A Methodology for Estimating Credit Ratings and the Cost of Debt for Business Units and Privately-held Companies

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Abstract

The cost of capital literature contains ample discussions of cost of equity estimation, but little attention is given to the cost of debt. In this paper, we develop a methodology for estimating the market cost of debt for business units and privately-held companies. Better cost of debt estimates should result in more accurate WACC estimates. Therefore, we should be able to make better capital budgeting decisions and create a better alignment between executive compensation and shareholder value creation. The first step in our methodology is the development of a credit rating model for business units and privately-held companies. We collected corporate credit rating grades from Standard & Poor's and Moody's and accounting information for a sample of 627 American companies. We ran a stepwise ordered logit model to select the variables that better explain agency credit ratings. We found that the most important variables in determining credit ratings are size, financial leverage, operating performance and volatility. The model classified correctly 58.14% of the sample in the appropriate rating class, 19.3% in the immediately higher category, and 19.3% in the immediately lower neighbor. Only 3.26% of the sample was classified in a rating grade distant from the right category. Using as an input the Business Unit's or Privately-held Company's accounting variables and running the credit rating model, we are able to classify the Business Unit or Privately-held Company in a rating category. The second step is to attribute a cost of debt to the estimated rating according to its maturity. We do so by using the Bloomberg industrial corporate yield curve by rating category. These yield curves are updated daily, so our cost of debt can be constantly updated.

Key words: cost of debt, credit rating, ordered logit, market cost of debt

I. Introduction

Cost of capital is strategically important to companies. Management accepts or rejects a project based on its Net Present Value (NPV). If the manager estimates a very low cost of capital, she tends to accept risky projects without a proper remuneration, and the overall company risk may increase. If she estimates a very high cost of capital, she will tend to reject strategically interesting projects. Both situations can damage company long term performance and survival. Nowadays many companies compensate their executives based on shareholder's value creation metrics, as for instance EVA® Cost of capital is one of the key drivers of value creation metrics. Therefore it is important to make properly risk adjusted estimates of cost of capital in order to align corporate strategy to shareholder's value creation and executive compensation.

The most popular framework to value projects consists on discounting free cash flows to firm by the Weighted-Average Cost of Capital (WACC). WACC is composed by the cost of equity and the cost of debt, and the weight of each component is defined by the capital structure. The cost of capital literature contains ample discussions of cost of equity estimation, but little attention is given to the cost of debt. In order to estimate the cost of debt, managers usually use historical interest rates of existing debt. The historical cost of debt may differ significantly of the interest rate managers would face to finance new projects with debt, that is, the market cost of debt.

In this paper, we develop a credit rating model for Business Units and Privately-held companies. We collected corporate credit rating grades from Standard & Poor's and Moody's and accounting information for a sample of 627 American companies. We ran a stepwise ordered logit model to select the variables that better explain agency credit ratings. Using input Business Unit's or Privately-held Company's accounting variables, and running the credit rating model, we are able to classify the Business Unit or Privately-held Company in a rating category. We then attribute a cost of debt to the estimated rating according to its maturity. We do so by using the Bloomberg industrial corporate yield curve by rating category.

The remainder of this paper is organized as follows. In Section II we discuss the relationship among credit rating, credit quality and cost of debt. In Section III we present the methodology used to estimate credit ratings based on accounting and market variables. In Section IV we illustrate how we can relate the credit rating to the cost of debt and maturity and in Section V we conclude the paper.

II. Credit rating, credit quality and the cost of debt

Players in the financial market rely heavily on the rating agencies opinion about the creditworthiness of a company or a security. Very seldom a company issues debt without a previous assessment of its credit quality by a rating agency. Credit ratings are public information, and represent the judgment of experienced and presumably well informed analysts. Yield-to-maturity of fixed income securities are strongly correlated to credit rating. Well rated securities have significantly lower yields than poor rated securities.

Standard& Poor's (2006) defines issuer credit rating as a current opinion about an obligor's overall capacity to meet its financial commitments. This opinion takes into account the capacity and willingness to meet financial commitments as they come due, and it is not specific to any particular financial obligation. This opinion does not take into account the specific nature or provision of any particular obligation, statutory or regulatory preferences, creditworthiness of guarantors, insurers, or other forms of credit enhancement. Moody's Investor Service (2007) defines issuer credit rating as an opinion about the ability of entities to honor senior unsecured financial obligations and contracts. Agencies provide corporate issuer credit rating for companies and sovereign issuer credit rating for countries. Agencies also take into account country risk factors. External obligations have a higher default probability than domestic obligations, and therefore agencies provide ratings in local currency and ratings in foreign currency. Usually the foreign currency rating of a specific obligor is lower than the local currency rating. Issue credit rating is an opinion that relates to a specific financial obligation and that takes into account the specific nature or provision of this obligation.

