

# A CREDIT RISK MODEL FOR CONSUMER LOANS PORTFOLIOS

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## ABSTRACT

The work presented in this paper is the development of a portfolio credit risk model for consumer credit. The proposed model is intended to be an instrument to obtain distributions of loss in the consumer credit portfolios in Brazil. The application of the proposed model can be divided into two main steps: division of the portfolio into segments and simulation of the loss distribution.

The consumer profiles and the risk classification of the credit operations are used to segment the portfolio. After the segmentation, credit loss distributions for each segment are selected. The portfolio segments and their respective marginal distributions of credit loss are used in a Monte Carlo simulation process to generate the portfolio loss distribution. The dependence among the credit loss in the different segments of the portfolio is modelled through an elliptical copula function. Statistical tests are done and show that the proposed model is adequate to represent credit loss distributions in consumer credit in Brazil.

## 1 - INTRODUCTION

Models of Portfolio Credit Risk predict the distribution of credit loss or value for a portfolio. The loss or value distribution can be used for many applications in risk management as Economic Capital and Value-at-Risk assessment, performance evaluation and marginal risk estimation. Popular credit risk models such as Creditmetrics (Gupton, Finger, and Bhatia,1997) and Moody's-KMV model (KMV Corporation, 1993a and 1993b) as well as most of the work done in credit portfolio modeling have been orientated to corporate credit. Some recent works have been focused on consumer credit as Jacobson and Roszbach (2003) and Perli and Nayda (2004).

Credit risk models are usually classified as structural or reduced-form models. Structural models rely on the original work of Merton (1974) and assumes that a firm goes into default if the value of its assets is less than the value of its debts. Reduced-form models do not explicitly model the default process in a single debtor, but the intensity of the occurrences default events independently of its causes. Structural modelling for consumer credit requires a theory of default for consumer credit that could allow an option based approach in the same way it is done for corporate credit. Andrade and Thomas (2004) proposed such a theory and a model to generate the distribution of a portfolio default rate by Monte Carlo simulation. But that approach implies on simulating joint defaults for the elements of a consumer credit portfolio and the time and computational power that is necessary for that imposes a limit for its use in big portfolios.

We propose in this article a reduced-form credit model for consumer loans portfolios and make an application and validation using consumer credit data from the Brazilian market. We model distributions of credit loss in the different segments that compose a portfolio and use these marginal loss distributions to obtain the loss distribution of the whole portfolio. The proposed model does not follow the traditional way of modeling default rates and loss given default (or recovery rate) separately, but the loss rate is directly modelled. This approach simplifies the model and makes it unnecessary to make premises like independence between default occurrence and recovery.

Section 2 describes the empirical data used for the model development. In section 3 we propose a methodology for portfolio segmentation with the objective to divide the portfolio into segments that can be considered homogeneous from the credit risk perspective. In section 4 we analyse the pattern of credit loss in each of the segments and select statistical distribution functions that best fit the loss data for each of the portfolio segments. The modelling of the joint pattern of credit loss among portfolio segments is done by an elliptical copula function. The simulation of the distribution of the losses of the portfolio is done by an algorithm that generates joint realizations of loss in each segment that are weighted to generate portfolio loss realizations. The dependence and simulation issues are discussed in section 5. Section 6 presents results of model tests and our final conclusions are presented in section 7.

## **2 - EMPIRICAL DATA**

We used data of installment loans of two million consumers, which was supplied by Serasa, a major credit bureau in Brazil. The sample was randomly selected from Serasa's database and the data included all payment behaviour registered for the consumers between January 1999 and January 2002. The available data comprised of detailed payment information with due dates and amounts as well as amounts paid and the dates of payment.

Additional data was supplied for the construction of the consumer attributes, including negative data, socio-demographic data, number of credit bureau inquiries and Serasa's credit bureau score and credit segmentation<sup>1</sup>. The data supplied allowed us to build variables that were current on specific dates in the past. Thus it was possible to obtain the value of a specific variable for a specific consumer in each one of the months within the period of analysis.

