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## **STRUCTURAL MODEL OF REVOLVING CONSUMER CREDIT RISK**

### **Abstract**

Basel II requires banks to estimate probability of default, loss given default and exposure at default for their retail lending. These measures are taken directly from corporate models of credit risk, which can not be directly transformed into retail lending due to the differences in loan repayment behaviour of individuals and corporations. Personal shocks (e.g. loss of income or marriage distress) are far more important for the individuals than the continuous process of revaluation of their liabilities and assets. It also appears that conventional credit scores can not be directly translated into the Basel II framework.

This paper proposes a framework for developing a model of consumer credit risk that is closely linked with analogous option-pricing based corporate credit risk models, and is capable of estimating risk measures required by the regulator. In order to develop a structural model of consumer default we analyse the volatility of consumer's disposable income and two stochastic variables that can be affected by idiosyncratic shocks: amount of money available to the consumer for repaying the debt (subject to consumer's willingness to pay), and the structure and level of consumer liabilities.

*Key words: Basel II, Consumer credit, Credit risk*

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## 1. Introduction

The advanced IRB approach of Basel II accord requires banks to estimate the probability of default (*PD*), loss given default (*LGD*) and exposure at default (*EAD*) for their retail lending (BIS, 2004). These measures are taken from corporate models of credit risk, which can not be directly applied in retail lending due to the differences in loan repayment behaviour of individuals and corporations. Personal shocks (e.g. loss of income or marriage distress) are far more important for individuals than is the continuous process of revaluation of their liabilities and assets (Thomas, 2003). It also appears that conventional credit scores cannot be directly translated into the Basel II framework (Allen, DeLong and Saunders, 2004; Thomas, 2003).

Many studies investigate actual reasons for consumer defaults. For example, Chakravarty and Rhee (1999) find that credit mis-management and life shocks are the main causes of default. There is, however, no systematic development of a causal theory evaluating consumer credit risk based on these.

In this paper we introduce a causal model of credit risk associated with qualified revolving retail credit risk. The model is based on two main assumptions. The first is that a consumer defaults when servicing of debt becomes unaffordable (or not important for the consumer), i.e. the lender's demand for debt repayment exceeds the amount of money allocated by the consumer for the servicing of it. The second is that the ability (and willingness) of the consumer to repay debt and the actual level of indebtedness are affected by two major processes: slow and predictable changes in income and expenses, and unexpected personal shocks affecting both income and expenses.

We propose that the credit card issuer (which we will refer to as the "lender" and the "bank" interchangeably) writes a revolving put option which gives the borrower a right to put some of the future expenses to the lender. Since the lender prevents further borrowing when the credit card limit is reached, the maximum total expenses that can be put to the lender is the credit limit (although in rare circumstances the credit limit can be breached). Cost of the option is determined by the lender, it increases as the amount of non-repaid debt increased. Consumer uses disposable income (and liquid wealth, if any) to repay the debt, but when it becomes lower than required loan repayment (cost of the

new option) the credit card account becomes delinquent, which may subsequently lead to a default. The direct exercise price is the amount that can be claimed by the lender from the borrower in case of default, however it is generally observed in practice that in many cases banks cannot recover anything from the defaulted consumer (Chatterjee, Corbae, Nakajima and Rios-Rull, 2002), but there are also indirect cost of default, so that the strike price of the default option is exogenous and the consumer default is not a zero-sum game. In this paper we focus on evaluating PD and LGD, which does not require valuing the option to default itself.

The proposed framework is in some respect analogous to the option-pricing framework proposed by Merton (1974), where a company defaults when the market value of its assets, which can be used to repay debts, becomes less than the value of debts. This framework was further developed by Black and Cox (1976), Margrabe (1978), Longstaff and Schwartz (1995), Vasicek (1991), Zhou (1997), Das, Freed, Geng and Kapadia (2002), Giesecke (2004) and others. Based on the academic refinement of the original Merton model, KMV has successfully developed a commercial product for measuring corporate credit risks (Allen, DeLong and Saunders, 2004). As the value of the borrower's assets can not in general readily be marked-to-market and therefore can not serve as a reliable predictor of default, Merton-like models cannot be directly applied in consumer lending, yet lenders need to have a model that provides an explanation of the consumer default and which conforms to the regulatory standards.

