THE BEST PRACTICE CB
INNOVATIVELY & EXCITINGLY THE BEST!


# Study On the Determination of the Optimum Credit Card Limit

**Yonho Song, Ph.D.**
**Korea Credit Bureau**
**yhsong@koreacb.com**

# > Content

# I. Optimum Limit Model

- To determine the optimum limit that maximizes the profit,
  - Solve following equation:

$$P(x) = \frac{u^G(x) \times N^G \times g - u^B(x) \times N^B \times l}{N}$$

$$= g \frac{N^G}{N} \times \left( u^G(x) - \frac{w_0}{w^*} \times u^B(x) \right)$$

$$Max\ P(x)\ w.r.t.\ x$$

$N^G$, $N^B$ : Number of good and bad customers

$u^G(x)$, $u^B(x)$ : Usage function w.r.t the limit for good and bad customers

$w_0 = l/g$ : (losses from bad customers)/(revenue from good customers)

$w^* = N^G/N^B$ : Population Odds

# I. Optimum Limit Model

## Common sense on the optimum limit

- Allow higher limits to “Good” customers with high usage amount, lower limits to “Bad” customers who are delinquent
- The optimum limit is the limit that maximizes the profit for a homogeneous customer group with a similar usage amount and credit rating.
  - When limits are randomly given to customers in homogeneous customer group $A_i$, profit function will be maximized at the optimum limit value.
  - The optimum limit value is determined by the marginal utilization rate of good and bad customers.

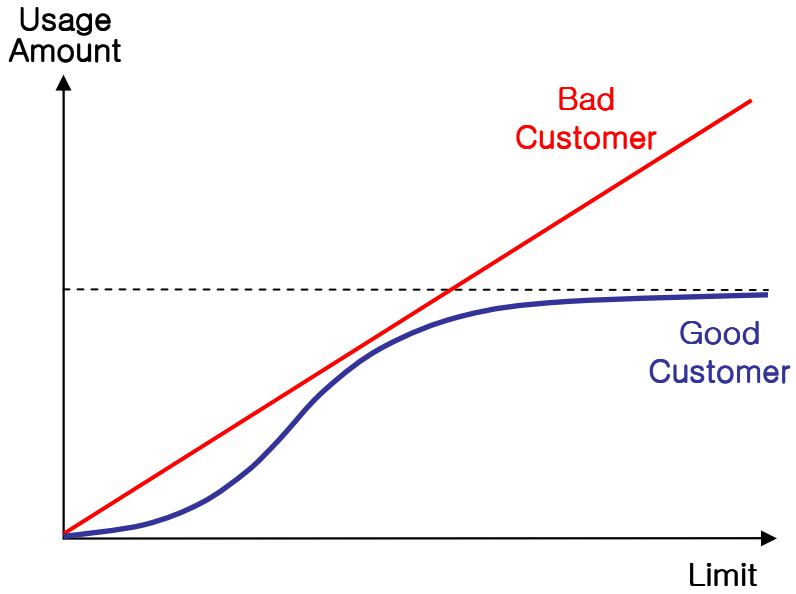
$$\frac{du_i^G(x)}{dx} = \frac{w_0}{w_i^*} \times \frac{du_i^B(x)}{dx} = \frac{w_0}{w_i^*} \times u_i^{B*}$$