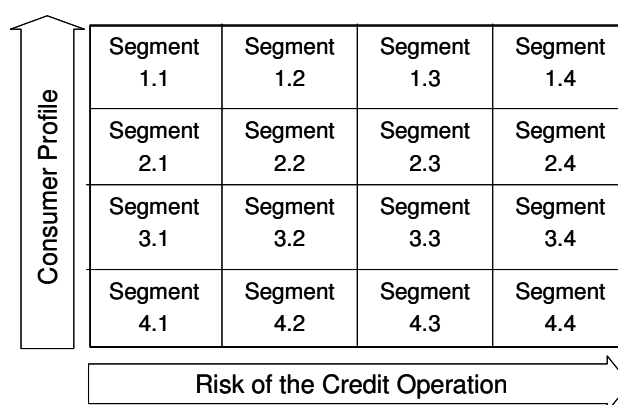
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<sup>1</sup> Serasa's credit segmentation allocates a specific consumer in 9 different categories according to the consumer's pattern of credit activity.

### 3 - PORTFOLIO SEGMENTATION

The first step of the proposed model is the division of the portfolio into segments that can be considered homogeneous from the credit risk perspective. We propose a grid segmentation of the portfolio using two dimensions: consumer profile and risk of the credit operation. Figure 1 illustrates the process. The expositions are grouped according to consumer profile and risk. Final segments used in the simulation process are obtained by crossing the categories of the two analysed dimensions.

Figure 1 – Portfolio Segmentation



With the segmentation approach, the Monte Carlo simulation to generate the loss distribution is not done for each element of the portfolio, but for each segment. It makes the use of the proposed model in a big portfolio with millions of elements as feasible as in a small portfolio.

#### 3.1 – Consumer profile segmentation

The objective of this step in the segmentation process is to obtain groups of consumers that are similar among them. However, this grouping process has to have sense under a portfolio credit risk perspective. These issues were considered for the choice of what was the definition of similarity used to cluster consumers, that was the correlation between the credit loss time series related to each kind of consumer. In this way it was possible to identify consumer profiles that present similar historical patterns of loss.

Since it is not feasible to build historical time series of losses for each consumer, we made a preliminary clustering of the consumers into preliminary groups based on socio-demographic and behavioural data. For each of those groups we calculated a credit loss time series that was used in a final clustering process by which the preliminary groups were merged according to the correlation between time series of credit losses. The credit loss time series were calculated using historical data of the credit operations done by the consumers that belong to the group.

The initial groups were obtained by K-means statistical clustering methodology (MacQuenn, 1967). The K-means method divides the individuals into a given number of clusters. We arbitrarily considered 300 preliminary groups. We used such a high number to obtain very homogeneous preliminary groups so that each calculated loss time series could be attributed to a well defined behavioural and socio-demographic profile. We used the following variables for the process:

- Socio-demographic: gender, age, income, job, zip code
- Behavioural: inquiries for credit reports in the last 6 months at Serasa, number of negative registers in negative lists, Serasa's credit bureau score, Serasa's credit segmentation.

All variables were codified as dummy variables. Variables with an order structure were codified as ordered dummies to keep their hierarchical structure. Table 1 shows an example of simple and ordered dummy codification. The codification gives an homogeneous treatment to all variables and avoids possible problems with scale differences.

**Table 1 – Variable Categorization**

Age	Dummy Codification					Ordered Dummy Codification				
	d1	d2	d3	d4	d5	d1	d2	d3	d4	d5
< 21	1	0	0	0	0	1	1	1	1	1
[21,30]	0	1	0	0	0	0	1	1	1	1
[31,40]	0	0	1	0	0	0	0	1	1	1
[41,50]	0	0	0	1	0	0	0	0	1	1
> 50	0	0	0	0	1	0	0	0	0	1

In order to build the loss time series we calculated the loss rate that can be attributed to each exposition in each period. To go further on the calculation of credit loss it is necessary to define what we considered an exposition. We use the term exposition not for the whole due amount of a loan, but for the amount corresponding to one loan installment in a period. In that way a instalment loan is divided into a number of expositions, each one corresponding to a time period.