There is a growing literature that adopts the option pricing approach so that it can be applied to the prediction of retail loans default. For example there are a significant number of proposals to apply the option pricing framework for the evaluation of the fixed rate residential mortgages, where the borrower has both call (refinance) and put (default) options, whose value depend on the level of interest rates and the value of the house. (Kau and Keenan, 1995; Ambrose, Capone and Deng, 2001). These models are difficult to use for measuring default risk associated with credit card accounts because credit card debts are generally not secured. If the value of the borrower's assets was the main determinant of default then every consumer whose assets do not have much value should default, a situation which does not generally apply.

Chatterjee et al (2002) propose an economic model where a consumer defaults on an unsecured credit facility if their earnings drop below some variable, which is dependent on the level of debt and level of the expected default punishments. Andrade and Thomas (2004) propose a set of models where a consumer has a call option on his/her good credit rating and the consumer's debt is the strike price. These papers demonstrate that structural models of consumer default can be developed despite the significant data limitations; they do not, however, provide an efficient framework for estimating risk parameters required by Basel II accord.

The remainder of this paper is organised as follows. In section 2 we design a model for studying revolving credit risk. Section 3 discusses the main default drivers and how they affect estimated probability of default and loss given default. Section 4 explores potential applications of the proposed model and Section 5 concludes.

## **Section 2. Model development**

Conventional credit scoring systems use borrowers' personal characteristics to determine the risk of default; these include age, occupation, place of residence, declared income, credit history and so on (Allen, DeLong and Saunders, 2004). Even supposing that these characteristics determine the likelihood of borrower default, the bank is still faced with the challenge of maintaining an accurate record of them. Basel II defines qualifying revolving retail exposures (QRRE) as revolving, unsecured and uncommitted credit facilities extended to the individuals (BIS, 2004). A consumer has a right (but not the obligation) to put expenses to their credit card without providing the bank with an update on their current personal circumstances (Gross and Souleles, 2002). In this situation, the bank can not directly observe changes in the consumer's individual characteristics, such as marital status or actual residential address, but it can observe changes on the consumer's credit card balance.

It is likely that adverse changes in the borrower's circumstances may be observed on the borrower's credit card statement prior to the borrower communicating potential problems to the bank or the actual account or the account becoming delinquent. For example, if the borrower's income drops or the amount of other debts increases, the amount of actually made monthly repayments may

decrease; a change from full time employment to self-employment may result in increased volatility of loan repayments and credit card expenses; or an increase in living expenses may appear on the account statement in the form of the increased volume of daily spending.

There are two main types of transactions that may appear on the borrower's credit card statement: expenses ( $CCE$ ) and credit repayments ( $CCR$ ). We shall think of expenses as a continuous process. Initial observation of a small number of credit card accounts suggests that expense ( $CCE_t$ ) can be presented as a lognormal random variable<sup>1</sup> with parameters  $(\log(\mu_{CCE}), \sigma_{CCE})$  and probability distribution function,  $f_{CCE}(CCE_t; \log(\mu_{CCE}), \sigma_{CCE}, CCB_{t-1}, L)$ , where any single expense transaction is generally limited by the current difference between credit card limit ( $L$ ) and credit card balance ( $CCB$ ).

In order to maintain access to the revolving credit facility the consumer has to make minimum monthly repayments ( $MMR$ ) and maintain the credit card balance ( $CCB$ ) below the credit limit.  $MMR$  is generally a fraction of account balance adjusted for any overlimit amount and any penalties applied to the account (which must be paid in full).

$$MMR_{DD} = kCCB_{DD-PP} + \max(CCB_{DD-PP} - L, 0) + Penalties(fee, interest)$$

$k$  - the specified (small) proportion of the outstanding account balance (e.g. 3%)

$DD$  - date when the payment is due

$PP$  - defined payment period, which is usually equal to 1 month (or 30 days)

(1)

We shall think of the actual repayments as a sequence of independent chance events (Poisson process -  $Y(\lambda)$ ) appearing on any day with average intensity  $\lambda = 1/T$  ( $T$  is length of average period between payments) with amplitudes denoted as  $CCR$ . Actual repayments, when they occur, are equal to the maximum amount the borrower affords/decides to pay<sup>2</sup>.