# I. Optimum Limit Model

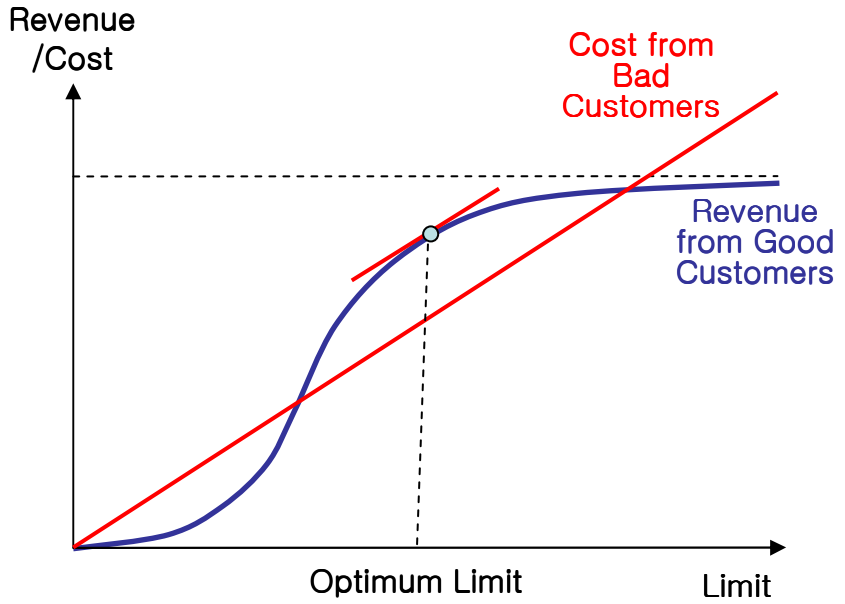
## Usage and Revenue/Cost function of Good and Bad customers

- **Usage functions for good and bad customers are different**
  - Usage function for good customers is S-shaped
  - Usage function for bad customers is linear with respect to the limit
- **The optimum limit can be obtained from Revenue and Cost functions**
  - Exists where the slope of cost function is the same as the tangent of revenue function

