Study on the Determination of the Optimum Credit Card Limit

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Abstract

It is essential for credit card companies to impose an optimum credit limit on card holders to boost revenue and manage delinquency. In our study, the optimum credit card limit model has been developed by understanding the impact of the credit card limit on usage, focusing on card purchase and cash advance facilities. The optimum limit model is designed to determine the limit value for which profit can be maximized by taking into account the fact that responses to the limit exhibited by good and bad customers are different.

For the optimum limit model to be applied to actual limit and performance data, the customer group should be homogenized to satisfy the assumptions of the model. The study has proven that this can be done by customer segmentation and resampling using the usage amount and creditworthiness. Over twenty one million newly opened credit card accounts in Korea Credit Bureau(KCB)'s database were used to calculate the profit as a function of credit limit for each segment. The optimum limit for card purchase, which has been calculated from the profit functions, demonstrates that the optimum limit increases in line with a customer's creditworthiness and usage amount, as expected.

It has been found that an optimum limit exists within the range of the currently assigned credit limit values for a mid-level risk grade customer who has a reasonable monthly usage amount (0 to 2500 USD). Therefore, it is possible to optimize credit limit policies of credit card companies by taking advantage of the methods developed in this study. As for the cash advance limit, it has been discovered that the optimum limit model can be applied to customers with a small amount of cash advance usage, while the customers' ability to repay and long-term profit/loss analysis should be accommodated for those with a higher usage amount.

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Ⅰ. Introduction

Granting an optimum credit limit to customers plays a core role in increasing sales and reducing delinquencies. Specifically, it can be considered the ultimate goal of the limit policy to increase revenues by granting suitable limits to meet the demands of good customers whilst discouraging use to lower delinquency levels by reducing the limits of less creditworthy (bad) customers. However, it is not a simple task to evaluate customers' creditworthiness and demands in order to decide on an appropriate credit limit to assign. It may be possible to find a limit that maximizes net profit by randomly assigning limits to customers using the Champion/Challenger system, but in reality the results of this type of study are not obtainable. Therefore, we have to utilize the limits imposed by actual policies and the subsequent usage performance to decide an optimum limit value. The problem is that the customer's credit rating and demands are reflected in the limit and this has an impact on the usage. Specifically, the limit policies currently in use are based on the credit rating and assign high limits to good customers, which gives rise to a situation where net profit increases as the limit increases. Therefore, the limit value that maximizes net profit cannot be found with the present data. In this study, we have proven that if we construct homogeneous customer segments by controlling the two factors which influence the customer performance (scale of use and credit rating), then only the limit, not any other customer properties, affects usage and delinquency and thereby the profit. However, even after segmentation, the limit has a residual dependency on the credit rating, which can be removed by resampling. By applying the optimum limit model to a homogenous customer base, the limit value that maximizes net profit can be found.

This paper is organized as follows: In Section II, the theoretical model that determines the optimum credit limit is introduced, and solutions to practical problems it presents are proposed. In Section III, KCB's credit card data is used to analyze the relationship between credit limit and customer performance, then through profitability analysis the optimum limits are determined for card purchase and cash advance. Finally, implications regarding credit limit policy drawn from this study are presented.

II. Optimum Credit Card Limit Policy Model Settings

1. Precedent studies

Similar to other financial products, credit cards have a credit risk and the limit serves as a source to produce revenue and reduce costs arising from customer usage and delinquency. In order to determine an optimum limit, the two opposing factors of usage level and creditworthiness must be considered. The credit rating is more important in cash advance than card purchase because insolvency occurs more with cash services. For this reason, creditworthiness in relation to determining credit limits was mainly focused on in past research with respect to cash services. Gross and Souleles (2001) found that an increase in the credit limit leads to an immediate increase in debt, and that not only customers with balances close to the credit limit, but also those with obligations far below the limit respond to the increase. According to this research, as in the case of savings, the credit card limit acts like a stock buffer in anticipating changes in income. Dey and Mumy (2005) found that the credit limit is positively correlated with variables related to the credit rating and the use of credit loans is negatively correlated with wealth and interest rate.

Most credit card companies focus more on creditworthiness than usage, assuming that the credit card limit can be used to control credit risk. Studying credit limit and delinquency data from a particular card company, Ko *et al.*(2009) concluded that increasing credit limits has a positive role in reducing delinquency rates. Their methodology of analyzing changes in delinquency rates caused by increasing credit limits to find the optimum limit for minimizing delinquency is basically correct. However, as discussed further on, the inherent relationship between risk and "the limit granted by the actual limit policy" was not fully considered. Also, in Gross and Souleles' (2000) research, when modeling the effect of a limit increase on the increase in debt, the bias in customers whose limits were increased was not removed. Since the credit rating and demands of customers are reflected in the limit increase, it is not valid to conclude that an increase in the customers' debt is due to an increase in credit limit. Application of such a model on a customer sample with different demands and credit rating will yield inconsistent results.

It seems that there has been no empirical research so far which reveals the precise dynamics of how the credit card limit impacts customers' usage and delinquency, due to limitations of the available data. The limits and usage data available from a particular card company does not give a wide range of limits granted to a homogenous customer group because the company assigns similar limit values to each particular customer group according to their limit policy. Credit card data from 11 card companies has been accumulated in KCB's database since 2006, which established a base to overcome these constraints. Information from over 21 million new card issuances was used to successfully remove the effects of customers' demands and creditworthiness on their usage and delinquency. Thereby we could make usage and delinquency solely depend on the limit and determine the optimum limit value that maximizes profit.

2. Definition of the Optimum Limit Problem

The problem of finding the optimum credit limit can be expressed as the following profit maximization problem:

$$\begin{aligned} P(x) &= \frac{u^G(x)\times N^G \times g - u^B(x)\times N^B \times l}{N} \\ &= g\,\frac{N^G}{N}\times\left(u^G(x)-\frac{w_0}{w*}\times u^B(x)\right) \end{aligned} \tag{2.1}$$

$$Max \quad P(x) \ \ w.r.t. \ \ x$$

where $P(x)$ is the average net profit function per customer at limit x, $u^G(x)$ and $u^B(x)$ are average usage functions with respect to the limit for good and bad customers respectively. N^G and N^B are defined as the number of good and bad customers in the population, g and l represent the revenue rate of good customers and loss rate of bad customers. $w_0 = l/g$ is the economic parameter that represents the ratio of losses from bad customers to revenue from good while $w^* = N^G / N^B$ is the population odds.