Agencies' opinions are built on qualitative and quantitative information furnished by the obligor or reliable sources. As long as the information changes, credit ratings may be changed. If there is no sufficient information, agencies may suspend or withdraw credit ratings.

Figure I contains credit rating symbols and summarizes their interpretation. According to Altman, Caouette and Narayanan (1998), Standard&Poor's credit opinion takes into consideration business risk (industry characteristics, competitive position, management) and financial risk (financial characteristics, capital structure, profitability, cash flow coverage ratios, financial flexibilities). Industry risk (attractivity and stability of the obligor's industry) has a significant weight in the rating process, according to Standard&Poor's. Moody's also emphasizes business fundamentals, as the supply and demand characteristics, market leadership and cost position. Standard&Poor's calculates and monitors in a regular basis many accounting ratios to assess financial risk: interest rate coverage, financial leverage and cash flow. Although there may be some divergences in ratings provided by Moody's and Standard&Poor's, in most cases they are convergent, at least at the level of the big letter. The mortality rates are also very similar for Moody's and Standard&Poor's rating categories.

Figure II contains the accumulated mortality rates by rating category built from the transition matrixes disclosed by Moody's Investor Service (2004) in the level of the big letter (Aaa, Aa, A, Baa, Ba, B, Caa). We can observe that the mortality rate is inversely related to the obligor credit quality and increase significantly for speculative ratings.

Interest rates are related to credit ratings. Bloomberg builds indexes of debt securities based on rating categories for industrial American companies and then estimates industrial yield curve by rating categories. Figure III contains the data of the yield curves in November 1st 2005. One can observe that the yield-to-maturity decreases as the credit rating improves, and this happens for any maturity. We can also note that yields were increasing with maturity.

The credit rating disclosure is an important event. Therefore it is important to understand what determines credit ratings.

III. Methodology

III.1. Literature Revision

Horrigan (1966) was one of the pioneers in developing models to estimate and forecast fixed income securities' credit ratings based on obligation's and obligor's characteristics. He used OLS regressions and codified the dependent variable (rating category) in a nine- points scale. Nine was assigned to the highest rating category (AAA or Aaa) and one to the lowest rating category (C). He selected the following explaining variables: total asset, book value of equity to total liability ratio, operating profit margin, working capital to sales ratio, sales to book value of equity ratio. He also used a dummy for representing obligation subordination status. The dummies and the total assets were the most significant explaining variables. The six independent variables together explained 65% of dependent variable variations. The model classified correctly 55% of the issues' ratings. Only few obligations were classified in a rating distant from their actual rating category.

West (1970) used the same dependent variables as Horrigan (1966), but ran logistic regressions. He tested the following explaining variables: profit variability coefficient (he used the profit historical series in the 9 previous years), confidence (number of years without losses to creditors), capital structure (market value of equity to total liability ratio) and market value of equity. The forecast power of the model was similar to Horrigan's.

Pinches and Mingo (1973) used multiple discriminant analysis to estimate issue's credit ratings. They built an estimation sample with 132 securities and a test sample with 48 securities issued between 1967 and 1968. The securities were classified by Moody's in rating categories between Aa and B. The authors ran a factor analysis to identify the financial and accounting variables that better explained rating categories. They selected the following characteristics: size, financial leverage, long term and short term capital intensity, return on investment, profit stability and interest coverage ratios. Long term and short term capital intensity were insignificant in explaining the rating categories. The forecast model used the following factors: issue size, long term liability to asset ratio (five year average), return on asset, consecutive years with dividend payments, net profit plus interest expenses to interest expenses ratio and a dummy variable for the subordination status. The subordination status dummy was the most relevant variable in estimating credit rating, followed by consecutive years of dividend payment and issue size. In the test sample, 65% of the obligations were correctly classified and none of them were classified in a rating category distant of more than the next neighbor rating category.

Altman and Katz (1976) used multiple discriminant analysis to estimate credit ratings for electric distribution companies. The variables that contributed the most to the discriminant function were interest coverage ratio, profit variability, interest coverage ratio variability, return on investment, and maintenance and depreciation expenses to operational revenue ratio. The model classified correctly 80% to 90% of the estimation sample securities.

