„Rating Properties and their Implications for Basel II – Capital“

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Agenda

- Validation of default probabilities
- Properties of Default Probabilities
- Impact on Basel II-Capital
- Inferential Statistic: “Spiegelhalter test”
- Empirical analysis
- Discussion

Validation of Default Probabilities

- Basel Committee on Banking Supervision proposes new framework for banking regulatory capital based on the inherent credit risk, which depends on forecasted parameters such as
  - Probability of default,
  - Exposure at default,
  - Loss given default and
  - Maturity.

- Challenge: Validation of the forecasted default probabilities ($\hat{\pi}_i$) by comparing the forecasts with the observed outcomes (y_i):
  - Default ($y_i = 1$)
  - No default ($y_i = 0$)

Validation of Default Probabilities

- Accuracy (also known as "Calibration-in-the-Small") is defined as:
  $\hat{\pi}_i = \pi_i$ for all i.
- Accuracy is measured by the Mean Square Error (MSE):

$$MSE = \frac{1}{N}\sum_{i=1}^{N}\left(y_i - \hat{\pi}_i\right)^2$$

- The smaller the MSE, the higher the forecasts' accuracy.
- Weighted penalty function, in which discrepancies between observations and forecasts are weighted by their severity through the quadratic function.
- MSE is the basic element of the further considerations

Properties of Default Probabilities

- By decomposing the MSE, different properties of default probabilities can be shown.

- Decomposition I of MSE:

$$MSE = \underbrace{\overbrace{\left(\bar{y} - \bar{\hat{\pi}}\right)^2}^{\text{Over-All-Calibration}}}_{} + \underbrace{\overbrace{s_y^2}^{\text{Uncertainty}} + \overbrace{s_{\hat{\pi}}^2}^{\text{Refinement}} - 2 \cdot s_y \cdot s_{\hat{\pi}} \cdot \overbrace{r_{y\hat{\pi}}}^{\text{Association}}}_{\text{Variance of Forecast Error}}$$

- Decomposition II of MSE :

