Predictive Sequential Debt Collection Management with z-similarities

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This is a project within the *Quantitative Financial Risk Management Centre*
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1. Debt collection management strategies are tailored to individual behaviour

e.g. Might have casework teams:

paid

missed

AWOL

2. Decision support tools use most recent information to determine actions

e.g. to predict

1. RISK of missed payment next month
2. conditional upon ACTION this month

3. Some problems constructing risk-management models for debt-collection management

1. SCOPE FOR REGRESSION
   - type of debt often better predictor than individual characteristics
2. POPULATION DRIFT
   - Rapidly changing environment, *e.g.* changing economic, competition, market and regulatory factors

4. Predictive sequential management involves tracking the individual sequentially and adjusting predictions

To predict we consider

1. INDIVIDUAL'S HISTORY
   - *e.g.* repayment history, letters sent, telephone calls received
2. OTHERS in portfolio

5. Approach through simple example: monthly repayment histories

i - debtor
$\boldsymbol{D}_i$ - repayment history
$\hat{u}_i$ - proportion paid to date

i	$\boldsymbol{D}_i$					$\hat{u}_i$
1	1	1	0	0	1	3/5
2	1	1	0			2/3
3	1					1

6. Approach through example: others with similar histories

Define 'similarity' between i and j as

$$z_{ij} = \frac{p(\boldsymbol{D}_i|\hat{\boldsymbol{u}}_j)}{\sum_{k=1}^{n} p(\boldsymbol{D}_i|\hat{\boldsymbol{u}}_k)} \qquad (1)$$

$\boldsymbol{D}_i$ data
$\boldsymbol{u}$ parameters
$p(\boldsymbol{D}|\boldsymbol{u})$ likelihood

7. Approach through example: similarity calculation with binomial model for likelihood

z_{ij} i	j 1	2	3
1	0.512	0.488	0.000
2	0.493	0.507	0.000
3	0.265	0.294	0.441

e.g.

$$z_{11} = \frac{\left(\frac{3}{5}\right)^3\left(\frac{2}{5}\right)^2}{\left(\frac{3}{5}\right)^3\left(\frac{2}{5}\right)^2 + \left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^2 + (1)^3(0)^2} = 0.512$$

8. Approach through example: predict payment next month C

$$p_e(C|\boldsymbol{D}_i) = \sum_{j=1}^{n} z_{ij} p(C|\boldsymbol{D}_i, \hat{\boldsymbol{u}}_j) \quad (2)$$

$$\begin{aligned} P_e(D_{16} = 1|\boldsymbol{D}) &= (0.512 \times 3/5) + (0.488 \times 2/3) + (1 \times 0) \\ &= 0.633 \\ P_e(D_{24} = 1|\boldsymbol{D}) &= 0.634 \\ P_e(D_{32} = 1|\boldsymbol{D}) &= 0.796 \end{aligned}$$

Population mean = $2/3$

9. Interpretation of sequentially applying this approach

1. Initially predictions based on portfolio average
2. As more data is gathered about the individual, predictions down weight those with different histories

10. Approach can be extended for actions: will the debtor pay next month?

Model: hazard for month s since last payment

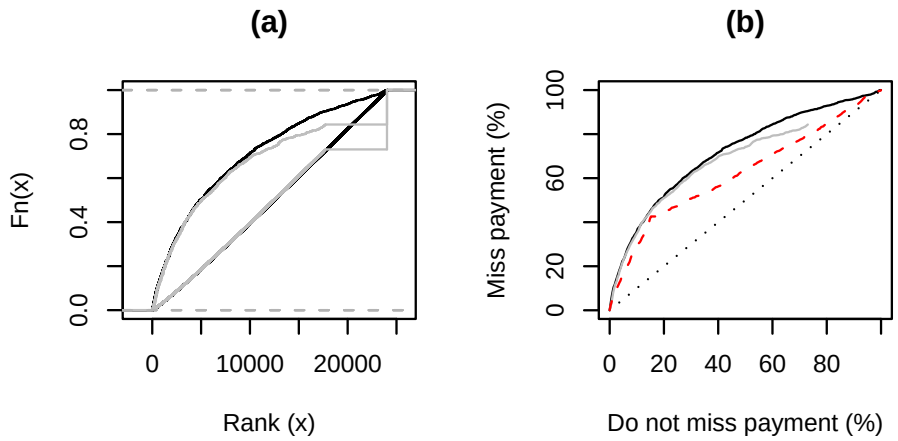
$$h_s = \rho^{1+(s-1)^u}, \quad (3)$$

$u > 0$ is a parameter, and

$$\rho = \frac{\exp(w_1 + w_2 a_s)}{1 + \exp(w_1 + w_2 a_s)} \quad (4)$$

action level a_s (number of phone calls); w_1 and w_2 unconstrained parameters.

11. Will the debtor pay next month?



black - z_{ij} predictions, grey - individual, red - population.

12. How late is late? Probability q pay in next week, given not paid in last 32 days

Model: logistic regression

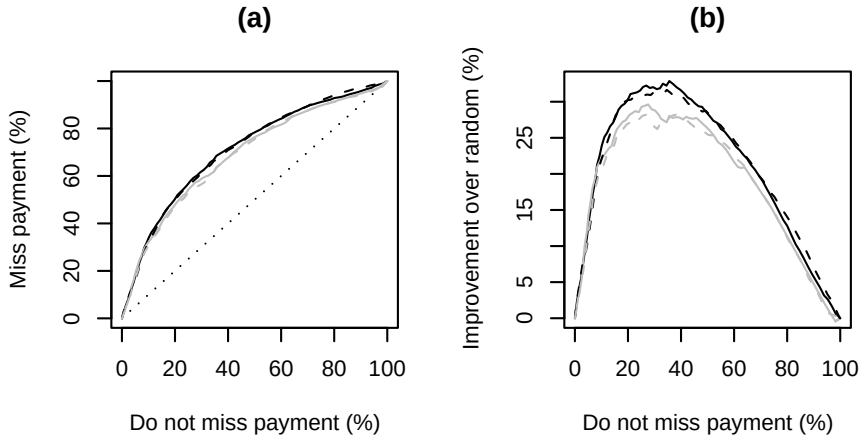
$$q = \exp(u_1 + u_2 a_{39})/\{1 + \exp(u_1 + u_2 a_{39})\} \quad (5)$$

Alternative, hazard model:

$$h_s = \exp(u_3 + u_4 a_s)/\{1 + \exp(u_3 + u_4 a_s)\} \quad (6)$$

for $32 < s \leq 39$, $\boldsymbol{u}$ unconstrained parameters, a_s action level to day s.

13. How late is late? Probability q_i pay in next week, given not paid in last 32 days



black - z_{ij} predictions, grey - individual; solid - logit, dashed - hazard

14. Predictive sequential risk management with likelihood-based similarities

Sequentially:

1. Use individual history
2. Take into account others
3. Predict

15. Thank you

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