According to Kaplan and Urwitz (1979), ordinary least square (OLS) and multiple discriminant analysis have relevant limitations to estimate credit ratings. OLS regressions assume that ratings represent equal intervals in a measurement scale, but this assumption does not hold in real life. Multiple discriminant analysis assumes that ratings are measured in a nominal scale, what is not a satisfactory assumption either according to the rating process. The authors understand that, in establishing a rating to an obligation, analysts try to measure the default risk or probability of default. It is not possible for the analysts to measure the default risk in a scale interval, but they make an ordinal ranking of the issues. This ranking implies that securities classified as AAA are less risky than securities classified as AA, and so on. They expect a higher number of defaults in worst rating categories than in better rating categories. Therefore it is not likely that the rating process results in equal intervals as assumed by OLS. The problem with discriminant analysis is the assumption that ratings contain only nominal information. Besides that, discriminant analysis assumes multivariate normality for independent variables, and the technique does not provide appropriate significant tests for the coefficients.

Therefore, Kaplan and Urwitz (1979) used ordered logit models to estimate credit ratings. The dependent variable is treated as a latent variable, because we observe the rating, but we do not observe the credit quality or default probability. The authors analyzed the following variables: (i) Interest coverage ratios: cash flow before interest expenses and taxes divided by interest expenses; cash flow before interest expenses and taxes divided by total debt; (ii) Capitalization indexes: total debt divided by total assets; long term debt divided by book equity; (iii) Size variables: total assets; issue size;(iv) Stability variables: coefficient of total asset variability; coefficient of profit variability;(v) Subordination: dummy variable indicating the subordination status; (vi) Market variables: beta coefficient and residual of the market model regression. According to the authors, the specific risk or regression error can be interpreted as a proxy for management ability.

The subordination dummy and size were very relevant to the model. The coverage ratios were insignificant. Beta was significant, while regression residual was insignificant. The model classified correctly 74% of the sample.

III.2. The logistic ordered model

The logistic ordered model, as explained by Greene (2002) is a latent variable model. According to the model it is not possible to observe the true value of the interest dependent variable Y, but it is possible to observe the dependent variable Z, which contains information about variable Y. The model assumes that the interest variable (default risk) is in a scale interval, and if it were possible to measure it, it would satisfy a linear model. But it is only possible to observe an ordinal version of Y which we denominate Z (credit rating). Z does not satisfy a linear model. Formally we have that:

$$Y = X\beta + \varepsilon \qquad (1)$$

Where ε is a vector of the error terms independent and identically normal distributed, that is, $\varepsilon \sim N(0, \sigma I)$. Suppose Z is a categorical variable with M categories (each M corresponds to a rating category), denominated $R_1, \ldots, R_M$, derived from the non observable variable Y. There are M + 1 postulated numbers, $\mu_0, \mu_1, \ldots, \mu_M$, with $\mu_0 = -\infty$ and $\mu_M = +\infty$ and $\mu_0 \leq \mu_1 \leq \ldots \leq \mu_M$ in such a way that $\mu_{k-1} \leq Y_j \leq \mu_k \leftrightarrow Z_j \in R_k$ for $1 \leq j \leq N$. X_j is the independent variable vector $(k+1) \times 1$ of company j ($X_{0j} = 1$). We have that:

$$\mu_{k-1} < Y_j \leq \mu_k \Leftrightarrow \mu_{k-1} < \beta X_j + \varepsilon_j \leq \mu_k \Leftrightarrow \frac{\mu_{k-1} - \beta X_j}{\sigma} < \frac{\varepsilon_j}{\sigma} \leq \frac{\mu_k - \beta X_j}{\sigma} \qquad (2)$$

and

$$\Pr(\mu_{k-1} < Y_j \leq \mu_k) = \Phi\left(\frac{\mu_k - \beta X_j}{\sigma}\right) - \Phi\left(\frac{\mu_{k-1} - \beta X_j}{\sigma}\right) \quad (3)$$

Where Φ (.) is a cumulative distribution function for a standardized random variable. The model is super identified, because any linear transformation of the underlying scale variable Y, if also applied to parameters μ_0, μ_1,…, μ_M, will result in the same model. In order to identify the model, we assume, without lost of generalization, that $\mu_1 = 0$ and $\sigma = 1$. The estimated model will be:

$$\Pr(\mu_{k-1} < Y_j \leq \mu_k) = \Phi(\mu_k - \beta X_j) - \Phi(\mu_{k-1} - \beta X_j) \quad (4)$$