Given an exposition  $i$ , its loss was calculated as:

$$L_i = 1 - \frac{DPA_i}{EA_i} \quad (1)$$

where  $L_i$  is the credit loss of exposition  $i$ ,  $EA_i$  is the amount of exposition  $i$  and  $DPA_i$  is the discounted payment amount related to exposition  $i$ . The payment amount is discounted from the payment date to the due date of the exposition. The discount rate used was the average cost of raising funds that financial institutions had in the period of analysis (1.5% p.m.).

A segment of the portfolio can be viewed as a group of expositions and its loss in a specific period can be obtained calculating the weighted average of the loss of the expositions in that period, using the exposition amounts for weighting. Using this process we constructed time series of losses for each of the 300 preliminary groups gotten by K-means segmentation. In each time period the expositions of the credit operations done by the consumers that belong to the group were used to calculate the value of the loss for the group in the period. A variable clustering methodology was used to merge the initial groups that have high correlation among their credit loss time series into four segments. This number of segments was arbitrarily chosen. For that task we used the variable clustering algorithm available at SAS software (SAS Institute, 1990).

### **3.2 - Risk segmentation**

To group the expositions according to their risk we modelled the expected loss for an exposition  $i$  as:

$$L_i = \frac{1}{1+e^{-f_i}} \quad (2)$$

where:

- $f_i = a_0 + a_1X_{1i} + \dots + a_nX_{ni}$
- $L_i$  is the fractional credit loss for exposition  $i$ , calculated using equation 1;
- $X_{1i}$  to  $X_{ni}$  are predictive variables;
- $a_0$  is an intercept and  $a_1$  to  $a_n$  are parameters of the model.

We used a random sample of 100,000 expositions (each exposition represents one installment of a loan) to estimate the parameters of the equation 2. The predictive variables were socio-demographic and credit behavior information of the consumer and characteristics of the exposition. The parameters were obtained by maximum likelihood estimation. The area under ROC curve (Receiver Operating Curve) was used to evaluate the model. Its value for the regression model was 0.942 that, according to Hosmer e Lemeshow (2000), is a excellent predictive power.

The predictive variables in the risk classification model were:

- Socio-demographic: income, age, gender, educational status, type of job and time at current job.
- Behavioral: Number of credit bureau inquiries, number of stopped checks, numbers of post-dated checks registered in the credit bureau, number of on time and late payments registered in the credit bureau, number and amount of derogatory information, time since last occurrence of a derogatory and amount of the last occurrence of a derogatory information.
- Characteristics of the exposition: type and number of installments of the credit operation, maturity of the exposition, number of prior installments of the credit operation with payment on time and with late payment, maximum number of days of delinquency in prior installments of the credit operation.

Using the predicted loss as a measure of risk it is possible to define different categories of risk to be used in the grid segmentation process. In our work we used four risk categories according to Table 3. The limits between the risk categories were arbitrarily defined.

Table 3 – Risk Categories

Risk Category	Predicted loss
1	[0;0.01]
2	]0.01;0.03]
3	]0.03;0.50]
4	]0.50;1.00]

### 3.3 - Final segments

All expositions of the sample were classified according to their consumer profiles and risk. Final segments of the portfolio were obtained by crossing the results obtained for both dimensions of segmentation resulting in 16 portfolio segments as represented in Table 4.

Table 4 - Portfolio segmentation

<b>Final Segment</b>	Consumer profile classification	Risk classification	<b>Final Segment</b>	Consumer profile classification	Risk classification
<b>1.1</b>	1	1	<b>3.1</b>	3	1
<b>1.2</b>	1	2	<b>3.2</b>	3	2
<b>1.3</b>	1	3	<b>3.3</b>	3	3
<b>1.4</b>	1	4	<b>3.4</b>	3	4
<b>2.1</b>	2	1	<b>4.1</b>	4	1
<b>2.2</b>	2	2	<b>4.2</b>	4	2
<b>2.3</b>	2	3	<b>4.3</b>	4	3
<b>2.4</b>	2	4	<b>4.4</b>	4	4

The expositions that were classified into a specific segment in each period were used to analyze the loss pattern of that segment. So, analyzing the expositions that were classified into segment 1.1 in period 1, period 2 and so on, we get a historical pattern of loss in the segment 1.1. The segmentation allows the dynamics of the portfolio composition to be inserted into the model. Effects of the portfolio management actions that can affect the portfolio profile, as new

acquisition strategies, for example, can be captured through changes in the portfolio composition among the segments. The same happens with external factors that can cause changes in the portfolio profile.

