A new model to measure the creditworthiness of borrowers in fixed term loans and revolving financial products

Lorenzo Quirini
Consum.it Risk Manager

Edinburgh, 29-31 August 2007

Consum.it

The company of the group "Monte dei Paschi di Siena" specialized in consumer credit market

Consum.it

-1999 Consum.it began its business in purpose loans (especially new and used cars), 91 million euros granted, 0.5% Italian market share
-2002 revolving credit cards
-2003 unsecured personal loans
-2006 charge cards, 2.7 billion euros granted, 5.1% Italian market share

An important issue:
the development of internal models and procedures to control and to manage the credit risk of the incoming production

The basic idea…

… Credit Reliability Index (CRI)

The CRI measures how a customer respects his financial obligations

The Credit Reliability Index for fixed term loans

Definition

Given the application time = 0

$$X_t = \frac{\sum_{h=1}^{t} R_h (1+i)^{-h}}{\sum_{h=1}^{t} r_h (1+i)^{-h}}$$

R_h the random payment at time h
r_h the contractual obligation at time h
i the contractual monthly interest rate
t the observation time

1
CRI as the fixed (non random) amount of each instalment

If $R_h = a r_h \ (a \leq 1)$ then

$$X_t = \frac{\sum_{h=1}^{t} R_h (1+i)^{-h}}{\sum_{h=1}^{t} r_h (1+i)^{-h}} = \ldots = a$$

2

The average CRI as the probability to pay, at any time, each instalment

p = Probability in paying the instalment

$$E(X_t) = E\left(\frac{\sum_{h=1}^{t} R_h (1+i)^{-h}}{\sum_{h=1}^{t} r_h (1+i)^{-h}}\right) = \ldots = p$$

3

$\log_v(CRI)$ as the number of the (non random) systematic delayed monthly payment

$v = \frac{1}{1+i}$, i the contractual i.i.r.

$b =$ number of delayed months

$$X_t = \frac{\sum_{h=1}^{t} r_h (1+i)^{-(h+b)}}{\sum_{h=1}^{t} r_h (1+i)^{-h}} = v^b$$

4
A relationship between CRI and default

$$E(X_T) = (1 - p_D) \cdot 1 + p_D \cdot \left(\frac{\delta \cdot (1+i)^{-T} + \sum_{h=1}^{m_1} r_h \cdot (1+i)^{-h}}{f} \right)$$

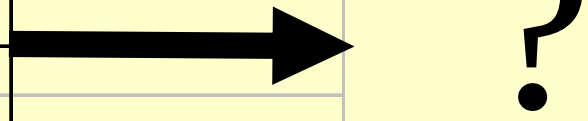
T fixed term
P_D default probability in [0, T]
m_1 last payment
f loan amount
δ recovery rate
i monthly interest rate