Since deciding the credit limit depends on the customer's properties such as usage scale and credit rating, it is not possible to assign a single optimum limit for the population containing a diverse range of customers. Therefore, in order to determine a single, efficient optimum credit limit, the customer groups must be configured to be as homogenous as possible. The profit function above shows how the profit of homogeneous customer group A_i changes in response to different credit limits. To decide the limit that maximizes net profit of customer group A_i, the profit function is differentiated with respect to the limit and the result set to 0 to give:

$$\frac{dP_i(x)}{dx} = g\frac{N_i^G}{N_i}\left(\frac{du_i^G(x)}{dx} - \frac{w_0}{w^*}\frac{du_i^B(x)}{dx}\right) = 0 \tag{2.2}$$

where du/dx represents the change in usage amount when the limit is increased by a unit amount, which can be called the marginal utilization rate. Because the usage function of bad customers is a linear function proportional to the limit as shown below, $du_i^B(x)/dx$ is a constant independent of the limit, which is the average usage rate of all bad customers (call it u_i^{B*}). Therefore, the limit value that maximizes net profit of customer group A_i satisfies the following equation:

$$\frac{du_i^G(x)}{dx} = \frac{w_0}{w^*} \times u_i^{B*} \tag{2.3}$$

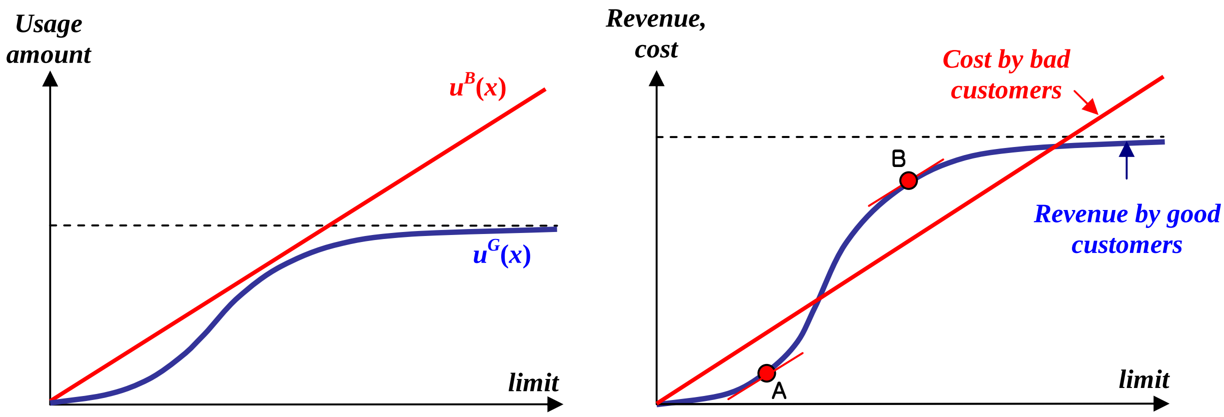
Although the solution to this equation requires precise knowledge of the usage function with regard to credit limit of customers belonging to group A_i, from the characteristics of the reaction of customers to limits, the approximate form of the function and the existence of an optimum limit can be inferred as follows.

3. Setup of the Optimum Limit Model

The optimum credit limit model is built on the foundation that the reaction to the limit is different between good and bad customers[1]. Good customers will not use a credit card if its limit lies below their

[1] Good customers are card users without delinquency and bad customers are those whose card usage leads to credit

usual usage amount, instead choosing another company that offers a sufficient limit. Granting the desired limit to a customer who is willing to use a specific card company leads to the customer signing up. However, regardless of how much the limit is increased, the customer usage amount (especially for card purchase) does not increase above the amount usually used. On the other hand, because bad customers will use up the entire limit available and end up delinquent regardless of limit, a linear relationship is formed between the usage amount and limit. Therefore, the curve of the usage amount with respect to the limit values of good and bad customers for a group of “homogenous” customers can be illustrated as in <Fig 1.A>. Although the profit curve for good customers and the cost curve for bad customers have the same form as with their respective usage curves, the cost curve appears below the profit curve, as in <Fig 1.B>, since there are substantially more good customers than bad.



A. Usage function B. Revenue and cost function

<Fig. 1> Response of good and bad customers to limit

The optimum limit given by Eq. 2.3, can also be described as the point where the good customers’ marginal revenue is equal to the bad customers’ marginal cost. Therefore, the optimum limit is at the point where the slope of the revenue curve is equal to the slope of the cost curve. There exist two such points: Point B, where the second derivative of the revenue curve is negative, gives the limit that maximizes net profit. Limits below B lead to a greater decrease in the good customers’ usage amount than the bad customers’ usage amount, lowering net profit. In the same way, when the limit is excessively high, revenue from the good customers does not become any greater, whereas the losses incurred by bad customers increase proportionately to the increase in limit, thereby reducing net profit.

Granting a single, optimum limit to a heterogeneous group of different customers is not efficient. In order to grant a single, efficient optimum limit, the group must be configured to be as homogenous as possible.

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loss. This distinction is theoretical and in reality, customers exist somewhere between these good and bad extremes. In this study, bad customers are defined as those who are 30 days in arrears within the first 12 months of new card issuance.

Here, homogeneity implies that the group has been arranged to include customers with the same revenue and cost curves, as in <Fig 1>. As revenue is determined by the customer group's average usage scale and cost determined by the proportion of bad customers within the group, constructing a homogenous group is the same as separating customers by usage scale and probability of being bad. At the point when a customer signs up for a credit card, the best predictors of usage amount and bad customer probability are the recent usage amount and the current credit rating. Assuming that customers are separated into groups with same usage amount and credit rating, A_1 and A_2 for example, each group will yield different optimum limits. In the case where group A_2 has a greater usage scale than A_1, the inflection point of the good customer usage curve of A_2 will move up and to the right making the optimum limit higher. On the other hand, if A_2 has a worse credit rating than A_1, the bad rate will increase so that the associated cost curve becomes steeper, making the optimum limit lower. Therefore, from the optimum limit model we can logically deduce that customers with large usage amount and good credit rating should be given high limits. Also, it is worthwhile to note that to the left of the optimum limit point B in <Fig. 1>, competition between card providers is important in preventing reduction of use while to the right, it becomes important to prevent higher delinquency rates causing an increase in costs.

4. The Relationship Between Credit Limit and Delinquency Rate in the Optimum Limit Model

There are two main methods to measure delinquency rates; the first involves measuring the proportion of delinquent accounts from those with a balance, and the second looks at the proportion of delinquency amounts from the total balance. These two are called "account-based delinquency rate" and "balance-based delinquency rate" respectively. Looking at the relationship between the limit and the delinquency rate in the previously described optimum limit model, the account-based delinquency rate must be a constant number without any relation to the limit. Though an excessive limit induces overspending and excessive debt so that occurrence of instances of good customers turning into bad customers may not be impossible, being good or bad is usually considered as an inherent property of the customer, which is decided by ability and willingness to repay. The optimum limit model is a function that describes how the usage amount changes when a variety of different limits are randomly given to the same customers, so the account-based delinquency rate does not change as a function of the limit. Assuming that a given population has a bad customer proportion of 5% and limits are randomly assigned, the proportion of bad customers for each limit will be 5% as well. Therefore, the account-based delinquency rate can be written as follows:

$$D_a(x) = \frac{N^B(x)}{N^B(x) + N^G(x)} = \frac{1}{1 + w*} \tag{2.4}$$

where $w*$ is the population odds which is independent of the limit.

