

„Rating Properties and their Implications for Basel II – Capital“

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Agenda

- Validation of default probabilities
- Properties of Default Probabilities
- Impact on Basel II-Capital
- Inferential Statistic: “Spiegelhalter test”
- Empirical analysis
- Discussion

Validation of Default Probabilities

- Basel Committee on Banking Supervision proposes new framework for banking regulatory capital based on the inherent credit risk, which depends on forecasted parameters such as
 - Probability of default,
 - Exposure at default,
 - Loss given default and
 - Maturity.
- Challenge: Validation of the forecasted default probabilities ($\hat{\pi}_i$) by comparing the forecasts with the observed outcomes (y_i):
 - Default ($y_i = 1$)
 - No default ($y_i = 0$)

Validation of Default Probabilities

- Accuracy (also known as “Calibration-in-the-Small”) is defined as:
 $\hat{\pi}_i = \pi_i$ for all i .
- Accuracy is measured by the Mean Square Error (MSE):

$$MSE = \frac{1}{N} \sum_{i=1}^N (y_i - \hat{\pi}_i)^2$$

- The smaller the MSE, the higher the forecasts’ accuracy.
- Weighted penalty function, in which discrepancies between observations and forecasts are weighted by their severity through the quadratic function.
- MSE is the basic element of the further considerations

Properties of Default Probabilities

- By decomposing the MSE, different properties of default probabilities can be shown.
- Decomposition I of MSE:

$$MSE = \overbrace{(\bar{y} - \hat{\pi})^2}^{\text{Over-All-Calibration}} + \underbrace{\overbrace{s_y^2}^{\text{Uncertainty}} + \overbrace{s_{\hat{\pi}}^2}^{\text{Refinement}} - 2 \cdot s_y \cdot s_{\hat{\pi}} \cdot \overbrace{r_{y\hat{\pi}}}^{\text{Association}}}_{\text{Variance of Forecast Error}}$$

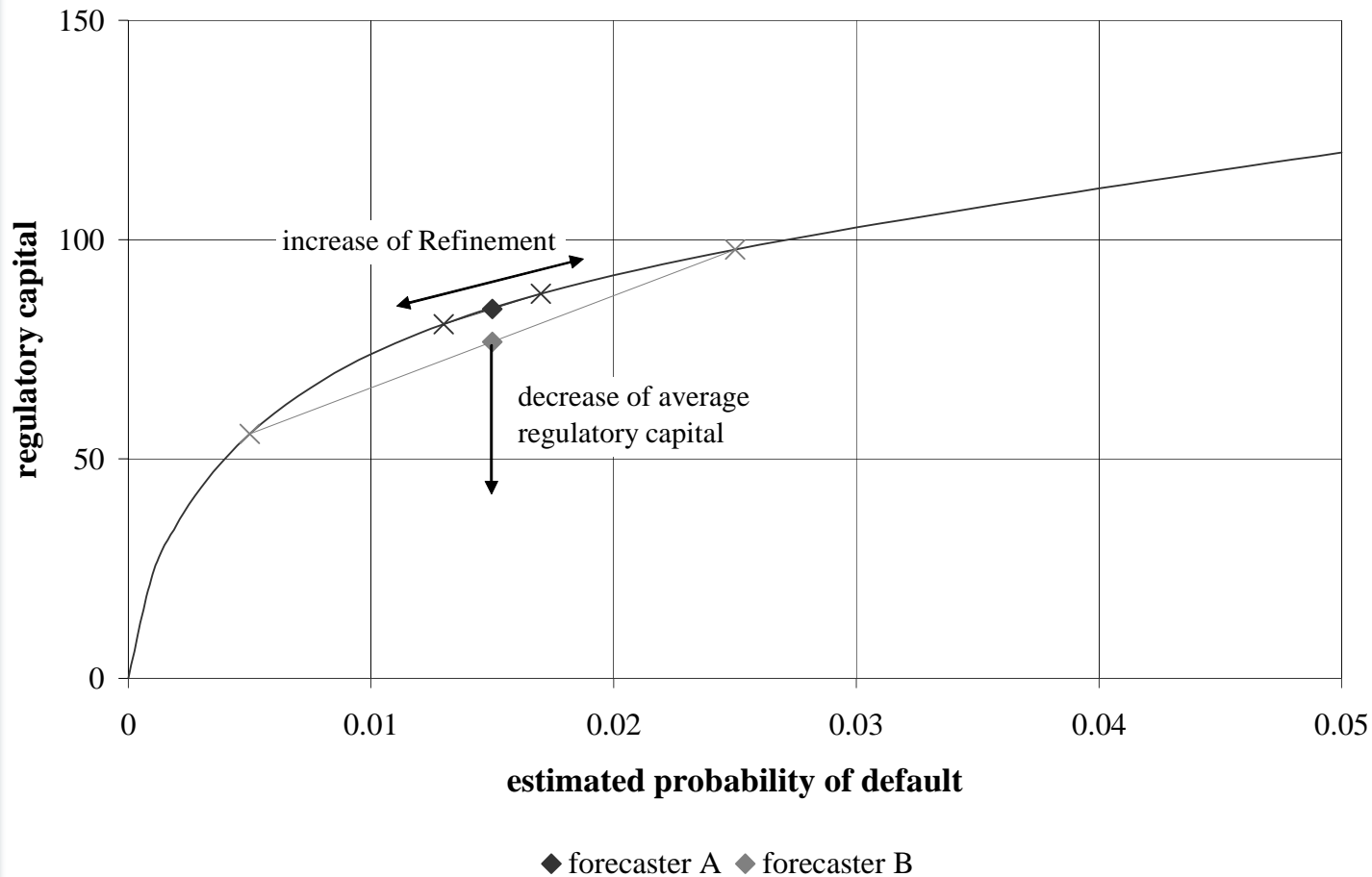
- Decomposition II of MSE :