The Credit Reliability Index for Revolving Credit Cards

The revolving credit card as a stochastic process: an example

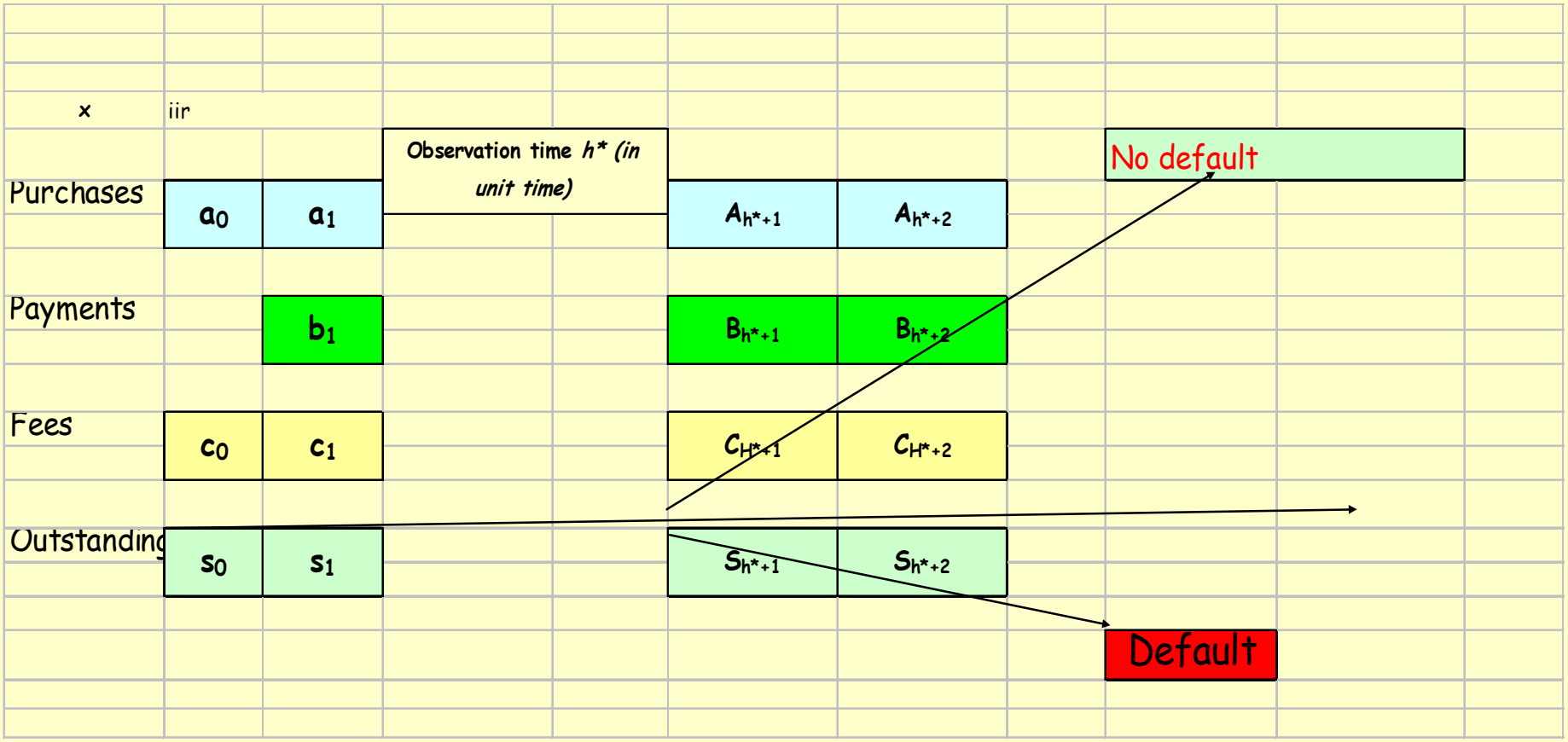
1.33%	monthly interest internal rate			
Purchases	**1589.70**	**437.20**	**0.00**	**100.00**
Payments		**80.00**	**80.00**	**80.00**
Fees	**32.29**	**14.29**	**0**	**5.29**
Outstanding	**1639.24**	**2036.12**	**1983.40**	**2034.20**

Observation time *h*=4 month*



?

The revolving credit card as a stochastic process: the model



The model

deterministic

$$s_0 = a_0 + c_0$$

$$s_h = s_{h-1}(1+x) + a_h + c_h + b_h$$

$$a_0 < 0, a_h \leq 0; c_0 < 0, c_h \leq 0; b_0 = 0, b_h \geq 0; s_0 < 0, s_h \leq 0$$

$$for\ h = 1, 2, \ldots, h^*$$

random

$$S_h = S_{h-1}(1+x) + A_h + C_h + B_h$$

$$A_h \leq 0, C_h \leq 0, B_h \geq 0, S_h \leq 0$$

$$for\ h = h^* + 1, h^* + 2, \ldots, t$$

The model

h^* the observation time

There are only two possible cases:

1) The cardholder will complete regularly his lifecycle at a random time T

2) The cardholder will fall into a default (absorbing) state at a random time T

$T = h^*+1, h^*+2, \ldots, \omega$

Case 1): no default

The CRI's value is 1, *id est*, the contractual obligations will be fulfilled

$$X_{eh^*}(t)=1$$

Case 2): the cardholder will become defaulter at time $T = t$

$$X_{dh^*}(t) = \frac{+\sum_{h=0}^{h^*} b_h (1+x)^{t-h} + \sum_{h=h^*+1}^{t} B_h (1+x)^{t-h} - K_t S_t}{-\sum_{h=0}^{h^*} a_h (1+x)^{t-h} - \sum_{h=h^*+1}^{t} A_h (1+x)^{t-h}}$$

CRI (the expectation)

$$cri_{h^*} = 1 \cdot p_{eh^*} + \sum_{t=h^*+1}^{\omega} p_{dh^*}(t) \cdot E(X_{dh^*}(t))$$

with

$$p_{eh^*} = \sum_{t=h^*+1}^{\omega} p_{eh^*}(t)$$

$p_{dh^*}(t)$ the probability that the cardholder, regular at time h^* ,will become a defaulter at time t

$p_{eh^*}(t)$ the probability that the cardholder, regular at time h^*, will end his lifecycle at time t

How to estimate
$p_{dh^*}(t)$, $p_{eh^*}(t)$, K_t ?