On the other hand, if $b(x)$ is the average balance for the limit x, the associated balance-based delinquency rate can be written as follows:

$$D_b(x) = \frac{b^B(x)}{b^B(x) + b^G(x)} \approx \frac{u^B(x)}{u^B(x) + u^G(x)} \tag{2.5}$$

While the account-based delinquency rate is not a function of the limit, the balance-based delinquency rate increases towards either side of the function where the ratio of “bad customer usage amount” against “good customer usage amount” increases. And although it does not match precisely, the minimum value of the balance-based delinquency rate occurs near the optimum limit value. Specifically, the balance-based delinquency rate becomes a function convex towards the bottom with regards to credit limit. In addition, the average balance of the bad customers is larger than that of the good customers, and so the balance-based delinquency rate is greater than the account-based delinquency rate.

If we take look at the actual data, however, we find that both the account-based delinquency rate and balance-based delinquency rate decrease as the limit increases. This all stems from the differences between the theoretical situation that the optimum limit model describes and the real situation where the model is practically applied. Real-life application of the optimum limit model should give a single value for all customers in each group. As previously explained, a customer group with a good credit rating will be given a high limit, causing the delinquency rate to fall as the limit increases. Therefore, the delinquency rate described by the optimum limit model does not exist in reality. The relationship between the limit and the delinquency rate will match the prediction of the optimum limit model only when the limits are randomly assigned to the customers.

Nevertheless, the reason this study is possible is because KCB has access to the limit and usage data from 11 card companies whose limit policies are all different. However, because the limit policies are basically based on credit ratings, regardless of how well the customers are split up, it is not possible to completely remove the dependence of the limit on the credit rating. The residual dependency after constructing homogenous customer groups can be overcome by readjusting the sample odds for each limit.

III. The Relationship Between Credit Limit and Card Purchase Usage

1. Analysis Data

The data used for this study was collected from the 11 card companies which represent more than 90% of the market share in Korea. KCB has collected credit data (opening of card and loan accounts and their performance statistics including usage, repayment, arrears, etc) from January 2006. Twenty one million newly opened credit card accounts between June 2006 and February 2009 were prepared for analysis. The performance for 12 months after signing up was aggregated by account while the demographic and credit information for 3 months before signing up was aggregated by customer. In the following discussion, some of the frequently used variables are defined as follows:

- U_L : pre-signup monthly total purchase amount of a **customer** (average over 3 months before signup)
- U_N : post-signup total purchase amount of a **customer** (average over 3 months after signup)
- u_N : post-signup purchase amount of a new **account** (average over 3 months after signup)
- S_L : pre-signup credit rating of a **customer**. The credit ratings lie on a scale of 1 to 10, with smaller numbers representing more creditworthy customers.

The upper cases represent the characteristics of a customer while lower case those of an account.

2. Determining the Optimum Purchase Limit

1) The Relationship Between Purchase Limit and Usage Amount

When analyzing the relationship between card limit and usage amount, the biggest problem comes from the fact that the limit itself is determined by (past) usage. Regardless of how the customer group is configured, the post-signup usage amounts are positively correlated with the limit. However, this is not necessarily evidence that granting a higher limit will increase the usage amount. The reason for this is because the customers who are granted high limits by actual limit policy are those who already have high pre-signup usage amounts. From the given population, the correlation coefficient between "initial purchase limit" (x) and "post-signup account usage amount" (u_N) is 0.11 whereas "pre-signup customer usage amount" (U_L) has correlation coefficient values of 0.13 and 0.49 with "initial purchase limit" (x) and "post-signup account usage amount" (u_N) respectively. Therefore, we can find that the effects of the pre-signup usage amount on the limit and post-signup usage amount are greater than the effect of the limit on the post-signup usage amount. The post-signup usage amount is also affected by the customer's attributes, giving the following expression:

$$u_N = f(x \mid \vec{X}) \tag{3.1}$$

Here, $\vec{X}$ represents the customer's attributes that affect the post-signup usage amount such as U_L and S_L , etc. In order to precisely determine how the limit affects usage, the properties of customers must be controlled perfectly.

The post-signup **account** usage amount(u_N) can be decomposed into the post-signup **customer** usage amount(U_N) and the ratio of the account usage amount to the customer usage amount, which is called the share of wallet(SOW). The limit mainly affects the SOW whereas the post-signup customer usage amount is the customer's intrinsic property and not affected by limit. So Eq. 3.1 can be rewritten as:

$$u_N = U_N(\vec{X}) \times SOW(x \mid \vec{X}) \tag{3.2}$$

In the case of card purchases, the post-signup customer usage amount(U_N) is mostly dependent on the pre-signup customer usage amount(U_L). Looking for the characteristic items that affect the customer usage amount(U_N) and SOW through univariate analysis, items such as the credit rating, number of cards held and annual income are identified as significant in addition to the pre-signup customer usage amount. These items affect the account usage amounts in the manner shown in <Table 1>.

<Table 1> Relation between pre-signup customer properties & post-signup limit/usage[2]

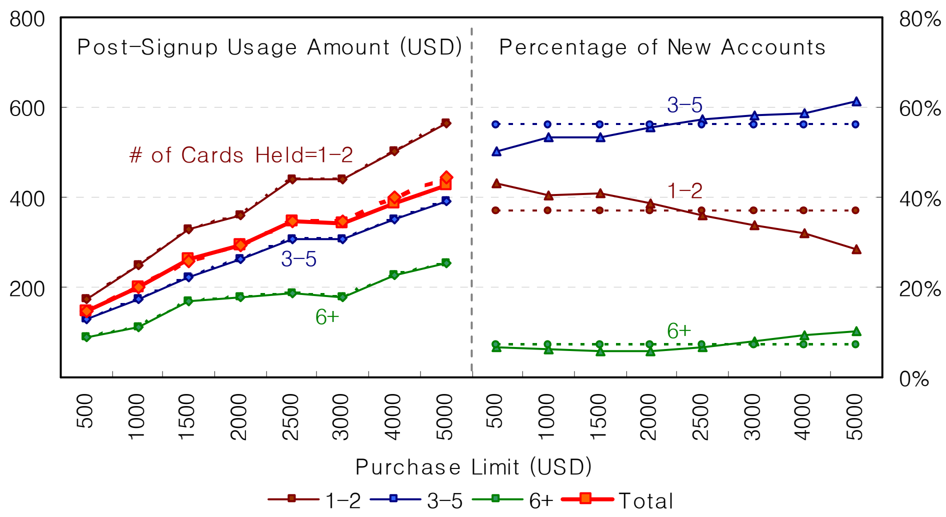
Variable	With no variables fixed					With U_L, S_L Fixed (for S_L, only fix U_L)				
	U_N	Usage Rate	SOW	u_N	Limit	U_N	Usage Rate	SOW	u_N	limit
U_L	+++	0	-	+	++					
S_L	0+	-	-	--	+++	0-	-	-	--	++
# of Cards Held	+	-	--	0-	++	0+	-	--	--	+
Annual Income	++	-	-	+	++	0+	-	0-	0-	+