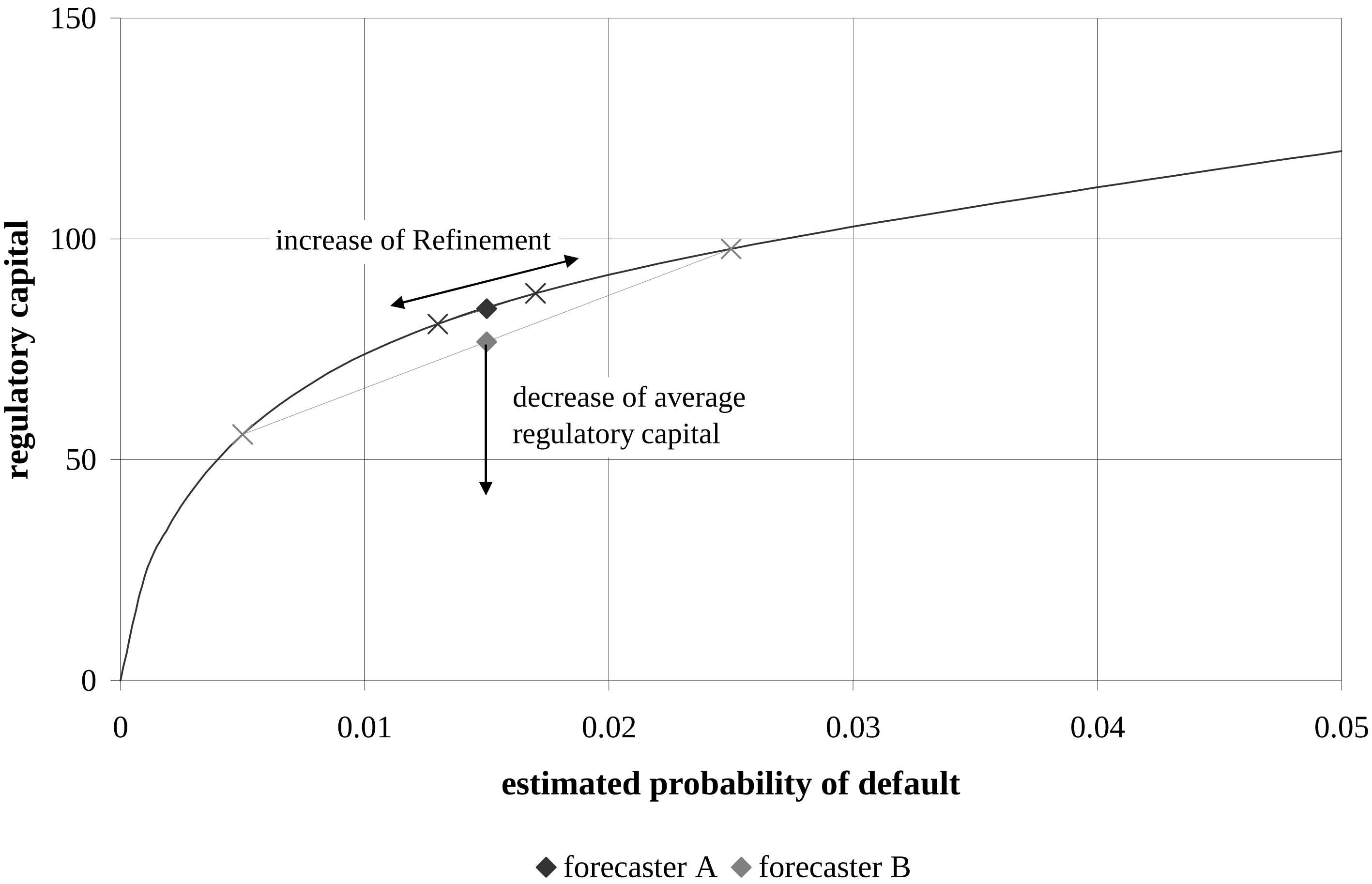
$$MSE = \overbrace{s_{\hat{\pi}}^2}^{\text{Refinement}} + \overbrace{\sum_y \frac{N_y}{N} \left(\bar{\hat{\pi}}_y - y\right)^2}^{\text{Discrimination I}} - \overbrace{\sum_y \frac{N_y}{N} \left(\bar{\hat{\pi}}_y - \bar{\hat{\pi}}\right)^2}^{\text{Discrimination II}}$$

Properties of Default Probabilities

- The following properties of default probabilities can be derived from the accuracy measure MSE:

  - Over-All-Calibration,
  - Uncertainty,
  - Refinement,
  - Association,
  - Discrimination I and
  - Discrimination II.

Impact on Basel II - Capital



$\hat{\pi}_i$	**Obligor 1**	**Obligor 2**
Forecaster A	0.013	0.017
Forecaster B	0.005	0.025

- Note: a higher refinement leads to a lower average capital requirement.

Inferencial Statistics

- Up to now: descriptive statistics for definition of rating properties
- Defaults or non-defaults are realisations of a random variable:

  y_i is Bernoulli distributed with parameter π_i
- Thus, MSE is a random variable itself
- The best a forecaster can do is to predict each default probability correct:

  $H_0 : \hat{\pi}_i = \pi_i$ for all i : tests on "to be calibrated-in-the-small"
- Then: $E\left(MSE_{\hat{\pi}_i = \pi_i}\right) = \frac{1}{N}\sum_{i=1}^{N} \pi_i (1 - \pi_i)$
- Test statistic: $Z_S = \frac{MSE - E\left(MSE_{\hat{\pi}_i = \pi_i}\right)}{Var\left(MSE_{\hat{\pi}_i = \pi_i}\right)^{0.5}}$ approx. $\sim N(0;1)$
- See Spiegelhalter (1988)

Empirical Analysis

- Comparison of three probit models for the default probabilities. The Models differ in the number of significant risk factors and therefore included information:
  - Model 1:
    - Working capital to total assets.
  - Model 2:
    - Working capital to total assets and
    - Long-term debt to total assets.
  - Model 3:
    - Working capital to total assets (WCA),
    - Long-term debt to total assets (LDA) and
    - Change in US real gross domestic product (GDP).