**Average Usage Amount per person**



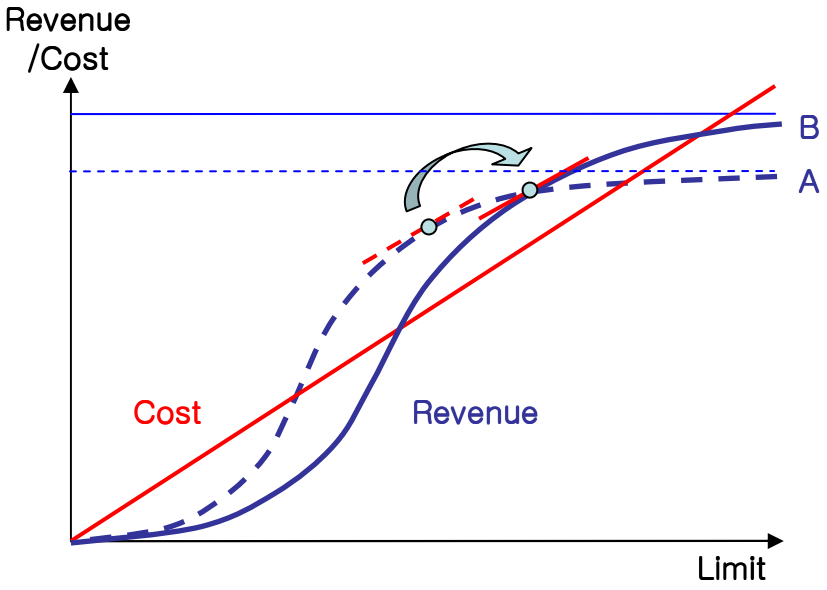

**Revenue/Cost function per person**



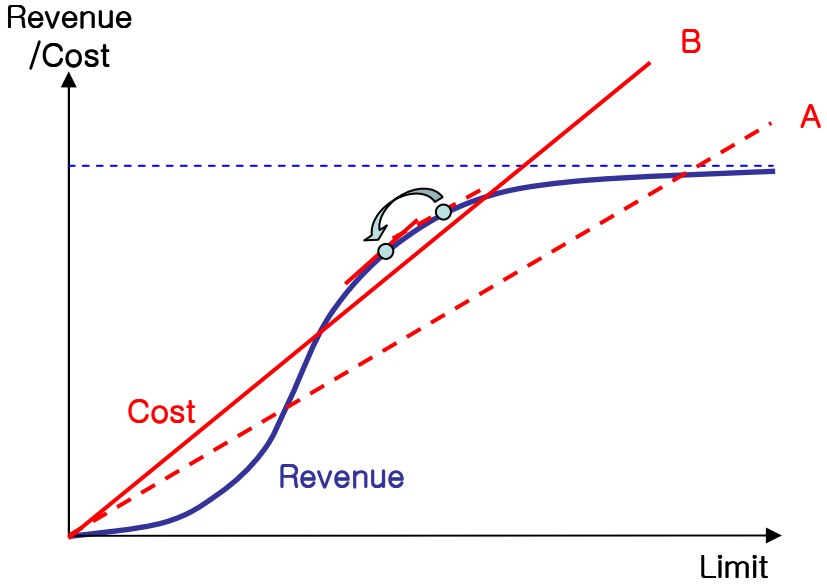
# I. Optimum Limit Model

- The optimum limit depends on the customer group
  - Optimum limit is higher for customer groups with higher usage amount and better credit
    - For a group with higher usage amount, the revenue curve moves up and to the right
      → The optimum limit becomes higher
    - For a group with worse credit rating, the cost curve becomes steeper
      → The optimum limit becomes lower

**A group with higher usage amount**




**A group with worse credit rating**



## Delinquency Rate according to the Optimum Limit Model

### Account-based delinquency rate

- Defined as (# of delinquent customers)/(# of customers with balance)
- Property of the customer group, independent of the limit

$$D_c(x) = \frac{N^B}{N^B + N^G} = \frac{1}{1 + w^*}$$

### Balance-based delinquency rate

- Defined as (delinquent amount)/(total balance)
- U-shaped and has a minimum value near the optimum limit

$$D_b(x) = \frac{b^B(x)}{b^B(x) + b^G(x)} \approx \frac{u^B(x)}{u^B(x) + u^G(x)} = \frac{1}{1 + u^G(x)/u^B(x)}$$

# I. Optimum Limit Model

- Can the Optimum Limit Model be applied to the actual data?
  - **The situation described by the Optimum Limit model**
    - Limits are randomly given in a homogeneous customer group
    - Customers at different limit values are of same properties
    - Usage amount and delinquency rate are only dependent on the limit, not the customer properties
  - **The situation described by actual data**
    - Higher limits are given to the customers with greater usage scale and better credit ratings
    - Customers at different limit values have different properties
    - Usage amount and delinquency rate are dependent on not only the limit but also the customer properties
  - **To apply the optimum limit model to actual data, effects of the customer properties on the limit need to be controlled**
    - Customers are segmented into homogeneous customer groups
    - In the customer groups, it must be verified that usage and delinquency depend only on the limit

# II. Determination of the Optimum Purchase Limit

## Analysis Data

- The data used for the study consisted of 21 million newly opened credit card accounts with detailed customer properties
  - The data was collected from 11 card companies by Korea Credit Bureau since Jan. 2006
  - Customers of the same properties were given different limits by different card companies due to differing limit policies
- Frequently used variables in the study include:
  - $\boldsymbol{U_L}$ : pre-signup monthly total usage amount of a **customer** (average over 3 months before signup)
  - $\boldsymbol{U_N}$ : post-signup total usage amount of a **customer** (average over 3 months after signup)
  - $\boldsymbol{u_N}$ : post-signup usage amount of a new **account** (average over 3 months after signup)
  - $\boldsymbol{S_L}$ : pre-signup credit rating of a **customer**. The credit ratings lie on a scale of 1 to 10, with smaller numbers representing more creditworthy customers.

