A statistical model for the temporal pattern of individual ATM withdrawals

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1. ATM withdrawal times show evidence of weekly cycles and quick-succession behaviour

Data:

- Random sample of one thousand accounts from a UK bank over a four-month period in 2005
- For each ATM withdrawal (cash or mobile-phone top-up), the time (second) of transaction recorded

Features:

1. Variation in the overall rate
2. Cyclical rate pattern within weeks
3. Constant long-run rate of withdrawal within accounts
4. Quick-succession withdrawals

2. A self-exciting point process model was developed to incorporate these features

Let $b(s; a_1, a_2) = B(a_1, a_2)^{-1} s^{a_1 - 1}(1-s)^{a_2-1}$ where $B(.,.)$ is the Beta function, and $w_1(t; u_2, u_3) = \int_{I_t} b(s; u_2, u_3) ds$, where I_t denotes the interval $(\{d(t)-1\}/7, d(t)/7)$ and $d(t) = 1 + t \bmod 7$ (the day of week).

Let $w_2(t; u_4, u_5) = b(a(t); u_4, u_5)$ where $a(t) = t \bmod 1$ (the time in day).

For $t > t_r$, we define $\lambda(t|H_t) =$

$$w_1(t; u_2, u_3) \times w_2(t; u_4, u_5) \times \exp[u_1 + u_6 \exp\{u_7(t_r - t)\}]$$

where u_1 and u_6 are unrestricted while the other parameters are greater than zero.

3. The parameters in the model have direct interpretations

Each individual has a propensity to make withdrawals linked to the calendar. A constant rate, $\exp(u_1)$, is scaled during the week to take into account the hour of the day, through $w_2(t; u_4, u_5)$, and the day-of-week proportion $w_1(t; u_2, u_3)$.

Each individual also has a propensity to make withdrawals linked to their history H_t, through the previous withdrawal time. Withdrawal rate may be boosted (or reduced) by a multiplying factor $\exp(u_6)$, decaying by $\exp[u_7(t_r - t)]$ following a withdrawal at time t_r. This allows for a form of quick-succession or delayed-succession behaviour.

If u_6 is greater (less) than zero then $\exp(u_1)$ may be interpreted as a lower (upper) bound on the cumulative rate over a week.

If u_2 is greater (less) than u_3 then relatively more (less) withdrawals will occur at the start of the week. If both parameters are equal then the day-of-week distribution will be symmetric around the middle of the week, and if they are both unity then there is no day-of-week effect.

4. Fits are by maximum likelihood, predictions use an empirical random-effects distribution

Fit:

- Fit maximum likelihood parameters $\hat{\boldsymbol{u}}_i$ to data $\boldsymbol{D}_i$ on each individual i
- Use general search algorithm, *e.g.* Nelder-Mead Simplex algorithm such as `nmsmax` in GNU Octave

Predict:

- Approximate random-effects distribution by $\pi(\hat{\boldsymbol{u}})$, a discrete probability mass function with atoms $1/n$ at each $\hat{\boldsymbol{u}}_i$
- Posterior:

$$\hat{p}(\boldsymbol{u}|\boldsymbol{D}_i) = \frac{p(\boldsymbol{D}_i|\boldsymbol{u})\hat{\pi}(\boldsymbol{u})}{\sum_u p(\boldsymbol{D}_i|\boldsymbol{u})\hat{\pi}(\boldsymbol{u})}$$

- Predict some future aspect C of behaviour for $t > T$:

$$p(C|\boldsymbol{D}) = \sum_u p(C|\boldsymbol{D}, \boldsymbol{u})\hat{p}(\boldsymbol{u}|\boldsymbol{D})$$

5. Two methods might be used to detect change in behaviour

Suppose we have observed data $\boldsymbol{D}^1$ on individual i over time period 1 and then new data $\boldsymbol{D}^2$ arrives. Then the observed cumulative rate function in the new time period may be compared to the estimated function $\Lambda(t|\boldsymbol{D}^2, \hat{\boldsymbol{u}})$, where $\hat{\boldsymbol{u}}$ is based on $\boldsymbol{D}^1$.

An alternative approach is to use changepoint detection methods. *i.e.* A changepoint model with 14 parameters, allowing for different behaviour in $\boldsymbol{D}^1$ and $\boldsymbol{D}^2$, may be compared to one with 7 parameters fitted to the complete data $(\boldsymbol{D}^1, \boldsymbol{D}^2)$.

6. The model has been applied to the data sample

1. Estimation: Fits show a range of patterns of behaviour.
2. Prediction: Relative to a Poisson model, a prediction test shows improved performance.
3. Change in behaviour: Martingale residuals provide a way of visualising behaviour. Changepoint methods flagged accounts with changes in rate, time in day and weekday.

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