It will be necessary to estimate M + K – 1 parameters: μ_1, …, μ_{M-1} and β_0, β_1, …, β_k. Suppose Φ (.) is a logistic distribution. In this case we have that:

$$\Pr(Y_j \leq \mu_k) = 1/(1+e^{X\beta-\mu k}) \quad (5)$$

$$\Pr(Y_j > \mu_k) = 1-1/(1+e^{X\beta-\mu k}) \quad (6)$$

$$\Pr(\mu_{k-1} < Y_j \leq \mu_k) = 1/(1+e^{X\beta-\mu k}) - 1/(1+e^{X\beta-\mu k-1}) \quad (7)$$

The log-likelihood function is:

$$\ln L = \sum_{j=1}^{n} \sum_{k=1}^{m} Z_{jk} \ln(1/(1+e^{X\beta-\mu_k}) - 1/(1+e^{X\beta-\mu_k-1})) \quad (8)$$

III. 3. Sample and Results

We collected accounting and market information data from 627 American industrial companies, which were rated by Moody’s and Standard&Poor’s. We used Eonomatica database for accounting and market information and Moody’s and Standard Poor’s sites for credit rating grades.

We calculated and tested the following explanatory variables: (i) size variables: ln (total asset) and ln(book value of equity); (ii) Financial leverage variables: total liabilities/ total assets, total liabilities/ book value of equity, book value of equity/ total assets, total debt/ total assets, total debt/ book value of equity; (iii) Solvency ratios: EBIT/ net debt, EBITDA/ total liabilities, (current assets – current liabilities)/ total assets EBIT/ total assets, (iv) Operational performance variables: ROA = net Profit/ total assets, Asset turnover = operating income / total assets, Operating margin = EBIT/ net income; (v)Stability variables: Beta coefficient (β), volatility of stock returns (σ) = standard deviation of the past 12 monthly stock returns, specific risk = $\sigma_i^2 - \beta_i^2 \times \sigma_M^2$,where σ_M is the standard deviation of S&P 500 monthly, returns in the past 12 months, Unlevered beta coefficient = $\beta_{stock}/(1\text{-}D/E^*(1\text{-}T_C))$ where D/E is the debt to equity ratio and T_C is the corporate tax rate.

Although Kaplan and Urwitz (1979) analyzed issue ratings, we decided to analyze issuer rating and we did not include the dummy variable for seniority. The reason for this decision is that our main purpose is to estimate the market cost of debt of privately-held companies and Business Units. Higher seniority and higher credit enhancement increase rating grades and that cause lower interest rates. But by the other side, higher credit enhancement also increases the expenses of the debt. The savings are interest rate is roughly compensated by the increase in debt expenses. Therefore what the company will pay for the debt is roughly the same as if it were senior unsecured debt.

Figure IV relates the categorical variable Z to each credit rating. Z is the dependent variable. Ratings were consolidated at the big letter and we did not observe divergence in rating grades between Moody’s and Standard& Poor’s at this level.

We ran a stepwise analysis and the following explaining variables were selected: (i)Size variable: ln(assets); (ii)Financial leverage variable: debt/ total assets, (iii)Solvency ratio: EBIT/net debt, (iv)Operational performance variables: ROA and EBIT/Net income and (v)Stability variable: volatility.

Figure V contains the ordered logit model results. We can observe that ln(assets), debt/total assets, ROA and volatility were significant and the coefficient signals were in accordance with expectations. The higher the size and the better is the operating performance, the lower is the Z variable, indicating that the better is the rating. The higher is financial leverage and the higher is the volatility, the higher is Z, so the worst is the rating. Variable EBIT/ net debt and EBIT/ Net income were not significant at a 5% level. We expected for both variables negative signs, but they presented positive signs. These variables were not significant in Kaplan and Urwitz (1979) either.

The model classified correctly 58.14% of the sample, 19.30% of the sample was classified in the immediate superior neighbor category (for instance, if the correct rating was A, the observation was classified as AA), and 19.30% in the immediate lower neighbor category (for instance, if the correct rating was A, the observation was classified as BBB). Only 3.26% of the sample was classified in rating categories more distant than the immediate neighbors.

