Title:
VaR for low default portfolios based on a single factor hazard model

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Abstract:

The VaR and the distribution of loss is a key determinant in the securitisation of portfolios within the Basel II regulatory framework. The central limit theorem suggests the total loss from large portfolios may be modelled by the normal distribution when client defaults are independent, However, common systematic factors lead to default dependence and so require more detailed modelling. One standard approach is to apply the Vasicek formula based on the Merton model for a plausible range of the portfolio correlation coefficient, the key parameter measuring dependence. Rather than adopt this structural framework we suggest that a reduced form model within a survival analysis framework is appropriate.

We model the effect of a systematic factor by a dynamic baseline hazard function, with time independent client covariates entering proportionally. The distribution of the portfolio loss predicted over a finite time horizon is calculated from the distribution of the integral of exponentiated Brownian motion. We show how calculate VaR by analytic approximations and simulation using an importance sampler. This integral is a function of a parameter measuring the volatility of the baseline intercept. We show how the VaR calculation varies with the volatility and with the loss horizon, and compare it to the Vasicek formula. Kalman filtering provides both diagnostic tests for the proportional hazard assumption and the maximum likelihood estimate of the volatility parameter. From an empirical study of a mortgage portfolio we establish the proportional hazard assumption and apply our results to the VaR calculation for loss over a 12-month horizon distribution.

This model with a dynamic hazard function fits within the single risk factor framework discussed by Gordy. We draw some conclusions arguing that reduced form modelling is always richer than the structural counterpart.