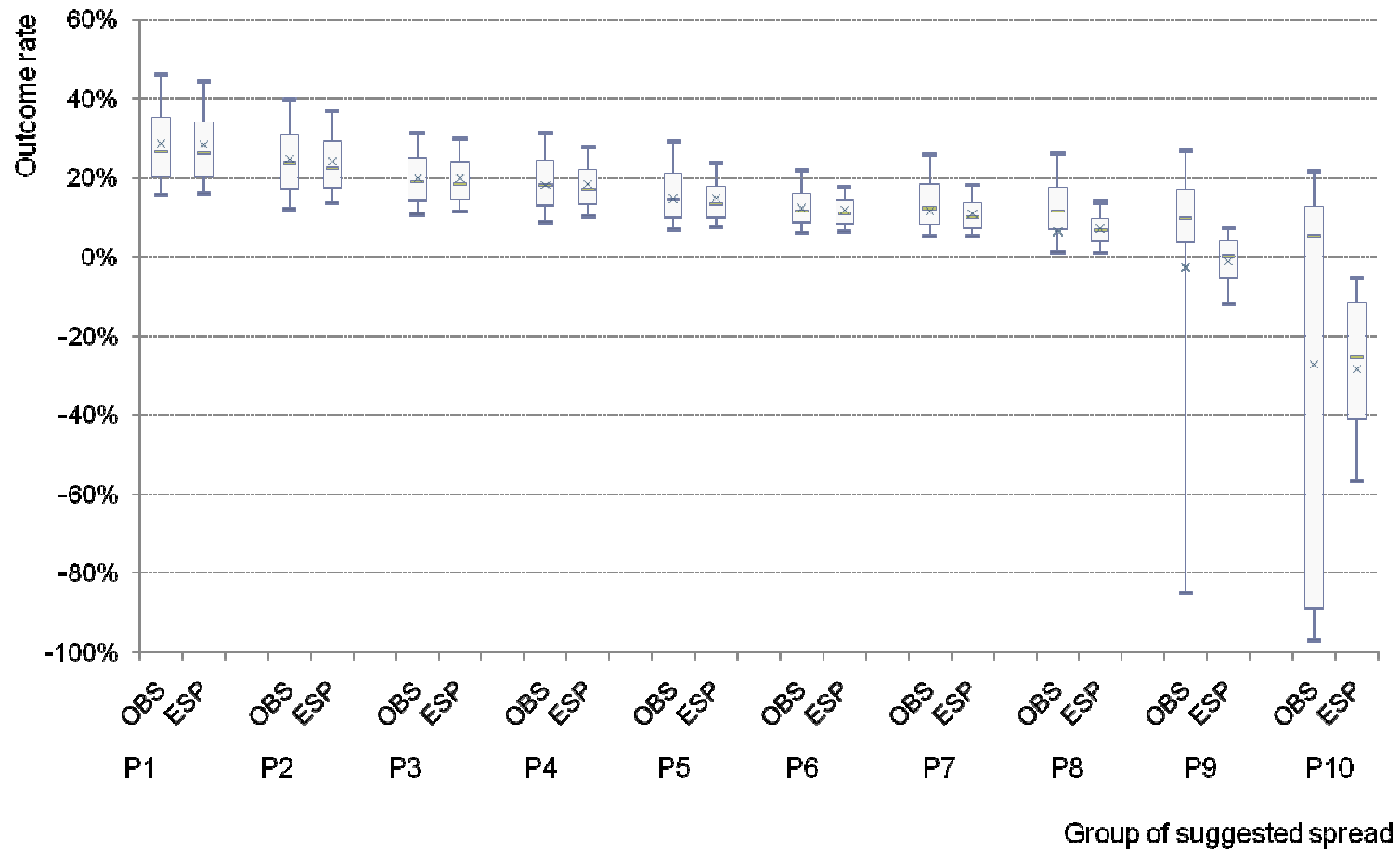
Target: 10%



Checking *old school* pricing



Checking profiles profitability





Final considerations

- How to use non finished loans?
- Alternative utility functions
- Use with bayesian inference
- How to perform rejected inference?
- How do we measure the effect of spread over credit risk?



Questions?

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