# II. Determination of the Optimum Purchase Limit

- How customer properties affect the usage?
  - When pre-signup usage amount is fixed, customer properties positively correlated with the limit are negatively correlated with the post-signup usage amount

$$u_N = f(x \mid \vec{X}) = U_N(U_L) \times SOW(x \mid \vec{X})$$

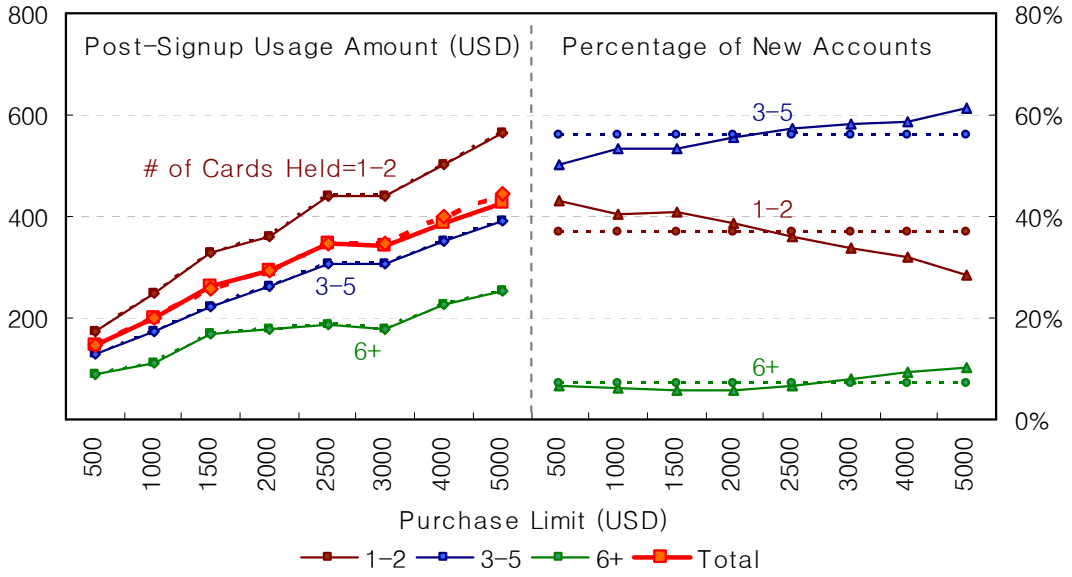
| Variable | With no variables fixed | | | | | With $U_L$, $S_L$ fixed (for $S_L$, only fix $U_L$) | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $U_N$ | *Usage Rate* | *SOW* | $u_N$ | *Limit* | $U_N$ | *Usage Rate* | *SOW* | $u_N$ | *Limit* |
| $U_L$ | +++ | 0 | − | + | ++ | | | | | |
| *Credit Rating*($S_L$) | 0+ | − | − | −− | +++ | 0− | − | − | −− | ++ |
| *# of Cards Held* | + | − | −− | 0− | ++ | 0+ | − | −− | −− | + |
| *Annual Income* | ++ | − | − | + | ++ | 0+ | − | 0− | 0− | + |

***SOW*** : Share Of Wallet, $\vec{X}$ : Customer Properties

***Usage Rate*** = (# of customers who used the new card)/(# of new customers)

# How customer properties affect the usage?

- Customers with a large number of credit cards have good credit ratings and large pre-signup usage amounts, so are given higher limits
  - But, *usage rate* and *SOW* decreases since their usage is distributed across many cards, yielding negative correlation between limit and post-signup usage amount
- Giving higher limits to customers with more cards makes correlation between the usage amount and limits less strong
  - But, its effect is negligible as shown by the dotted lines.