#### **4 - DISTRIBUTION FITTING AND SELECTION**

The proposed model does not have any distributional assumptions for credit loss. Our approach is to seek which distribution best fits the empirical loss data in each of the segments of the portfolio.

To deal with the low number of credit loss observations that were available for the distribution fitting process, we used a resampling method proposed by Politis *et al.* (2001). The method is called subsampling and is a variation of the classical bootstrap method (Efron, 1979). Resampling methods generate a big number of observations from a dataset by extracting new random samples from the original sample. The main characteristic of the subsampling process is that it generates samples without reposition that have a number of observations lower than the original sample, while the classical bootstrap method generates samples with reposition that have the same number of observations that the original sample. According to Politis *et al.* (2001), the subsampling method is more robust than the classical bootstrap method as it does not require independence assumptions. They also show that the subsampling process works well for a wide range of subsample sizes.

The main application of resampling methods is the generation of the empirical distribution of a statistic. This is also our objective and the statistic we are interested in is the weighted average loss among the expositions, which represents the loss in the portfolio or segment of the portfolio when using exposition amounts as the weights. Resampling methods for credit portfolios were also used by Lopez and Saidenberg (1999) and Carey (1998).

The available database contained about 10.6 million expositions. We extracted 3,000 samples without reposition, each one with 300,000 elements. The number and size of the samples were arbitrarily chosen. For each subsample we calculated the loss in each of the 37 periods (from jan/99 to jan/02) in each

of the 16 portfolio segments, leading to a total of 111,000 observations of credit loss per segment, that were used for the distribution fitting and selection processes.

For the distribution fitting process we tested several traditional statistical distributions that have shape similar to the one expected in the loss distributions, with a large tail for higher losses. Furthermore, we also tested two distributions which can have a variety of shapes according to the parameters used to fit the data. They were Johnson's distributions (Johnson, 1949) and generalized lambda distributions (Dudewicz e Karian, 1996). The parameters of Johnson's and generalized lambda distributions were estimated by non-linear regression using SAS software. All other distributions were fitted by maximum likelihood methods using @Risk software. The tested distributions are presented in Table 5.

Table 5 - Fitted distributions

• Generalized beta	• Chi-squared
• Extreme value	• Gamma
• Lognormal	• Inverse gaussian
• Log-logistic	• Pearson type V
• Pearson type VI	• Rayleigh
• Weibull	• Johnson's distributions
• Generalized Lambdas	

We used the Anderson-Darling statistic (Anderson and Darling, 1952) to select the distribution that has the best fit for each segment of the portfolio. Table 6 presents the best fit for each segment. Assintotic critical values of Anderson-Darling (AD) statistic were obtained from Giles (2000).

Some segments have AD values superior than critical ones meaning that we can say that the best fitted distribution can not be considered the true distribution of the empirical data. As our main objective was the overall fit of the portfolio loss distribution, we used the best aproximation for the empirical data even for the segments with AD values higher than the critical values. Results show clearly the superiority of Johnson's distributions to fit our empirical data.

Table 6 – Distribution fitting and selection results

Segment	Best fitted distribution	A-D statistic
1.1	Johnson	4,2
1.2	Johnson	0,9*
1.3	Generalized Lambdas	1,1*
1.4	Pearson V	87,9
2.1	Generalized Lambdas	1,7*
2.2	Johnson	0,6*
2.3	Johnson	0,5*
2.4	Johnson	15,5
3.1	Inverse Gaussian	58,1
3.2	Generalized Lambdas	10,5
3.3	Johnson	0,2*
3.4	Johnson	40,9
4.1	Johnson	3,8*
4.2	Johnson	1,8*
4.3	Johnson	0,6*
4.4	Johnson	12,3

\* values lower then critical value of AD statistic with 1% significance level.