This methodology is a way of mirroring external rating. It is also useful to estimate Privately-held companies' ratings to satisfy regulatory needs for financial institutions. The New Basle Agreement (Basle II) accepts three approaches in order to determine Economic Capital (the minimal capital required to face credit, operational and market risks): Standard Approach, Basic Approach and Advanced Approach. The Standard Approach is based on agencies credit ratings to assess corporate credit risk, but there are many companies that are not rated by the agencies. One way to create internal rating systems, as suggested by Servigny and Renault (2004) is to mirror external rating, as we propose to do.

IV. Exemplifying the methodology for estimating the market cost of debt

In order to illustrate how we could estimate market cost of debt for Privately-held companies and Business Units, consider the fictitious company ZETA, with the following explaining variables: (i)ln(assets) = 16.4650; (ii)debt/ total assets = 0.3035; (iii)EBIT/net debt = 6.1212; (iv)ROA = 0.027 and EBIT/Net income = 0.0637; (v)volatility = 0.4.

The first five variables were calculated based on financial statements. The volatility of the value's returns of company ZETA was estimated by a Monte Carlo simulation as suggested by Copeland and Antikarov (2003).

Figure VI contains the probability that company ZETA belongs to each of the rating categories. We estimated these probabilities using equations (5), (6) and (7) with the estimated parameters disclosed in Figure V.

We classify company ZETA as BBB or Baa, since it is the rating that the company has the highest probability to belong to.

Suppose company ZETA would issue four years maturity zero coupon bonds in November 1st 2005. According to Figure III, the YTM for a BBB security with four years of maturity is 5.35%. As Bloomberg updates the industrial yield curves by rating categories daily, we could use them for estimating up-to-date market cost of debt.

V. Conclusion

The methodology presented in this paper is simple and can be used to estimate market cost of debt. It is possible to update the cost of debt in a regular base, since Bloomberg updates daily its yield curves per ratings categories. It is a strategically interesting tool, because better estimates of cost of capital lead to better capital budgeting decision and better alignment between shareholder's value creation and executive's compensation.

The most significant variables in explaining ratings were size (ln(assets)), financial leverage (debt/total assets), ROA (net income/ total assets) and volatility. This is in accordance with the existent literature.

Understanding rating determinants can be used to predict the impact of increasing the use of debt in the cost of capital, and to estimate the optimal capital structure.

The logit ordered model can also be used to estimate issue rating of credit obligations portfolio for regulatory purpose. But for this purpose, it would be advisable to include in the analysis variables related to credit enhancement and use as dependent variables issue's ratings instead of issuer's ratings.

One limitation of the methodology developed here is that we did not include in the analysis country risk. As the companies became more global, it should be interesting to consider the impact of multinationals business units. This is a possible expansion of this work, and it would be necessary to collect issuer's ratings from companies around the world and control for the countries' effects.

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Figure I. Ratings' Definition

Investment Grade			*Speculative Grade*		
S&P and other agencies	Moody's	Interpretation	S&P and other agencies	Moody's	Interpretation
AAA	Aaa	The highest credit quality, with minimal credit risk. The capacity to meet financial commitments is extremely strong.	BB+ BB BB-	Ba1 Ba2 Ba3	Speculative elements and subject to substantial credit risk. Less vulnerable to nonpayment than other speculative issues. However faces major ongoing uncertainties or exposure to adverse business, financial or economic conditions that could lead to inadequate capacity to meet its financial commitments
AA+ AA AA-	Aa1 Aa2 Aa3	High quality, with very low credit risk. The capacity to meet financial commitments is very strong.	B+ B B-	B1 B2 B3	Speculative and subject to high credit risk. The obligor currently has the capacity to meet its financial commitments. Adverse business, financial or economic conditions likely will impair the capacity or willingness to meet financial commitment.
A+ A A-	A1 A2 A3	Upper-medium grade and subject to low credit risk. Somewhat more susceptible to the adverse effects of changes in circumstances and economic conditions than obligations in higher rated categories. However, the capacity to meet its financial commitments is still strong.	CCC+ CCC CCC- CC	Caa1 Caa2 Caa3 Ca	Poor standing and subject to very high credit risk. Vulnerable to nonpayment and is dependent on favorable business, financial and economic conditions to meet financial commitment. In the event of adverse business, financial, or economic conditions will not likely have the capacity to meet financial commitment.
BBB+ BBB BBB-	Baa1 Baa2 Baa3	Subject to moderate credit risk. Considered medium-grade. Adequate protection parameters. However, adverse economic conditions or changing circumstances are more likely to lead to a weakened capacity to meet financial commitments.	C	Ca	Typically in default, with little prospect for recovery of principal or interest. Bankruptcy petition has been filed or similar action has been taken but payments on the obligation still are being continued.
			D		Default