$p_{dh*}(t)$, $p_{eh*}(t)$ can be estimated,for example, by mortality tables with several elimination causes

$$K_t = \frac{\sum_{k \geq t} R_k (1+x)^{-(k-t)}}{-S_t}$$

t default time

R_k payments at time k, with k≥t

x interest rate

S_t outstanding at time t

A simple simulation

The parameters for the simulation:
Annual contractual interest rate: 0.18
ω = 25th month
Number of revolving cards: 3000
K_t a uniform random variable in [0, 0.6]
$P_{dh*}(t)$ constant and equals to 0.01
$P_{eh*}(t)$ constant and equals to 0.04
A_0 first purchase as a uniform random variable in [-2400, 0)
A_h follow purchase as a uniform random variable in [-800, 0) h =1, …, ω-1

The double mortality table

Time	Pop	Def.	Reg.	K_t
0	3000	0	0	
1	3000	27	109	0.280
2	2864	41	119	0.286
3	2704	27	97	0.279
...				
12	1749	24	64	0.291
...				
23	982	13	39	0.214
24	930	11	28	0.279
25	891	0	891	

Cash flow for defaulters at time 12

Default Time		Purchases	Payments	Fees
12	0	-25923.17	0.00	-24.00
12	1	-10368.03	7414.09	-24.00
12	2	-8715.78	8230.73	-24.00
12	3	-10449.54	8157.81	-24.00
12	4	-10099.17	8473.60	-24.00
12	5	-10185.19	10257.58	-24.00
12	6	-10068.74	9181.53	-24.00
12	7	-11601.57	9178.37	-24.00
12	8	-9576.05	9342.18	-24.00
12	9	-10175.21	9285.29	-24.00
12	10	-11022.67	9566.73	-24.00
12	11	-8432.70	11150.70	-24.00
12	12	0.00	12720.69	0.00

CRI. 0.8018
prob. def. 0.0080
Yearly iir -0.6861
Loss -23935.74 euro

Cash flow for customers who have ended regularly by time 24

Time		Purchases	Payments	Fees
25	0	-2032366.00	0.00	-1660.00
25	1	-620213.50	656230.80	-1551.00
25	2	-579627.00	658261.30	-1432.00
25	3	-521423.70	626124.80	-1335.00
25	4	-501907.10	608484.60	-1237.00
25	5	-471739.10	538017.70	-1167.00
25	6	-448971.70	535698.00	-1075.00
25	7	-394624.70	521953.70	-982.00
25	8	-358213.90	480523.50	-891.00
25	9	-319892.40	467626.80	-804.00
25	10	-294004.90	408461.30	-726.00
25	11	-271436.10	352952.50	-666.00
25	12	-240165.90	334293.70	-602.00
25	13	-209034.70	348410.70	-524.00
25	14	-170471.70	315944.00	-450.00
25	15	-165551.60	272689.80	-392.00
25	16	-134363.20	264786.80	-336.00
25	17	-120863.50	196560.80	-292.00
25	18	-91797.71	212209.70	-241.00
25	19	-78529.86	174250.00	-196.00
25	20	-60319.35	157098.80	-155.00
25	21	-46617.63	133179.70	-115.00
25	22	-24691.77	134009.90	-67.00
25	23	-11167.57	107888.50	-28.00
25	24	0.00	59422.07	0.00

CRI. 1.0000

Yearly iir 18.00
Profit 380158.40 euro

Cash flow for customers at risk at time 24

		Purchases	Payments	Fees
26	0	-1058830.00	0.00	-891.00
26	1	-338978.30	295269.00	-891.00
26	2	-361308.20	304546.50	-891.00
26	3	-355251.80	314413.00	-891.00
26	4	-348015.00	317796.00	-891.00
26	5	-352028.50	319908.70	-891.00
26	6	-356228.10	320588.30	-891.00
26	7	-359548.50	322132.20	-891.00
26	8	-353006.40	330613.30	-891.00
26	9	-341590.00	332472.70	-891.00
26	10	-358746.00	340461.00	-891.00
26	11	-363867.50	336905.70	-891.00
26	12	-349981.30	330053.90	-891.00
26	13	-351234.30	345654.70	-891.00
26	14	-358983.20	341266.60	-891.00
26	15	-363280.60	336756.20	-891.00
26	16	-347761.80	349990.80	-891.00
26	17	-361133.70	347214.60	-891.00
26	18	-354062.50	335289.30	-891.00
26	19	-361459.00	341569.60	-891.00
26	20	-348569.50	340182.10	-891.00
26	21	-364060.00	338950.30	-891.00
26	22	-365704.70	335977.50	-891.00
26	23	-357075.40	341632.50	-891.00
26	24	-355226.90	353684.30	-891.00
26	25	0.00	2208285.00	0.00

CRI. 1.0000
Yearly iir 0.18
Profit 573406.00 euro

Economic and financial effects on simulated portfolio

Mean values by group data
Cumulative Prob. Default 0.1497
CRI 0.9547

Portfolio summaries
Interests 503445.80 euro
Yearly iir 0.0821
Yearly contractual iir 0.1800
Total CRI 0.9736

Bibliography

Quirini L., Vannucci L., (2004). "Un indicatore per misurare l'affidabilità di soggetti prenditori di finanziamenti di credito al consumo", in Studi e note di economia, n. 3/2004, pagg. 77- 88.

Quirini L., Vannucci L., (2007). "Un modello per la stima della qualità creditizia dei titolari di carte revolving", in "Basilea 2: cosa devono fare le banche adesso" Associazione Bancaria Italiana