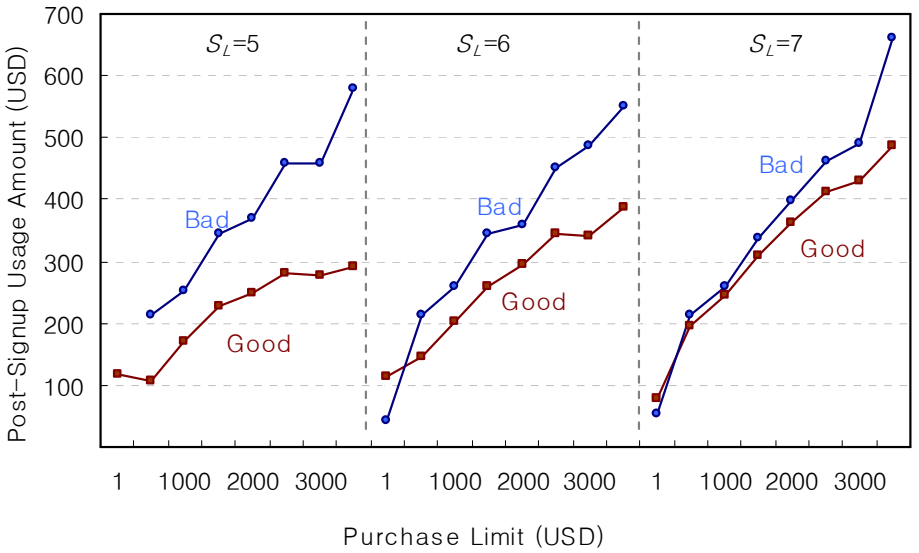
# II. Determination of the Optimum Purchase Limit

## Relation between the Limit and Usage Amount

- If pre-signup usage amount and credit rating are fixed, post-signup usage amount is a function of the limit only

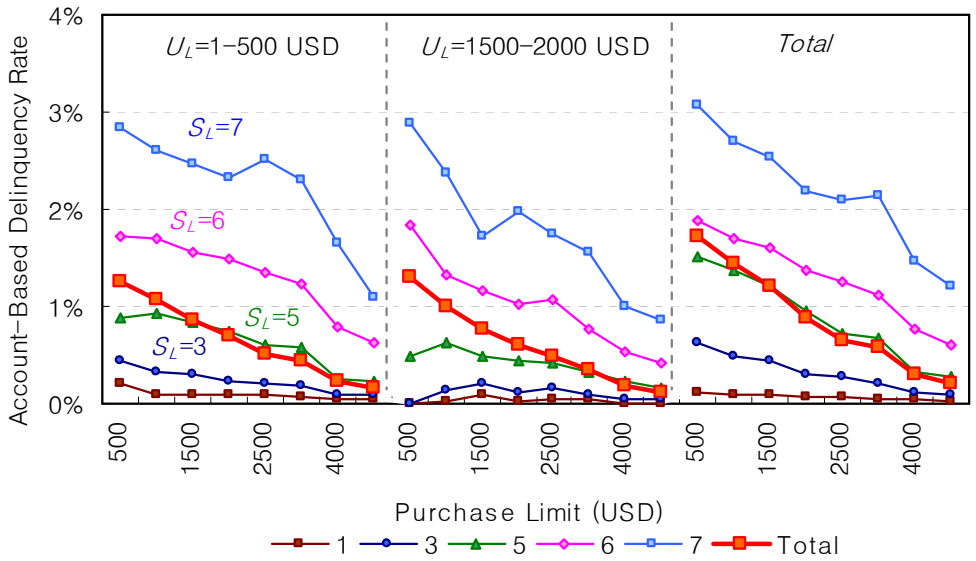
$$u_N = f(x \mid U_L, S_L)$$

- The relation between the limit and usage amount from actual data agrees with the assumption of the optimum limit model



# II. Determination of the Optimum Purchase Limit

- Relation between the limit and delinquency rate
  - **Even after fixing pre-signup usage amount and credit rating, account-based delinquency rate decreases as the limit increases.**
    - It is possible to subdivide customers with the same credit rating according to creditworthiness.
    - The dependence of the limit on creditworthiness cannot be completely removed by Credit Bureau data.



# II. Determination of the Optimum Purchase Limit

- Removing the dependence of the limit on creditworthiness
  - Account-based delinquency rate depends on the limit because the limit depends on creditworthiness