$$MSE = \overbrace{s_{\hat{\pi}}^2}^{\text{Refinement}} + \overbrace{\sum_y \frac{N_y}{N} (\hat{\pi}_y - y)^2}^{\text{Discrimination I}} - \overbrace{\sum_y \frac{N_y}{N} (\hat{\pi}_y - \hat{\pi})^2}^{\text{Discrimination II}}$$

Properties of Default Probabilities

- The following properties of default probabilities can be derived from the accuracy measure MSE:
 - Over-All-Calibration,
 - Uncertainty,
 - Refinement,
 - Association,
 - Discrimination I and
 - Discrimination II.

Impact on Basel II - Capital



$\hat{\pi}_i$	Obligor 1	Obligor 2
Forecaster A	0.013	0.017
Forecaster B	0.005	0.025

- Note: a higher refinement leads to a lower average capital requirement.

Inferential Statistics

- Up to now: descriptive statistics for definition of rating properties
- Defaults or non-defaults are realisations of a random variable:
 y_i is Bernoulli distributed with parameter π_i
- Thus, MSE is a random variable itself
- The best a forecaster can do is to predict each default probability correct:
 $H_0 : \hat{\pi}_i = \pi_i$ for all i : tests on “to be calibrated-in-the-small”
- Then:
$$E(MSE_{\hat{\pi}_i = \pi_i}) = \frac{1}{N} \sum_{i=1}^N \pi_i (1 - \pi_i)$$
- Test statistic:
$$Z_S = \frac{MSE - E(MSE_{\hat{\pi}_i = \pi_i})}{\text{Var}(MSE_{\hat{\pi}_i = \pi_i})^{0.5}} \quad \text{approx. } \sim N(0;1)$$
- See Spiegelhalter (1988)

Empirical Analysis

- Comparison of three probit models for the default probabilities. The Models differ in the number of significant risk factors and therefore included information:
 - Model 1:
 - Working capital to total assets.
 - Model 2:
 - Working capital to total assets and
 - Long-term debt to total assets.
 - Model 3:
 - Working capital to total assets (WCA),
 - Long-term debt to total assets (LDA) and
 - Change in US real gross domestic product (GDP).