As shown in the table, when pre-signup usage amount(U_L) is fixed, all the customer properties which show positive correlation with the limit value are negatively correlated with the post-signup account usage amount(u_N). The reason is as follows: If we fix the post-signup customer usage amount(U_N) by fixing the pre-signup customer usage amount(U_L) in Eq. 3.2, other variables affect the post-signup account usage amount(u_N) through SOW. The limit policies are mostly based on the credit rating rather than usage scale, and variables which affect the limit are indirectly reflected in the limit through correlation with the credit rating. Customers with good credit ratings generally have a large number of cards and are less likely to use the newly opened account. Therefore these customers have low share of wallet, and thereby low usage amount on the new account. As a result, if a variable is positively correlated with the limit, it is also positively correlated with the credit rating and negatively correlated with SOW and u_N.

The number of cards held by the customer before signup is the most typical example of such variables.

[2] More + signs implies a stronger positive correlation and more - signs implies a stronger negative correlation. 0+(or 0-) means very weak positive(or negative) correlation. Usage rate is defined as the proportion of the number of accounts whose usage amount is greater than zero to the number of new accounts.

When pre-signup usage amount and credit rating are fixed, post-signup account usage amount(u_N) increases as the limit increases, as seen in <Fig 2>. The customers who have a larger number of cards spend a smaller amount on the new accounts, as seen on the left, and the proportion of such customers increases as the limit increases, as seen on the right. The thick, sold line represents the total customers not separated by the number of cards. If we resample the number of customers to remove the correlation between the number of cards and limit, as designated by the dotted lines on the right, the thick, solid line on the left becomes the thick, dotted line. The slope of the solid line is less steep than that of the dotted line since a larger proportion of customers with a large number of cards means that the average usage amount at higher limit values is reduced. The difference between the two thick lines, however, is negligible. Therefore we can conclude that the positive correlation between the limit and the usage amount is not because the customers with a large number of cards are assigned higher limits. In the same manner, we can show that if the pre-signup usage amount and credit rating are fixed, the increase in the usage amount being in line with the increase in the limit is not due to the way the limit was assigned.

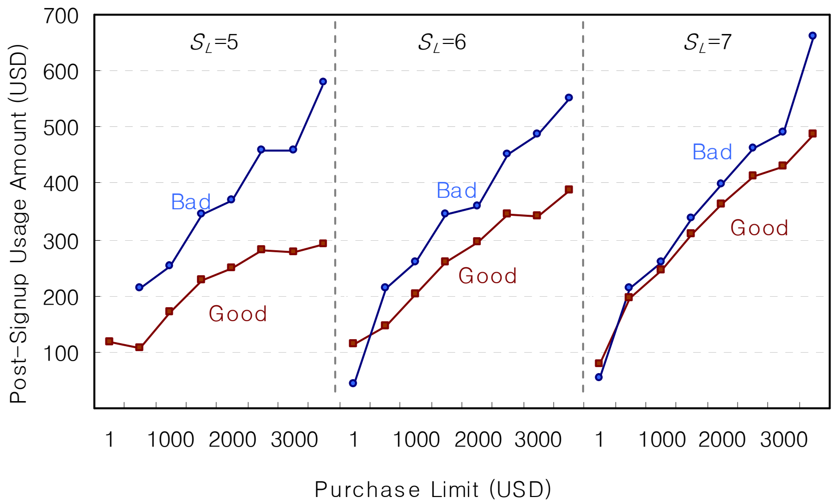


<Fig 2> Relationship between limit and usage amount for groups with a different number of cards held (post-signup usage amount=1,000-1,500 USD and credit rating=6)

If we make customer groups homogenous in terms of usage and delinquency by categorizing according to pre-signup customer usage amount(U_L) and credit rating(S_L), the effects of the remaining variables act in such a way that reduces the relationship between the limit and usage, the effect being almost insignificant. Therefore we can assert that the reason the post-signup usage amount appears high for customers with high limits is because they are granted high limits. Hence it can be concluded that granting higher limits induces higher post-signup account usage, and that if U_L and S_L are controlled, the usage amount is a function of only the limit. So Eq. 3.1 can be rewritten as:

$$u_N(x) = f(x \mid U_L, S_L) \tag{3.3}$$

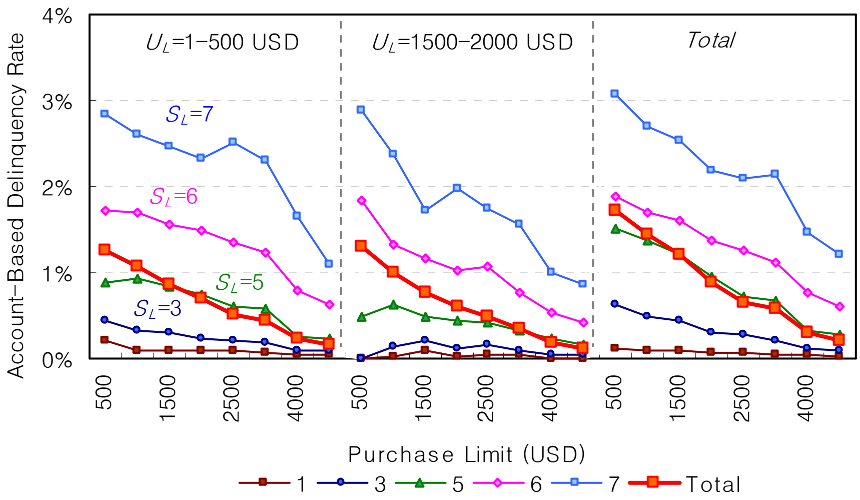
If we take look at the relationship between the post-signup account usage amount and the limit for the customer groups configured by pre-signup customer usage amounts and credit ratings, the usage patterns of good and bad customers assumed in the optimum limit model can be verified. As shown in <Fig 3>, for the customer group with U_L =1,000-1,500 USD and S_L = 5, the rate of increase of the post-signup usage amount for good customers decreases for limit values over around 2,500 USD while the usage amount of bad customers continues its growth even when given high limits. For the groups with worse credit ratings, the usage curve for good customers becomes similar to that of bad customers. Even though the average usage amount of good customers is larger for the group with S_L = 7 than the group with S_L = 5, the ratio of bad customers is also larger, causing a rise in the delinquency rate and a subsequent fall in profit.