More formally, the model of a single credit card balance can be expressed in the following terms:

$$CCB_t = CCB_{t-1} + CCE_t - CCR_t Y(\lambda)$$

$$CCB \in [0, L + Penalties]$$
(2)

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<sup>1</sup> This is consistent with the spending pattern, where the consumer continually makes small purchases, such as grocery shopping, and occasional large purchases, such as international airline tickets.

<sup>2</sup> Chatterjee et al, (2002) shows that when financially possible, a rational borrower should repay the loan and maintain access to the credit. By maximising  $CCR$  the borrower can minimise interest charges, we therefore assume that actual monthly repayments demonstrate individual's loan repayment capacity.

In this setup, if the borrower's total repayments ( $T\_CCR_{DD}$ ) within the payment period starting from a statement date are less than the minimum monthly repayment determined on the statement date ( $MMR_{DD}$ ), the account becomes delinquent

$$T\_CCR_{DD} = \sum_{t=DD-PP}^{DD} CCR_t Y(\lambda); \quad (3)$$

$$T\_CCR_{DD} < MMR_{DD} \rightarrow \text{delinquency}$$

Basel II stipulates that the account shall be considered as defaulted if it has been delinquent for 90 days (BIS, 2004). In terms of the typical credit card loan, a default is described as 3 consecutive account delinquencies.

$$\left\{ \begin{array}{l} T\_CCR_{DD} < MMR_{DD}, \\ T\_CCR_{DD+PP} < MMR_{DD+PP}, \\ T\_CCR_{DD+2PP} < MMR_{DD+2PP} \end{array} \right\} \rightarrow \text{default} \quad (4)$$

Under reasonable assumptions one can obtain a formula for probability of a one month account delinquency, with expected delinquent amount and total exposure at that date.

If the borrower makes a repayment with Poisson probability  $P_1$  and this repayment is at least equal to the  $MMR$  with probability  $P_2$ <sup>3</sup>, then the account will not be delinquent at the end of the month. We simplify the analysis by assuming the borrower will make only one single credit repayment during a given month.

It follows that the probability of account delinquency ( $P_{AD}$ ) shall be equal to 1 minus probability of occurrence of sufficient repayment.

$$P_{AD} = 1 - P_1 P_2 \quad (5)$$

In order to estimate the expected amount of account delinquency at the end of the repayment period ( $AoD_{DD}$ ), one needs to sum the current account balance and the amount of further expenses that may be incurred within the following month.

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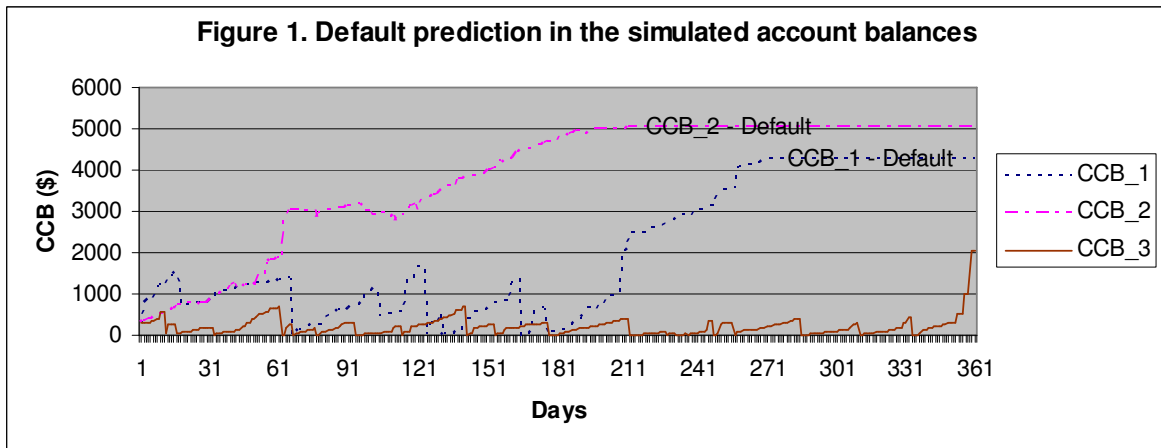
<sup>3</sup> If the assumption that  $CCR$  has a normal distribution holds, and its parameters can be estimated from the available dataset, the probability that at the end of the one month period consumer's  $CCR$  will be above  $MMR$  can be estimated as  $P_2 = prob(CCR_1 > MMR_1) = \Phi((MMR - \mu_{CCR})/\sigma_{CCR})$  where  $\Phi$  is CDF standard normal distribution.