Empirical Analysis

- Data set: typical small-sized corporate loan portfolio from 1997 to 2003
- 18,246 observations and 228 defaults
- Estimation Sample: 1997 to 2002
- Validation Sample: 2003 with 2,223 observations and 36 defaults
- Parameter estimates:

Model	Variable	Estimate	Standard Error	Standardised Estimates	p-Value
1	Intercept	-2.1808	0.0295	.	<.0001
	WCA	-0.4048	0.0666	-0.1329	<.0001
2	Intercept	-2.3532	0.0442	.	<.0001
	LDA	0.5186	0.0871	0.1324	<.0001
	WCA	-0.2875	0.0726	-0.0944	<.0001
3	Intercept	-2.1080	0.0663	.	<.0001
	GDP	-0.0807	0.0173	-0.1187	<.0001
	LDA	0.4951	0.0873	0.1264	<.0001
	WCA	-0.2695	0.0727	-0.0885	0.0002

Empirical Analysis

- Characteristics of the forecasts:

Model	\bar{y}	$\bar{\hat{\pi}}$	$\bar{\hat{\pi}}_{y=0}$	$\bar{\hat{\pi}}_{y=1}$	$s_{\hat{\pi}}$	Average Capital Requirement
1		0.01133	0.01131	0.01257	0.00254	76.39
2	0.01619	0.01221	0.01214	0.01669	0.00806	75.75
3		0.01393	0.01385	0.01864	0.00840	79.70

Empirical Analysis

- Decomposition I of MSE

Model	Accuracy	Over-All-Calibration	Variance of Forecast Error		
	$MSE =$	$(\bar{y} - \bar{\hat{\pi}})^2 +$	$s_{\hat{\pi}}^2 +$	$s_y^2 -$	$2s_{\hat{\pi}}s_y r_{y\hat{\pi}}$
1	0.0159221	0.0000237	0.0000064	0.0159321	0.0000401
2	0.0158680	0.0000159	0.0000650	0.0159321	0.0001450
3	0.0158550	0.0000051	0.0000705	0.0159321	0.0001527

Empirical Analysis

- Decomposition II of MSE

	Accuracy	Refinement	Discrimination I	Discrimination II
Model	$MSE =$	$s_{\hat{\pi}}^2 +$	$\sum_y \frac{N_y}{N} (\bar{\hat{\pi}}_y - y)^2 -$	$\sum_y \frac{N_y}{N} (\bar{\hat{\pi}}_y - \bar{\hat{\pi}})^2$
1	0.0159221	0.0000064	0.0159156	0.000000025
2	0.0158680	0.0000650	0.0158032	0.000000330
3	0.0158550	0.0000705	0.0157849	0.000000366

Empirical Analysis

- Spiegelhalter test: $H_0 : \hat{\pi}_i = \pi_i$ for all i

Model	MSE	p-Value
1	0.0159221	0.0310
2	0.0158680	0.0843
3	0.0158550	0.3591

- The null hypothesis can be rejected for model 1 (alpha = 0.05)
- Model 2 and 3 are assumed “to be calibrated-in-the-small”

Discussion

- Calibration-in-the-Small can be achieved (among others) by different dimensions:
 - Refinement,
 - Discrimination and
 - Over-All-Calibration.
- Importance of these properties depends especially on the type of portfolio.
- Measures are random like the default events themselves, i.e., inferential statistics based on the tests of hypotheses should be conducted.

Discussion

- Note: Validation is not solely a statistical exercise. All of the approaches including ours are especially limited with regard to assumptions and data.

- A broader validation approach should encompass due diligence of the
 - Model and its alternatives,
 - Development processes,
 - Conceptual soundness and
 - Previous statistical studies.

Thank you for your attention

Please

- Feel free to share comments or ask any questions you may have.
- Let me know if you are interested in the ongoing research.

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