<Fig 3> Relationship between limit and usage amount
for customer groups with U_L =1,000-1,500 USD and S_L = 5-7

2) The Relationship Between Purchase Limit and Delinquency Rate

The account-based delinquency rate and balance-based delinquency rate measured for the population are found to be decreasing functions of the limit. This is because customers with high credit ratings are given high limits. It is expected that the effect of the credit rating reflected in the limit can be removed by controlling the credit rating. However, even with the same credit rating, the account-based delinquency rate appears to decline with respect to the limit as seen in <Fig 4>.



<Fig 4> Relationship between limit and delinquency rate

In principle, because credit scores are sufficient statistics, the probabilities of good and bad customers should not change for a given score, even if customer properties are different. For example, the proportion of bad customers should be the same between male and female customer groups with the same scores of 600 points. Also, the probability of a bad customer with score s must always be the same, regardless of the customer's other properties($\vec{X}_1$), which can be expressed as follows:

$$p\left(bad \mid S = s, \vec{X}_1\right) = p\left(bad \mid S = s\right) \tag{3.4}$$

Since the credit rating represents the score category, the above equation must also apply to the credit rating. If we fix the credit rating and pre-signup usage amount and check whether Eq. 3.4 holds in relation to the remaining variables, we find that the statistical sufficiency of the credit rating no longer holds, although not to the extent that the rank ordering between ratings is destroyed. Actually, because the ratings are defined as intervals of the credit score, it is possible to separate the same rating group into more and less superior customers. The score is precisely an example of this.

Therefore, using S_L and U_L alone, we cannot construct customer groups homogenous enough for the delinquency rate not to be affected by the limit. Although customers can be separated to be more homogenous by using the credit score instead of the credit rating, this approach has no meaning because of the following reasons: Firstly, when using the score to make the customer groups sufficiently homogenous in terms of the creditworthiness, the sample size within each group becomes too small. Particularly the number of bad customers which goes into the delinquency rate calculation becomes too small to produce statistically reliable results. Secondly, no matter how precise the score is, it is impossible to completely explain the credit policy of the card company. When card companies grant limits, several types of extra internal information such as the in-house application score and the applicant information are used in addition to credit bureau information. For this reason, it is not possible to remove the dependence of the limit on the creditworthiness using only the credit bureau information.

The reason we try to configure a homogenous group by creditworthiness, such that the account-based delinquency rate is not dependent on the limit, is because the subset of customers at each limit value must have the same delinquency rate in order that the optimum limit model can be successfully applied. If the customers given high limits are originally "superior" customers than those given low limits, both the account-based delinquency rate and the balance-based delinquency rate decrease as the limit increases, which makes the profit function always increasing in the limit. In this case, the optimum limit model's key assumptions are not satisfied.

Within a customer group categorized according to the credit rating(S_L), customer usage amount(U_L) and other variables($\vec{X}_2$), the account-based delinquency rate (D_a) is not only the customer group's property as in Eq. 2.4, but it is also dependent on the limit values(x). Therefore in reality, Eq. 2.4 must be rewritten as follows:

$$D_a(x) = f\left(S_L, U_L\right) + \varepsilon\left(x(\vec{X}_2)\right) \tag{3.5}$$

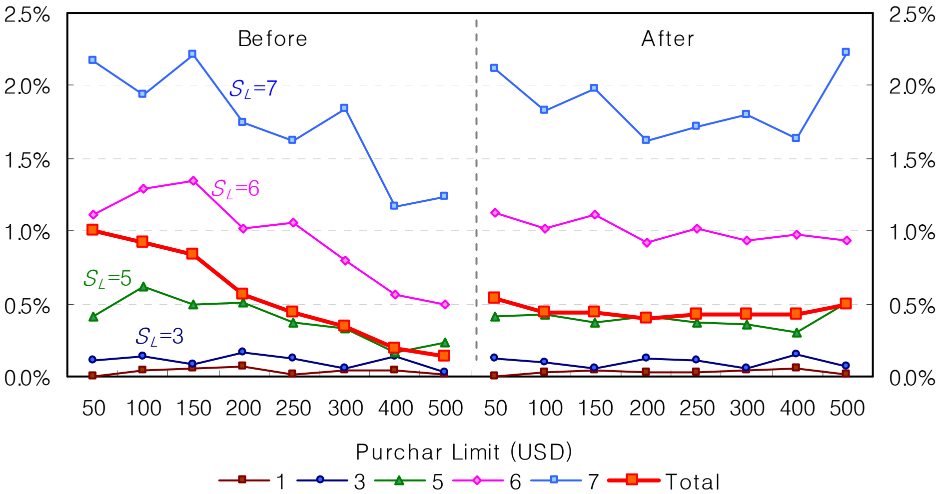
The dependence of the delinquency rate on the limit, corresponding to the residuals, cannot be removed with the credit bureau information. But this residual dependence can be removed by readjusting the good to bad customer ratio, called "odds", so that it is the same for each limit value.

Let us assume a homogenous customer group A, configured according to Equation 3.5. When the customers in this group sign up for new cards at company "B", the card company assigns limits to the customers according to their creditworthiness. For example, if A is a customer group of 9000 good and 1000 bad customers, company B will assign a high limit, of 5,000 dollars, to the most superior sample A_1, of 980 good and 20 bad customers, and a slightly lower limit, of 4,500 dollars, to the second-ranked sample A_2, of 1420 good and 80 bad customers, and so on. The result of assigning limits according to creditworthiness in this way is that the account-based delinquency rates decrease as the limit increases.

Now let us apply the sampling ratio used by company B backward so that the good to bad ratio of subgroups A_i (= SO_i =sample odds) becomes the same as the good to bad ratio of the population (= PO =population odds). This can be done by multiplying the number of bad customers of subgroup A_i by SO_i / PO, i.e.,

$$A_i = \left(N_i^G, N_i^B\right) \rightarrow A_i' = \left(N_i^G, N_i^B \times \frac{SO_i}{PO}\right) \tag{3.6}$$

Comparing the reconfigured subgroup A_i' to the original subgroup A_i, the average usage amount of good and bad customers has not changed. Instead, because the good to bad ratio has changed, the delinquency rate is different. <Fig. 5> shows the account-based delinquency rate before and after adjusting the good to bad ratio by resampling. As expected, the differences in delinquency rate caused by the limit have been removed.