Empirical Analysis

- Data set: typical small-sized corporate loan portfolio from 1997 to 2003
- 18,246 observations and 228 defaults
- Estimation Sample: 1997 to 2002
- Validation Sample: 2003 with 2,223 observations and 36 defaults
- Parameter estimates:

Model	Variable	Estimate	Standard Error	Standardised Estimates	p-Value
1	Intercept	-2.1808	0.0295	.	<.0001
	WCA	-0.4048	0.0666	-0.1329	<.0001
2	Intercept	-2.3532	0.0442	.	<.0001
	LDA	0.5186	0.0871	0.1324	<.0001
	WCA	-0.2875	0.0726	-0.0944	<.0001
3	Intercept	-2.1080	0.0663	.	<.0001
	GDP	-0.0807	0.0173	-0.1187	<.0001
	LDA	0.4951	0.0873	0.1264	<.0001
	WCA	-0.2695	0.0727	-0.0885	0.0002

Empirical Analysis

- Characteristics of the forecasts:

Model	$\bar{y}$	$\bar{\hat{\pi}}$	$\bar{\hat{\pi}}_{y=0}$	$\bar{\hat{\pi}}_{y=1}$	$s_{\hat{\pi}}$	Average Capital Requirement
1		0.01133	0.01131	0.01257	0.00254	76.39
2	0.01619	0.01221	0.01214	0.01669	0.00806	75.75
3		0.01393	0.01385	0.01864	0.00840	79.70

Empirical Analysis

- Decomposition I of MSE

Model	Accuracy	Over-All-Calibration	Variance of Forecast Error		
Model	$MSE =$	$(\bar{y} - \bar{\hat{\pi}})^2 +$	$s_{\hat{\pi}}^2 +$	$s_y^2 -$	$2 s_{\hat{\pi}} s_y r_{y\hat{\pi}}$
1	0.0159221	0.0000237	0.0000064	0.0159321	0.0000401
2	0.0158680	0.0000159	0.0000650	0.0159321	0.0001450
3	0.0158550	0.0000051	0.0000705	0.0159321	0.0001527

Empirical Analysis

- Decomposition II of MSE

Model	Accuracy	Refinement	Discrimination I	Discrimination II
	$MSE =$	$s^2_{\hat{\pi}} +$	$\sum_y \frac{N_y}{N}\left(\bar{\hat{\pi}}_y - y\right)^2 -$	$\sum_y \frac{N_y}{N}\left(\bar{\hat{\pi}}_y - \bar{\hat{\pi}}\right)^2$
1	0.0159221	0.0000064	0.0159156	0.000000025
2	0.0158680	0.0000650	0.0158032	0.000000330
3	0.0158550	0.0000705	0.0157849	0.000000366

Empirical Analysis

- Spiegelhalter test: $H_0 : \hat{\pi}_i = \pi_i$ for all *i*

Model	MSE	p-Value
1	0.0159221	0.0310
2	0.0158680	0.0843
3	0.0158550	0.3591

- The null hypothesis can be rejected for model 1 (alpha = 0.05)
- Model 2 and 3 are assumed "to be calibrated-in-the-small"

Discussion

- Calibration-in-the-Small can be achieved (among others) by different dimensions:
  - Refinement,
  - Discrimination and
  - Over-All-Calibration.

- Importance of these properties depends especially on the type of portfolio.

- Measures are random like the default events themselves, i.e., inferential statistics based on the tests of hypotheses should be conducted.

Discussion

- Note: Validation is not solely a statistical exercise. All of the approaches including ours are especially limited with regard to assumptions and data.

- A broader validation approach should encompass due diligence of the
  - Model and its alternatives,
  - Development processes,
  - Conceptual soundness and
  - Previous statistical studies.

Thank you for your attention

Please

- Feel free to share comments or ask any questions you may have.
- Let me know if you are interested in the ongoing research.

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