$$AoD_{DD} = CCB_{DD-PP} + \exp(\mu_{CCE} + \frac{1}{2}\sigma_{CCE}^2) \quad (6)$$

Because the borrower can reach the credit limit within one month, exposure at default is assumed to be equal to the borrower's personal credit limit. If we further assume that credit card companies write off all penalties incurred on the account when the borrower defaults, we can exclude penalties from the calculation, therefore:

$$Exposure\ at\ default = L \quad (7)$$

To conclude this section we illustrate dynamics of three simulated accounts (see Figure 1). Account 1 arrives to default because Poisson event of the repayment did not occur within 90 days, account 2 arrives to default because loan repayments were much less than it was demanded by the lender, and third account does not arrive to default.

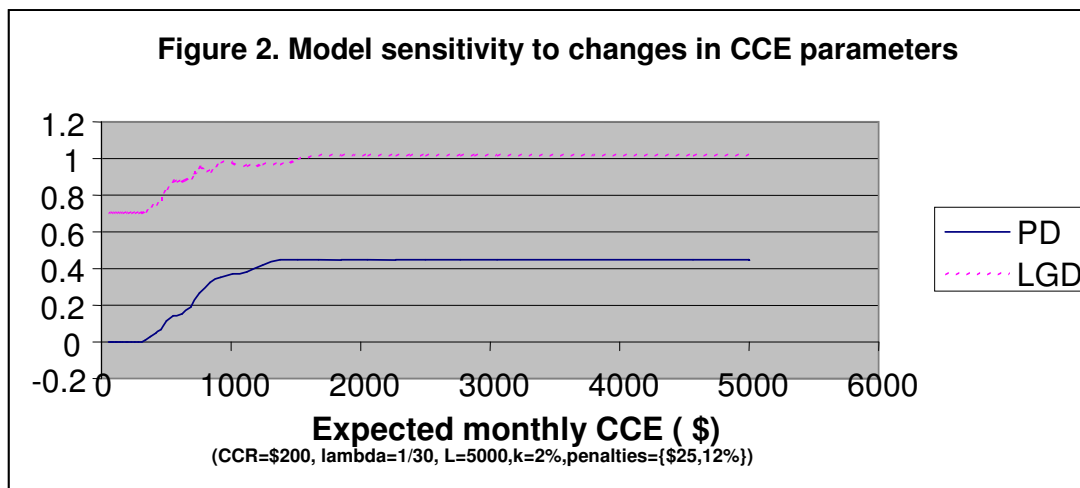


### Section 3. Default drivers.

Equations (2), (3) and (4) show that probability of default and loss given default can be written showing functional dependence on starting credit card balance, consumer expenses, credit card repayments, the Poisson process of occurrence of the credit card repayments, and other parameters specifying credit card specific parameters. In order to illustrate sensitivity of the proposed model to different risk drivers, we simulate the performance of the credit card account under different circumstances.