$$D_c = f(S_L, U_L) + \varepsilon\left(x(\vec{X}_2)\right)$$

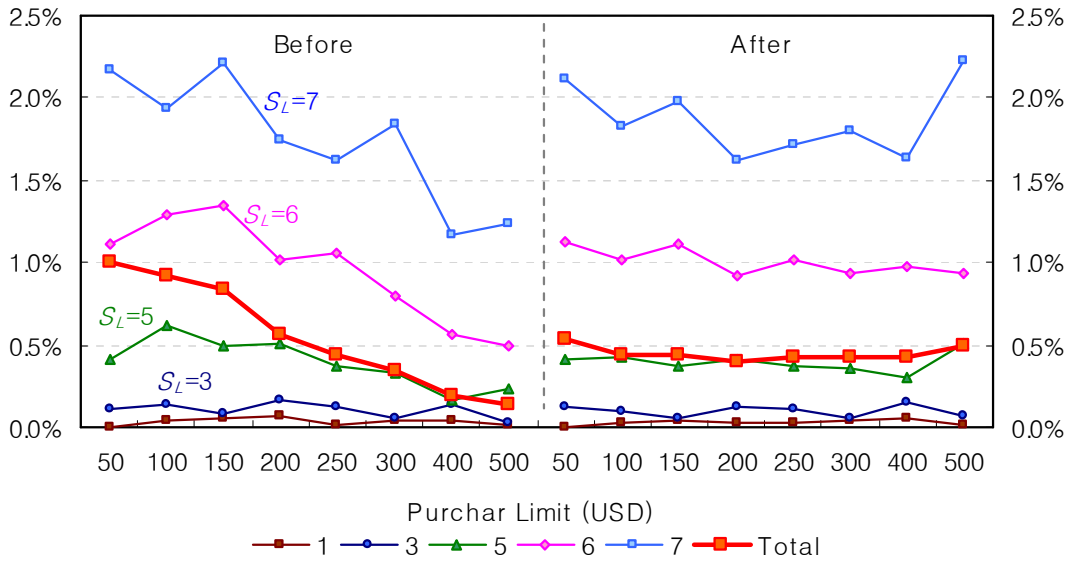
where $\vec{X}_2$ represents customer properties except pre-signup usage amount and credit rating

  - The dependence of the limit on creditworthiness can be removed by readjusting odds at each limit value
    - Resample customers of subgroup $A_i$ so that sample odds( $SO_i = N_i^G / N_i^B$ ) becomes the same as population odds( $PO = N^G / N^B$ )

$$A_i = (N_i^G, N_i^B) \rightarrow A_i'(N_i^G, N_i^B \times \frac{SO_i}{PO})$$

# II. Determination of the Optimum Purchase Limit

- After resampling, delinquency rate agrees with the assumption of the optimum limit model
  - Account-based delinquency rate is independent of the limit
  - Balance-based delinquency rate is a U-shaped function of the limit



# II. Determination of the Optimum Purchase Limit

- After resampling for homogeneous customer group $A$, net profit per person for limit $x$ is given by

$$P(x)=\frac{\left(u_{N12M}^{G}(x)\times N^{G}(x)+u_{N12M}^{B}(x)\times N^{B}(x)\times\frac{SO(x)}{PO}\right)\times(r_1-r_2)-A_D(x)\times R(2\rightarrow 7)\times N^{B}(x)\times\frac{SO(x)}{PO}}{N^{B}(x)+N^{B}(x)\times\frac{SO(x)}{PO}}$$

$N^G$, $N^B$ : Number of Good and Bad Customers

$u_{N12M}^{G}$, $u_{N12M}^{B}$ : Average usage amount per person for the first 12 months after opening the new account

$r_1$: Merchant fee (=2.45%)