## 5 - DEPENDENCE MODELLING AND PORTFOLIO LOSS SIMULATION

In our proposed model we segment the credit portfolio and find out which statistical distribution best fits the credit loss in each portfolio segment. To obtain the portfolio credit loss distribution it is still necessary to model the dependence among the portfolio segments and to use a simulation algorithm that incorporates the marginal loss distributions of the segments and the pattern of dependence among them.

Our aim in modelling the dependence among variables, in our case credit loss in the portfolio segments, is to get their joint distribution, which describes their joint behaviour. The joint distribution can be defined through the univariate distributions of each variable (called marginal distributions) and the dependence relation among them, represented by a copula function.

The copula function is a mathematical function that has, as its arguments, the marginal distributions of each variable and produces as a result the joint multivariate distribution of them. Copula functions and its properties are described by Nelsen (1999). Applications of copula functions in risk management can be found in Embrechts *et al.* (1999).

In the proposed model the joint behaviour of credit loss in the different segments of the portfolio is modelled by an elliptical copula function. This type of copula has as its parameters the correlations among the variables (credit loss of the portfolio segments in our case). We tested the two most commonly used copula functions for credit risk (Bluhm *et al.*, 2003), the Gaussian or normal copula and the Student copula. As the Student copula presents asymptotic tail dependence and the normal copula does not, Embrechts *et al.* (1999) suggests that the use of Student copula in credit portfolio models may be more suitable.

The Monte Carlo simulation process to obtain the loss distribution for a credit portfolio is composed of two steps: joint simulation of credit loss in the different segments of the portfolio and the weighting of the joint realizations of loss in the segments according to the exposition amount of each segment to generate simulated realizations of loss in the portfolio. For the execution of the simulation process we just need to have as inputs the correlation matrix among credit loss time series for the segments of the portfolio, the marginal distributions for each portfolio (as in section 4) and to choose a copula function. For the simulation of joint realizations of loss in the different portfolio segments we used the algorithm proposed by Romano (2001). We used two elliptical copula functions in the simulations: Gaussian and Student with 2 degrees of freedom.

## **6 - MODEL TEST AND VALIDATION**

In order to test the proposed model we created 3,000 hypothetical portfolios selecting randomly without reposition from the database described in section 2. Expositions already in default (more than 30 days past due) were not considered for the selection.

Each one of the hypothetical portfolios was composed of 300,000 expositions, which were selected randomly from the database of 10.6 million expositions available for the work. The proposed model was applied to each one of the selected portfolios using the Gaussian copula and the Student copula with 2 degrees of freedom. For each portfolio we simulated 370,000 realizations of credit loss (10,000 for each of the 37 time periods). The realizations were used to calculate the percentiles 95% and 99% of the loss distribution of each portfolio.

Considering all the 3,000 portfolios with 37 observations of credit loss (each observation related to one of the 37 periods), we had a total of 111,000 empirical observations of credit loss. The 37 loss observations of each portfolio were compared with its respective percentile 95% and 99% that were obtained from the proposed model. The percentual of those observations whose values were over the percentiles 95% and 99% gotten by the proposed model are displayed in Table 7. If the model is correctly evaluating these percentiles the percentage of loss observations above 95% and 99% predicted percentiles should be close to 5% and 1%, respectively.

Table 7 – Observations over predicted percentile

Copula function	Predicted percentile	Observations over predicted percentile (%)
Normal	95%	5,20%
Normal	99%	1,08%
Student-2	95%	5,16%
Student-2	99%	0,50%

### 6.1 - Berkowitz test (2001)

To evaluate the proposed model, we applied the version of the Berkowitz test (Berkowitz, 2001) that was used by Frerchs and Löffler (2002). The Berkowitz test evaluates if the empirical data follows the estimated distribution of loss. Table 8 shows the number and percentual of the 3,000 portfolios which the null hypothesis of the Berkowitz test was rejected using 1% and 5% significance

levels. Results show that the proposed model represents well the true loss distribution. Using 1% significance level we can not reject the null hypothesis in any of the tested portfolios. If we use 5% significance level we reject the null hypothesis in only 0.83% of the portfolios when using Gaussian copula and none with the Student-2 copula.