Source: Authors' compilation based on Standard& Poor's (2006) and Moody's Investor Service (2007)

Figure II – Accumulated Mortality Rates in % by Rating Grades (1970-2003)

S&P and others	Moody's	One year after issuance									
		1	2	3	4	5	6	7	8	9	10
AAA	Aaa	0	0	0	0.04	0.12	0.21	0.3	0.41	0.52	0.63
AA	Aa	0	0	0.03	0.12	0.2	0.29	0.37	0.47	0.54	0.61
A	A	0.02	0.08	0.22	0.36	0.5	0.67	0.85	1.04	1.25	1.48
BBB	Baa	0.19	0.54	0.98	1.55	2.08	2.59	3.12	3.65	4.25	4.89
BB	Ba	1.22	3.34	5.79	8.27	10.72	12.98	14.81	16.64	18.4	20.11
B	B	5.81	12.93	19.51	25.33	30.48	35.1	39.45	42.89	45.89	48.64
CCC-C	Caa-C	22.43	35.96	46.71	54.19	59.72	64.49	68.06	71.91	74.53	76.77

Source: Moody's Investor Service (2004)

Figure III. American Industrial Yield Curve by Rating Categories (November 1st 2005)

Maturity years	US T-Strip	US Ind. AAA	US Ind. AA	US Ind. A	US Ind. BBB	US Ind. BB	US Ind. B
0.25	3.91%	4.26%	4.36%	4.49%	4.82%	5.30%	5.91%
0.5	4.26%	4.39%	4.50%	4.60%	4.91%	5.33%	6.08%
1	4.36%	4.69%	4.70%	4.77%	5.02%	5.44%	6.40%
2	4.36%	4.71%	4.74%	4.86%	5.12%	5.81%	6.75%
3	4.43%	4.72%	4.76%	4.87%	5.25%	6.16%	7.09%
4	4.46%	4.75%	4.81%	4.93%	5.35%	6.40%	7.39%
5	4.47%	4.83%	4.89%	4.99%	5.39%	6.62%	7.54%
7	4.52%	4.93%	5.00%	5.11%	5.56%	6.91%	7.73%
8	4.60%	4.99%	5.05%	5.18%	5.64%	7.05%	7.79%
9	4.63%	5.04%	5.11%	5.24%	5.72%	7.11%	7.78%
10	4.69%	5.11%	5.17%	5.30%	5.83%	7.23%	7.77%
15	4.85%	5.37%	5.40%	5.58%	6.10%	7.39%	8.08%
20	4.86%	5.47%	5.51%	5.68%	6.21%	7.40%	8.04%
25	4.79%	5.43%	5.52%	5.69%	6.20%	7.33%	8.00%
30	4.66%	5.30%	5.54%	5.70%	6.25%	7.34%	8.03%

Source: Bloomberg

Figure IV – Categorical Variable Z Assigned to Rating Category

Rating S&P	Moody's	**Categorical Variable Z**
AAA	Aaa	**1**
AA	Aa	**2**
A	A	**3**
BBB	Baa	**4**
BB	Ba	**5**
B	B	**6**
CCC	**Caa**	**7**

Source: Authors' analysis

Figure V – Ordered Logit Result for Rating Classification

Explanatory Variable	Coefficient	t- statistic
ln(assets)	-0,6899	-7,23
Debt/ Total Assets	4,4294	6,40
EBIT/ net debt	0,0013	1,25
ROA	-13,3429	-7,18
EBIT/ Net income	0,2938	0,84
Volatility	9,3877	11,14
μ_1	-12,7225	
μ_2	-10,7605	
μ_3	-7,9505	
μ_4	-5,0147	
μ_5	-1,5903	
μ_6	2,2964	
Ln(maximum likelihood)	-432,0123	
LR	369,6700	

Figure VI. Probability of Company ZETA belonging to different rating categories

Categorical Variable Z	Rating Category	Probability
1	CCC or Caa	0.22%
2	B	1.31%
3	BB or Ba	18.94%
4	BBB or Baa	62.43%
5	A or A	16.43%
6	AA or Aa	0.65%
7	AAA or Aaa	0.01%

Source: Authors' analysis