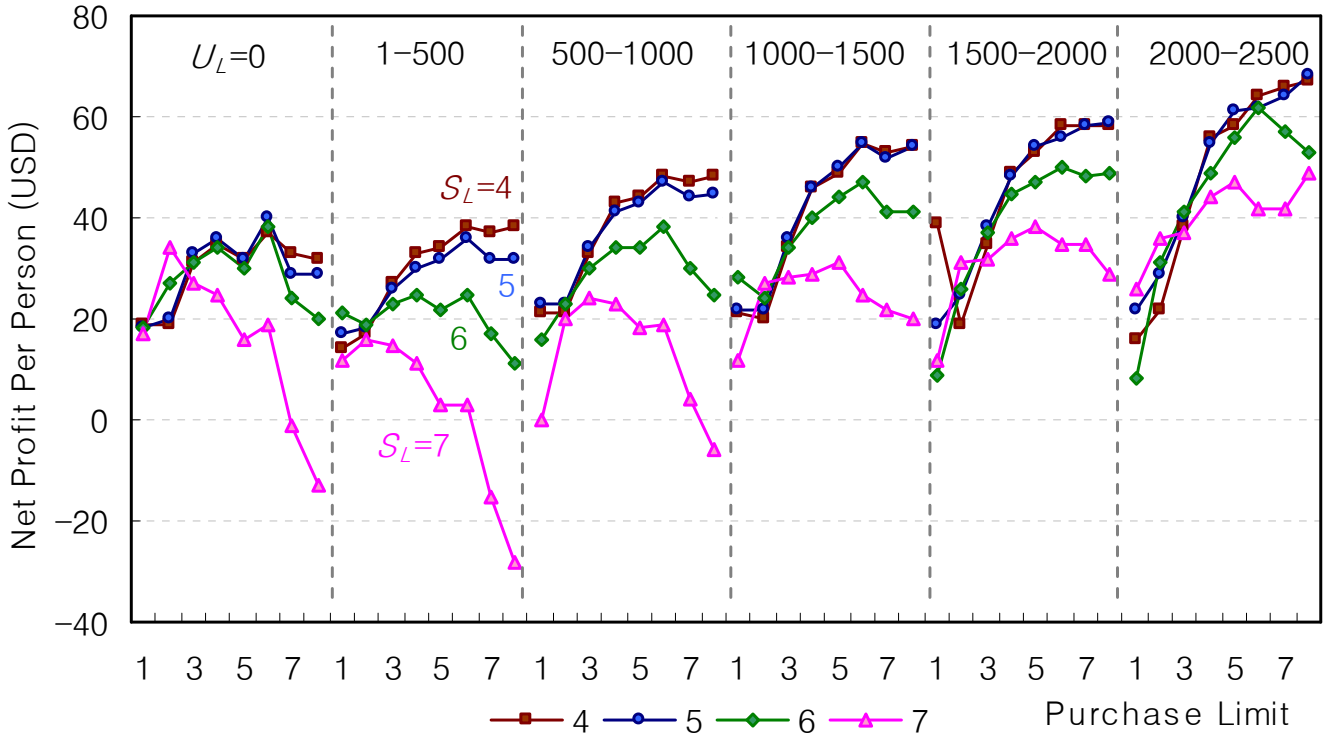
$r_2$: Financing interest rate (=0.46%)

$A_D$: Average 2-period(30-60 DPD) delinquent amount per person

$R(2\rightarrow 7)$ : The roll-rate at which the 2-period delinquent amount becomes a charge-off (=58%)

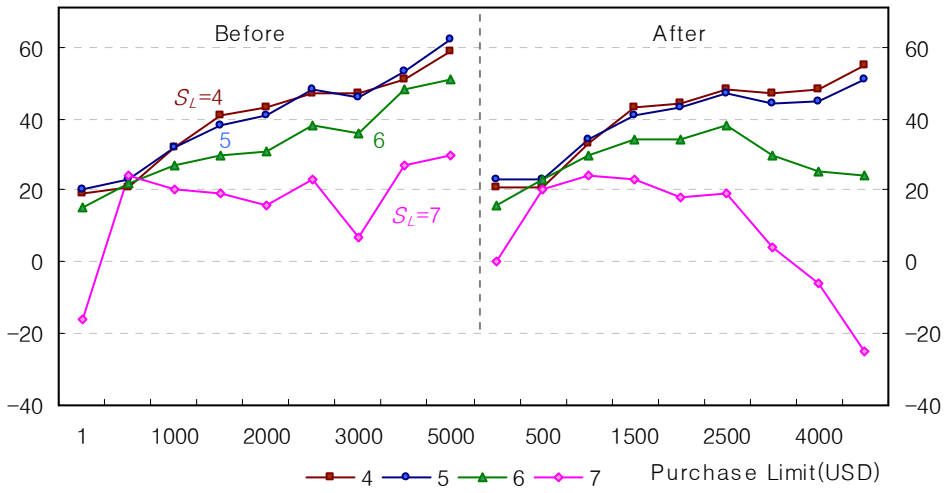
# II. Determination of the Optimum Purchase Limit