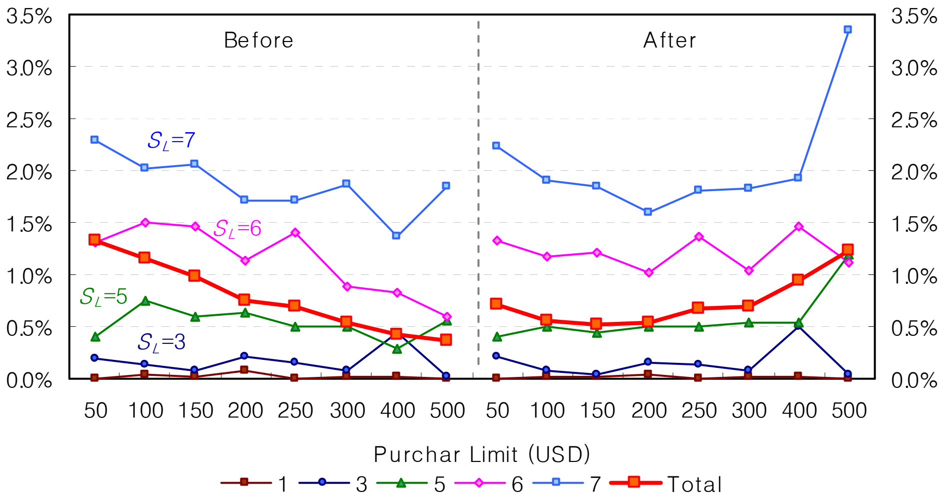


<Fig 5> Account-based delinquency rate, before and after adjusting good to bad ratio for customer groups with U_L =1,500-2,000 USD

In the optimum limit model, the account-based delinquency rate should be independent of the limit since it is a characteristic of the population. On the other hand, the balance-based delinquency rate takes a U-shaped form, and so the net profit per person can take a maximum value given the appropriate limit value. From <Fig 6>, it is seen that after resampling, the balance-based delinquency rate has changed its form from monotonically decreasing to increasing above a certain limit value.

3) Determination of the Optimum Limit Through Profitability Analysis

As discussed so far, after categorizing customers according to pre-signup usage amount and credit rating, followed by resampling to adjust good to bad ratios at each limit to match the population ratio, the effects of the customer properties on post-signup account usage and delinquency are removed. Therefore, having satisfied the optimum limit model's assumptions, it is possible to apply the model to the adjusted actual data to determine the optimum limit value that maximizes net profit.



<Fig 6> Balance-based delinquency rate, before and after adjusting good to bad ratio for customer groups with U_L =1,500-2,000 USD

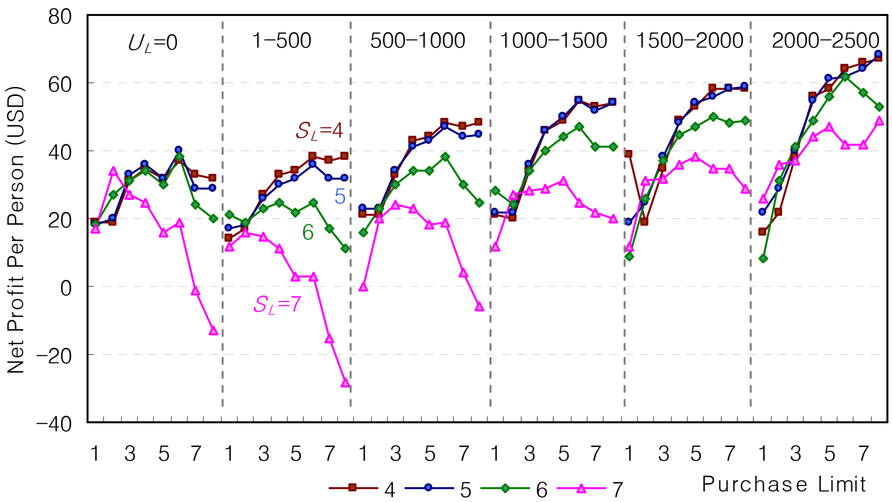
Analysis of the profitability of newly opened cards is dependent on the performance period. The usage ratio, monthly usage amount and delinquency rate change with respect to months on the book. Furthermore, distribution over time of the costs associated with acquisition, issuance and delivery makes exact calculation of the profit even more difficult. The purpose of this study is to determine the optimum limit so it is more important to understand how the profit changes with respect to the limit than the explicit profit value. Therefore, only the revenue and costs directly related to usage are considered, and indirect revenue and costs such as the annual fee, acquisition costs and fixed costs are all excluded.

Assuming that card purchases are all lump sums, the revenue is calculated by multiplying the merchant fee rate with the usage amount. Among various types of direct costs, only financing and charge-off costs will be considered in the profit calculation. Due to the mismatch between the periods in which revenue and costs occur, when analyzing profit of credit products over a period of time, future loss must be converted to the current period's cost. To this end, methods such as allowances for bad debt and expected loss can be used. In our study, we estimate losses using the roll-rate. According to the definition of the bad customer, all bad customers will have been delinquent by 2-periods (more than 30 days – less than 60 days) at some time. Therefore, multiplying the sum of all 2-period delinquency amounts occurring within the performance period by the ratio that changes the 2-period delinquency amount to the 7-period (depreciation) amount gives the loss value. After resampling of customer group A, net profit per person for the limit x can be written as follows:

$$P(x)=\frac{\left(u_{N12M}^{G}(x)\times N^{G}(x)+u_{N12M}^{B}(x)\times N^{B}(x)\times\frac{SO(x)}{PO}\right)\times(r_1-r_2)-A_D(x)\times R(2\to7)\times N^{B}(x)\times\frac{SO(x)}{PO}}{N^{B}(x)+N^{B}(x)\times\frac{SO(x)}{PO}} \tag{3.7}$$

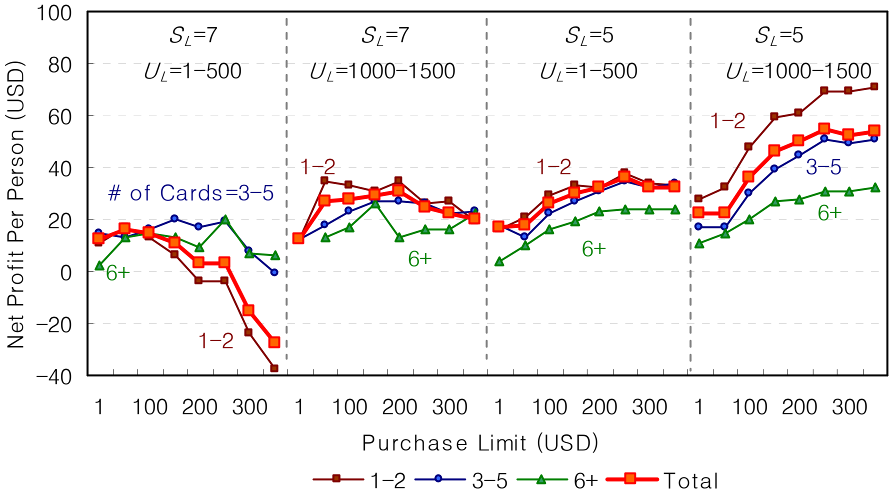
where u_{N12M}^{G} and u_{N12M}^{B} are usage amounts of good and bad customers for the first 12 month period after signup. The bad customers' usage is included because revenue is realized through merchant fees at the point of card purchases. r_1 represents the average merchant fee rate of 2.45%, and r_2 represents the financing interest rate of 0.46% for the credit granting period of 28 days. A_D is the average 2-period delinquent amount (including balances not due) per person. $R(2\to7)$ is the roll-rate at which the 2-period delinquent amount becomes a charge-off. It ranges between 50% and 60% in the case of card purchases; the most recently observed value of 58% from a particular card company was used.