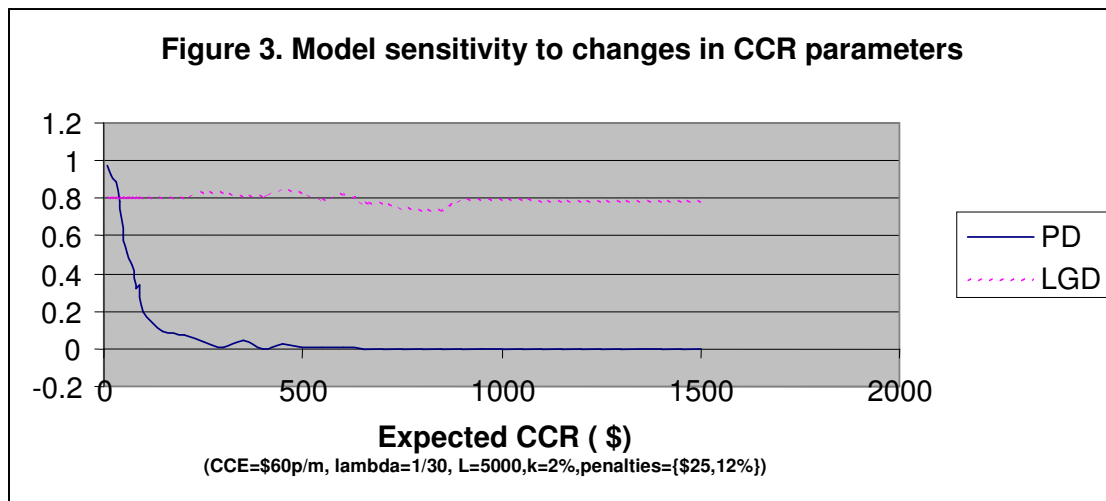
Normally credit card contracts may include additional arrangements. This, for example, can be an option to draw cash from the credit card and make minimum monthly repayments; because this opportunity is costly to the credit card holder, it can be assumed that the borrower will only use it if the account becomes 90 days past due. For the purpose of illustration we assume that only the borrowers whose credit balance is at least 30% below the limit may have this option. We demonstrate below that the results produced by the model are not dissimilar to what one would expect to observe in practice. In accordance with BIS (2004) requirements we constructed our model to estimate *PD* and *LGD* within a one year horizon. We therefore think of *PD* as the probability of 3 month consequent delinquency; and, because in many cases the lender is not able to recover anything from the consumer in default (, we treat the credit card balance at default as *LGD*..

Figure 2 demonstrates that growth in the expected monthly expenses, caused either by increased  $\mu_{CCE}$  or  $\sigma_{CCE}$  results in increased probability of default and loss given default. *PD* is limited to some figure less than 1, because maximum *CCR* is limited to *L* and the amount of monthly repayments is assumed to be constant. If, on the contrary, the credit card did not have a limit, then *PD* would approach 1 as *CCB*, and therefore *MMR*, would grow above the affordability level. With increased  $\mu_{CCE}$  and  $\sigma_{CCE}$ , because of the chance that the borrower will reach the credit limit faster and will incur more charges and penalties, the *LGD* rises together with the increased expected *CCR*.