Table 8 – Results of Berkowitz test

Copula	Significance level	Portfolios where the null hypothesis is rejected	
		#	%
Normal	5%	25	0,83%
Student-2	5%	0	0%
Normal	1%	0	0%
Student-2	1%	0	0%

We present in Figures 2 and 3 the distribution of the empirical and simulated values of credit loss for all the 3,000 portfolios using Gaussian and Student-2 copulas. These graphics allow a visual evaluation of the goodness of fit of the proposed model.

Figure 2 – Loss distributions with Gaussian copula

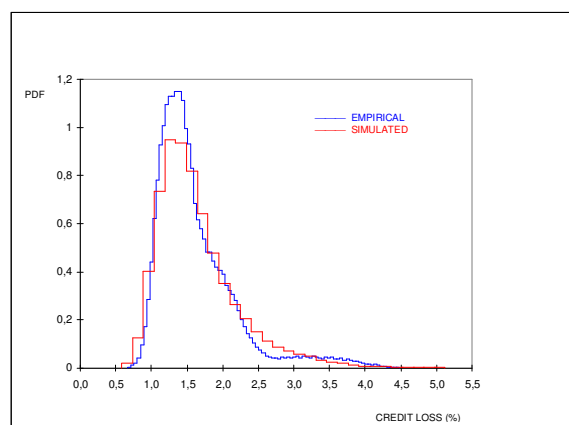
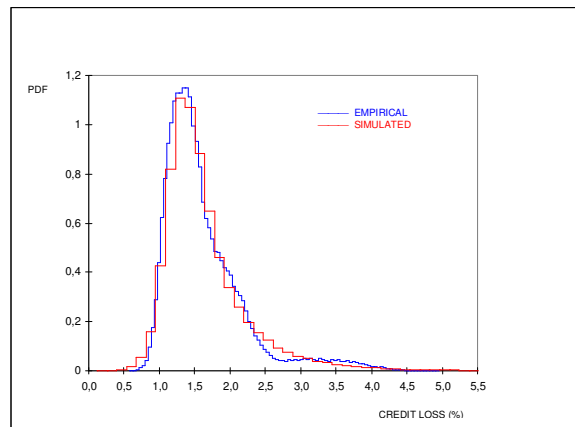


Figure 3 – Loss distributions with Student-2 copula



## 7 - FINAL REMARKS AND CONCLUSIONS

In this article we presented a model to assess credit risk of a consumer loans portfolio. The proposed model takes the reduced-form approach modelling the intensity of loss occurrence. The model was tested with Brazilian data and shown to be adherent to the empirical distribution of loss. We used credit bureau data to build 3,000 hypothetical portfolios for testing the model.

A version of the Berkowitz test was used to assess the quality of the model and results show that we can not reject the hypothesis that the estimated loss distribution is equal to the real loss distribution in the vast majority of the portfolios analysed. The tests included the use of Gaussian and Student-2 copula function for modelling the dependence among credit loss in different portfolio segments. The use of Student-2 copula performed better in the Berkowitz test but the use of Normal copula generated a number of empirical loss observations above the predicted 95% and 99% percentiles closer to the expected. Graphical analysis of Figures 2 and 3 visually confirms a good adherence to empirical data but it does not reveal any major difference in the tail fitting when comparing the Gaussian and the Student-2 copulas.

Although the proposed model is a flexible framework to modeling the loss distribution of a portfolio, allowing the use of different statistical distributions to model credit loss in different segments of the portfolio, we verified that the

Johnson's distributions had a better fit for most of the segments analysed. Thus suppressing the process of distribution selection and adopting the Johnson's distribution for all segments may not have a significant impact in the model performance.

The proposed model requires only socio-demographic and payment behaviour data from the debtors. These kind of data is widely used by financial institutions for development of credit and behaviour score models and represents no extra burden for data collection. Although the proposed model was developed using credit bureau data, it can be fully estimated using internal data of a financial institution.

The proposed model can also be used for portfolios of credit for small businesses which often do not have enough data to apply corporate credit risk models.

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