<Fig 7> shows the function of net profit per person with respect to the limit for several customer groups segmented by pre-signup usage amount and credit rating. The optimum limit that maximizes net profit is higher for the customer group with a larger value for pre-signup usage amount and a better credit rating, in good agreement with the result derived theoretically from the optimum limit model. For the S_L =7 groups, where the bad customer ratio is high, the change in net profit with respect to the limit is greatest, and the optimum limit increases by a large amount as the pre-signup usage amount increases. The bad customer ratio is low in the S_L =4-5 groups so the optimum limit is found at a much higher value. The optimum limits for customer groups with pre-signup usage amounts of more than 2,000 USD seem to exist in an area outside of the analyzed range (0-7,000 USD). The net profit is sometimes higher when pre-signup usage amount is 0 than when it is 1-500 USD, which is because there exists consumption that is not identified by the customer group's card usage. S_L =1-3 groups not shown in <Fig 7> basically overlap with S_L =4 groups, and for S_L =8-10 groups losses are prominent for lower limits.



<Fig 7> Function of net profit per person with respect to the limit for a sample of customer groups

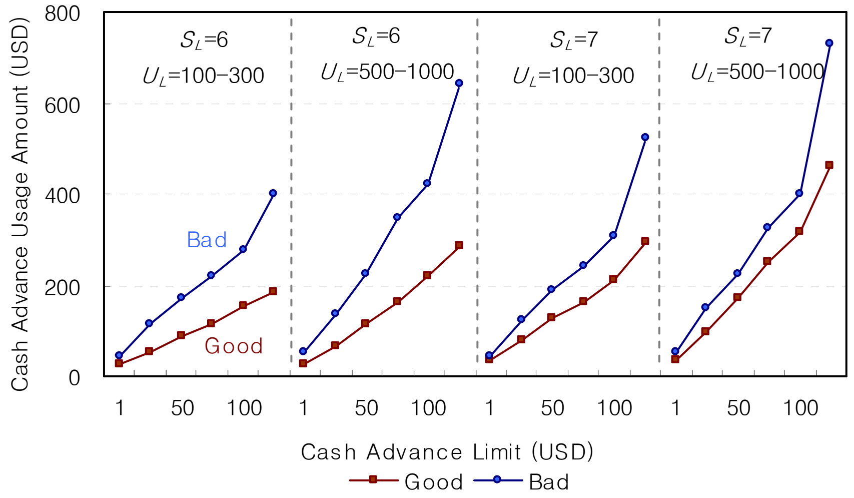
Separating customers further by utilizing number of cards held in addition to pre-signup usage amount and credit rating increases the homogeneity of each customer group, improving the efficiency of the optimum limit values observed. For example, although the optimum limit for the customer group with U_L =1-500 USD and S_L =7 is given at around 500-1,000 USD, if this group is segmented further by number of cards held, the optimum limit is given at around 2,000-2,500 USD for customers holding more than 3 cards, as shown in <Fig 8>. On the other hand, for the group with U_L =1,000-1,500 USD and S_L =5, the optimum limit decreases as the number of cards held becomes greater. For customer groups where usage amount is small, the previously latent consumption is more easily exposed as the number of cards held becomes larger. Meanwhile, for customer groups with large usage amounts, the SOW decreases as number of cards held increases.



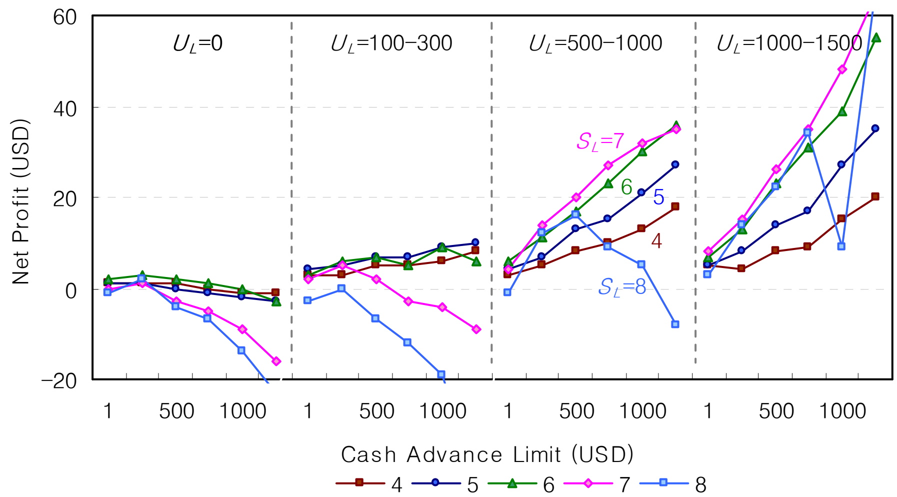
<Fig 8> Function of net profit per customer after segmenting further by number of cards held

3. Determination of the Optimum Cash Advance Limit

The analysis to determine the optimum cash advance limit is basically the same as with card purchases. After controlling the pre-signup cash advance usage amount(U_L) and credit rating(S_L), the remaining variables have almost no effect on the usage curve and even if they did, it would act in the manner of reducing the gradient. The biggest difference between card purchases and cash advance is that the good customer usage curve is asymptotic for card purchases, whereas the cash advance usage curve is not. As seen in <Fig 9>, not only the bad but also the good customers' post-signup cash advance usage amount keeps increasing as the limit increases. The slope becomes steeper for customer groups with worse credit ratings and larger pre-signup cash advance usage amounts. This is because high cash advance limits induce demands, unlike high purchase limits. For this reason, when finding the net profit curve after constructing homogeneous groups and resampling, the net profit is seen to be monotonically increasing for most customer groups except those which have low demands and bad credit ratings, as shown in <Fig 10>.