- Net profit function agrees with the result derived from the optimum limit model
  - The optimum limit that maximizes net profit is higher for customer groups with a better credit rating and a larger pre-signup usage amount



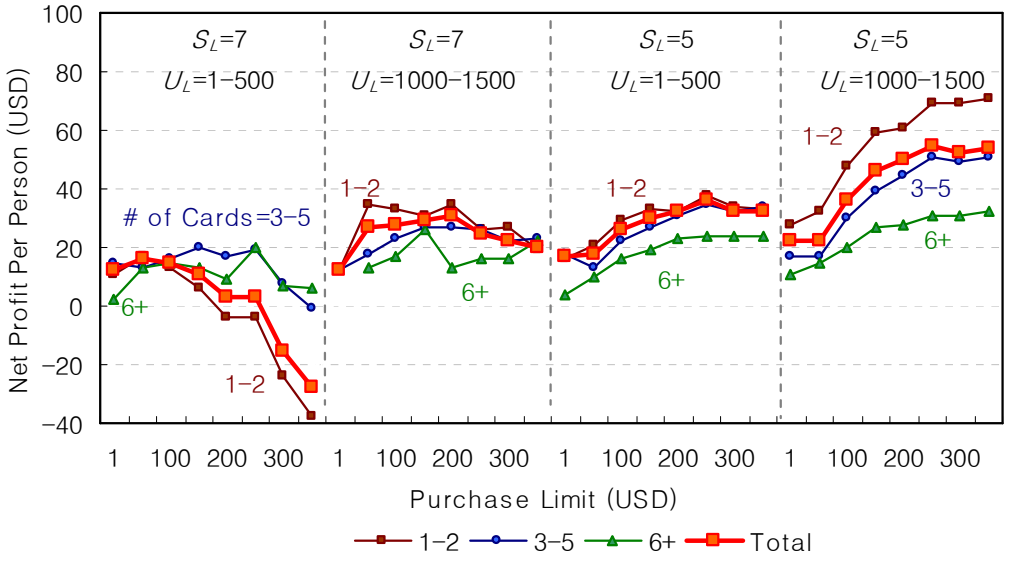
# II. Determination of the Optimum Purchase Limit

- Net profit function before and after resampling
  - Before resampling, net profit monotonically increases with the limit
    - Customers with different limit values are of different creditworthiness
    - The monotonic increase is a result of the dependence of the limit on creditworthiness
  - After resampling, net profit decreases beyond the optimum limit
    - the dependence of the limit on creditworthiness has been removed



# II. Determination of the Optimum Purchase Limit

- Subdividing customer groups using number of cards held increases efficiency of the optimum limit
  - For customer groups with small pre-signup usage amount, the post-signup usage amount increases as number of cards held increases, yielding higher optimum limit values
  - For customer groups with small pre-signup usage amount, SOW decreases as number of cards held increases, yielding lower optimum limit values



# III. Summary and Conclusion

- Establishment of the optimum limit model
  - Good and Bad customers respond to the limit in different manners
    - Usage functions are different
  - To apply the model to actual data,
    - Divide customers using credit rating and pre-signup usage amount
    - Remove the remaining dependence of limit on the creditworthiness by resampling
- Determination of the optimum limit from the net profit function
  - The optimum limit is higher for customers with good credit ratings and large usage amounts
  - The optimum limit model can be used effectively to determine the optimum limit policy for mid-range rated customers with a reasonable monthly usage amount
  - Dividing customers into more homogeneous groups yields a more efficient optimum limit