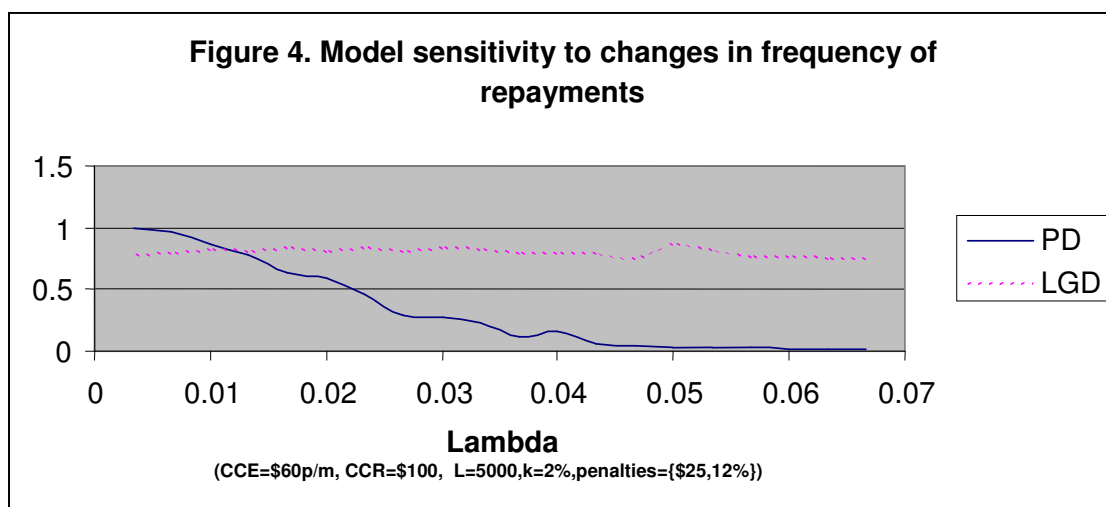


Another important model parameter is an amount of the expected consumer repayments. As the consumer's *CCR* grows the probability of default decreases (See Figure 3). As it was pointed out

earlier, consumer's *CCR* may drop for a number of reasons, which may include partial loss of income or the incurrence of the additional debt, which will require part of the consumer's previous *CCR* to be spent on servicing it. One can note that credits extended to consumers with bad credit history may perform well because the amount of additional credit available to these consumers is generally limited meaning that chances of reduction in *CCR* because extra debts is incurred are comparatively low.



A higher frequency of loan repayments ( $\lambda$ ) results in a lower probability of default (See Figure 4). The more often a consumer pays against a credit card balance, the less chance that the consumer will incur a large amount of debt, which they cannot afford to pay. It is consistent with conventional beliefs that longer maturity of debt implies a higher uncertainty of loan repayment. We think of the increase of  $\lambda$  as a potential result of shocks affecting the borrower's ability to maintain stable payments.



## Section 4. Applications

Planned changes in the regulatory capital requirements and related standards offers a new challenge to banks with a significant retail exposure. Retail loans that share similar risk characteristics must be divided into the pools, and for each pool, banks must estimate the probability of default, the loss given default and the exposure at default. In order to separate revolving loans into the pools, at minimum, banks need to consider individual borrowers and transactional risk characteristics and delinquency of exposures (BIS, 2004).

In response to these Basel II requirements, we propose that consumers may be dynamically allocated to different pools according to the values of the risk drivers and credit card characteristics (e.g. *L*, *structure of penalties*) discussed in the previous sections. Then the bank can do simulations and estimate probability of *PD* and *LGD* for each pool separately. In this way the lender can review accounts on a monthly or quarterly basis, without being dependent on the consumer updating personal details. Then the lender may be able to reallocate consumers to the pools according to their risk characteristics. Basel II requires each pool to be of a reasonably large size (BIS, 2004) so it can be expected that the distribution of the risk determining variables can be approximated to normal and some approximated estimates of the risk measures required by the Basel II accord can be obtained analytically based on the information contained in the borrowers' transactional data.

Although many behavioural scoring models use transactional data for determining default risk, we suggest that the proposed model offers additional capabilities in terms of adjusting the risk factors for the effects of macroeconomic and legal changes. For example, it allows estimation of changes in *PD* and *LGD* based on the assumption that the expected \$*X* rise in fuel prices will result in \$*Z* increase in the consumers' expenses. Impacts of the expected changes in government welfare policies may be measured by altering *CCR* used for estimating the risk parameters.

We also suggest that the proposed model can be used in other spheres of consumer credit risk management and economics. For example, one can use the results in the collections process, where based on the risk parameters used by the model, different credit control techniques can be allocated to

the different pools of loans. Although historic data may not be available for new credit products or products expanded to the emerging markets, one can use the model for assessing expected performance of these loans by altering credit card parameters and tuning the model to the new economic conditions.

#### **4. Conclusion**

We have proposed a framework for measuring credit risk associated with qualifying revolving retail credit exposures. The proposed framework allows for dynamic monitoring of credit risk associated with individual consumers and pools of consumers. As compared to the conventional credit scoring techniques the proposed model obtains risk estimates required by the Basel II directly from the consumer's transactional history. It can be also supposed to be less dependent on the sample data used for developing the model.

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