<Fig 9> Cash advance usage curve for good and bad customers



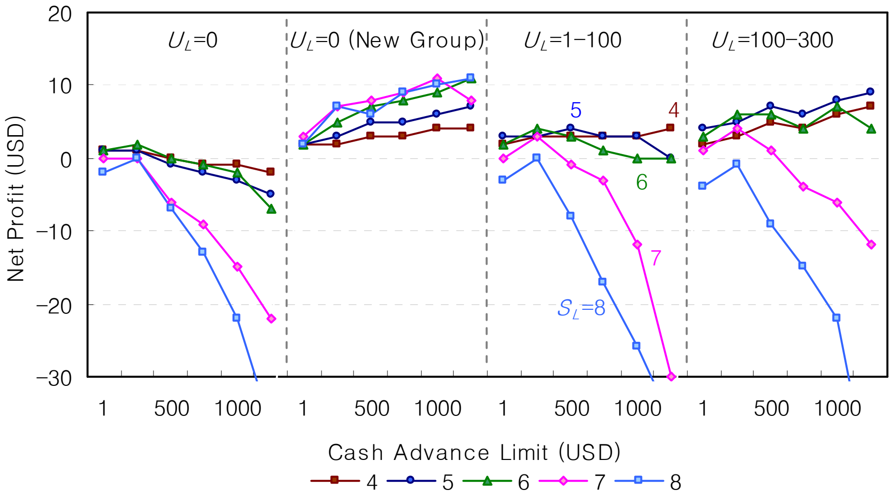
<Fig 10> Cash advance net profit function for a sample of customer groups

Another feature of cash advance which is different from card purchases is that the net profit is higher from customers with low credit ratings and high demands. This comes about because the cash advance fees between a minimum of 16.5% and up to a maximum of 24.5% are assigned according to credit ratings. The roll-rate with respect to cash advance balance from period 2 to 7 was applied at 68%, as was recently measured at a particular card company. Excluding the customer groups with low credit ratings and low demands, optimum cash advance limits take large values outside the range of analysis(0-2,000 USD). This means that the best limit policy is to grant a very high cash advance limit to the mid-range rated customers who frequently use the cash advance facility, which is a difficult idea for the card companies to accept.

The discrepancy between the analysis results and common sense arises from the assumption of the optimum limit model that the goodness of a customer is an intrinsic property which is not affected by the limit. This assumption, however, does not hold for the customers who utilize cash advance excessively. It has already been experienced in the credit card crisis of 2003[3] that excessive debt beyond the ability to repay forces customers into delinquency. In addition, it could also be said that this discrepancy was caused partly by the short performance period. Conducting a profitability analysis on the cash advance shows that negative net profit is observed in the long term (eg. 2 years) while positive net profit is observed in the short term. Therefore, in order to determine a precise and reasonable value for the optimum cash advance limit, a long-term profitability analysis is necessary and an optimum cash advance limit model that reflects the ability to repay needs to be established. Despite these issues, it is possible to find an optimum limit which maximizes the net profit of customer groups with low demands and low credit ratings, which proves that the optimum limit model can be successfully applied to the cash advance scenario within these groups.

As in the case of card purchases, more precise segmentation gives a more efficient optimum limit value. Approximately 85% of customers did not use cash advance before signup, but a small portion of these customers did after opening a new account. However, because the number of these customers is so much greater than the pre-signup cash advance users, most of the post-signup cash advance usage is accounted for by them. Therefore, in order to effectively instigate cash advance use, a more elaborate separation of these customers is required. Looking at the characteristics of customers who go from not using to using cash advance, the demands for the loan can be identified by looking at other types of loans they possess. Specifically, the new cash advance users transfer their existing loans to the cash advance balance or increase their debt liabilities using cash advance. Consequently, if we separate the customers who use overdrafts or card company loans from the non pre-signup cash advance users, we can obtain different net profit functions, as shown in <Fig 11>. It is found that this new group which accounts for 9% of all customers yields much better profitability compared to groups with small pre-signup cash advance usage. If we grant appropriate limit values that meet the demands of the customers, it may be possible to create additional profits from this new group.

[3] The card companies in Korea greatly promoted cash advance usage since 2000, which resulted in great losses in 2003. Korea Credit Bureau was established upon this card crisis.



<Fig 11> Segmentation of customers with cash advance usage = 0

Ⅳ. Summary and Conclusion

In this study, a theoretical model to determine the optimum credit card limit was established and based on this, it was demonstrated that finding optimum limit values using actual data is possible. The existence of the optimum limit occurs from the fact that the good and bad customers respond to limits in a different manner; i.e. if the optimum limit is exceeded, the revenue from good customers does not increase any further, whereas the costs from bad customers continuously increase, reducing net profit. However, because the actual limit and usage data collected from card companies does not satisfy the basic assumption of the optimum limit model that “the customers given different limits must be of the same properties”, it is not possible to apply the optimum limit model to the data as is. In order to configure the customer groups to satisfy the assumptions of the optimum limit model, customers were separated into homogenous groups by usage amount and credit rating, which are the two components of the model. Nevertheless, the dependence of the limit on the credit rating is not totally removed. This problem was resolved by readjusting the good to bad ratio of each limit to the good to bad ratio of the population. In customer groups configured in this way, it was also demonstrated that the function of the usage amount with respect to the limit is not affected by customer properties that affect the limit.

This method of segmenting and resampling was applied to limit and performance data of new card accounts available in KCB’s database in order to satisfy the assumptions of the optimum limit model, then profit was calculated for each limit to determine the optimum limit value of the customer groups. As a result, it was verified that the larger the pre-signup usage amount and the better the credit rating, the greater the optimum limit value should be, as predicted theoretically. Although this method yields an optimum limit value for any customer group, the optimum limit becomes less efficient as homogeneity is reduced.

Although the optimum limit model used in this study can be applied to cash advance, an optimum limit cannot be derived for all customer groups due to the characteristics of cash advance being different from card purchases. The optimum limit model's assumptions are valid for customer groups with low demands in cash advance, generating reasonable results. For customers with high demands, however, the results do not make much sense. In order to obtain a reasonable optimum limit value for cash advance, a long-term profitability analysis and an optimum model that reflects the ability to repay are required.

Lastly, regarding competition, the result of the optimum limit model suggests that card companies should refrain from attracting customers by granting limits higher than the optimum value since this only causes losses. The optimum limit derived from the model cannot be applied alike to all situations. If the economic parameters used and characteristics of the population change, the optimum limit will be different even for the same customer group. This is because the profit is affected by many other factors related to marketing and customer management, in addition to the limit. Therefore optimizing the limit alone cannot achieve maximization of net profit. For customers with high credit ratings and large usage amounts, the net profit curve does not increase any further beyond a certain limit value and a very wide range is given for the optimum limit. Within this range, it is more important for card companies to enhance competitiveness by offering better products and services than purely by controlling the limit.

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