Incorporating Stochastic Dominance Constraints in Credit Scoring Methodology –Getting More from Models

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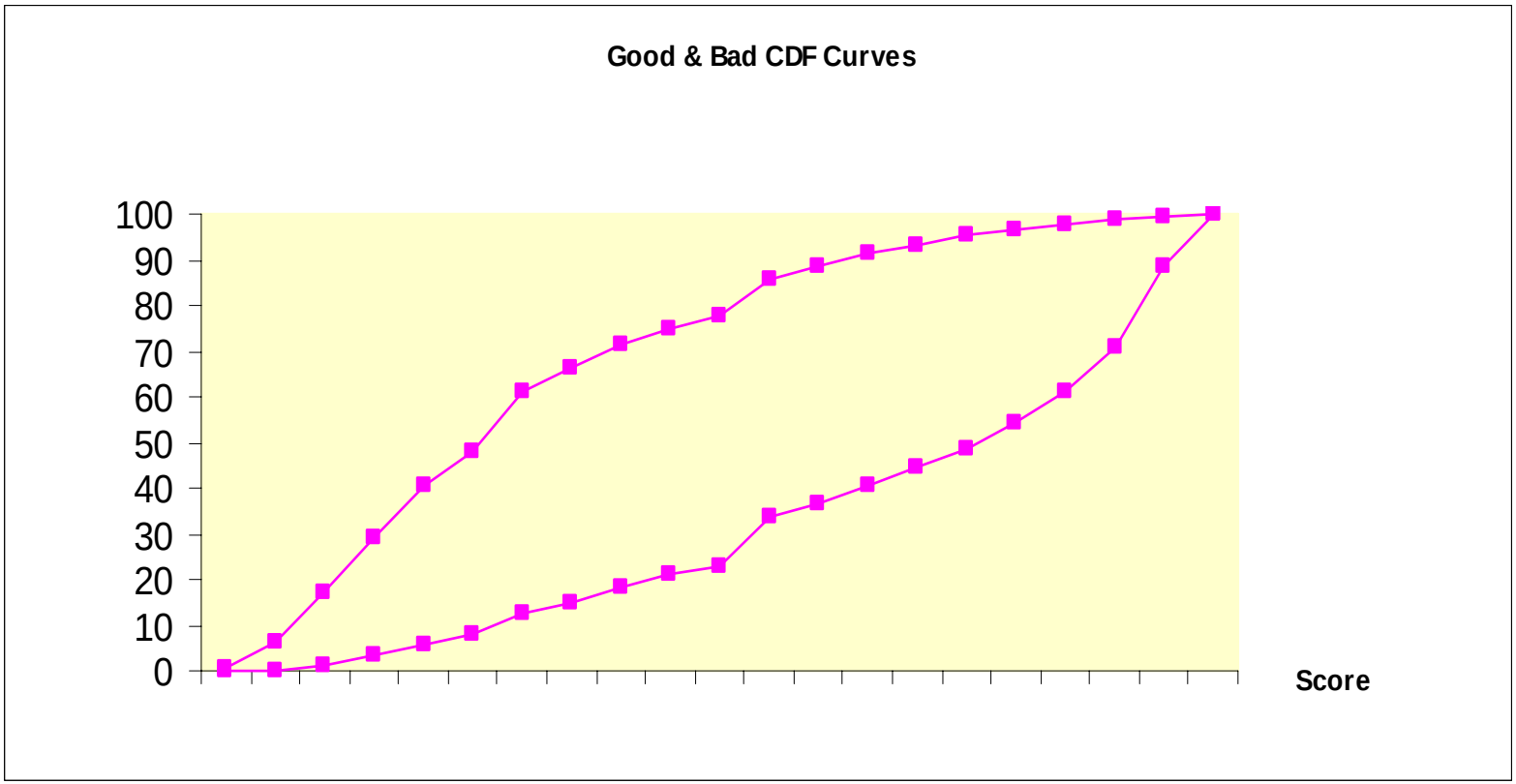
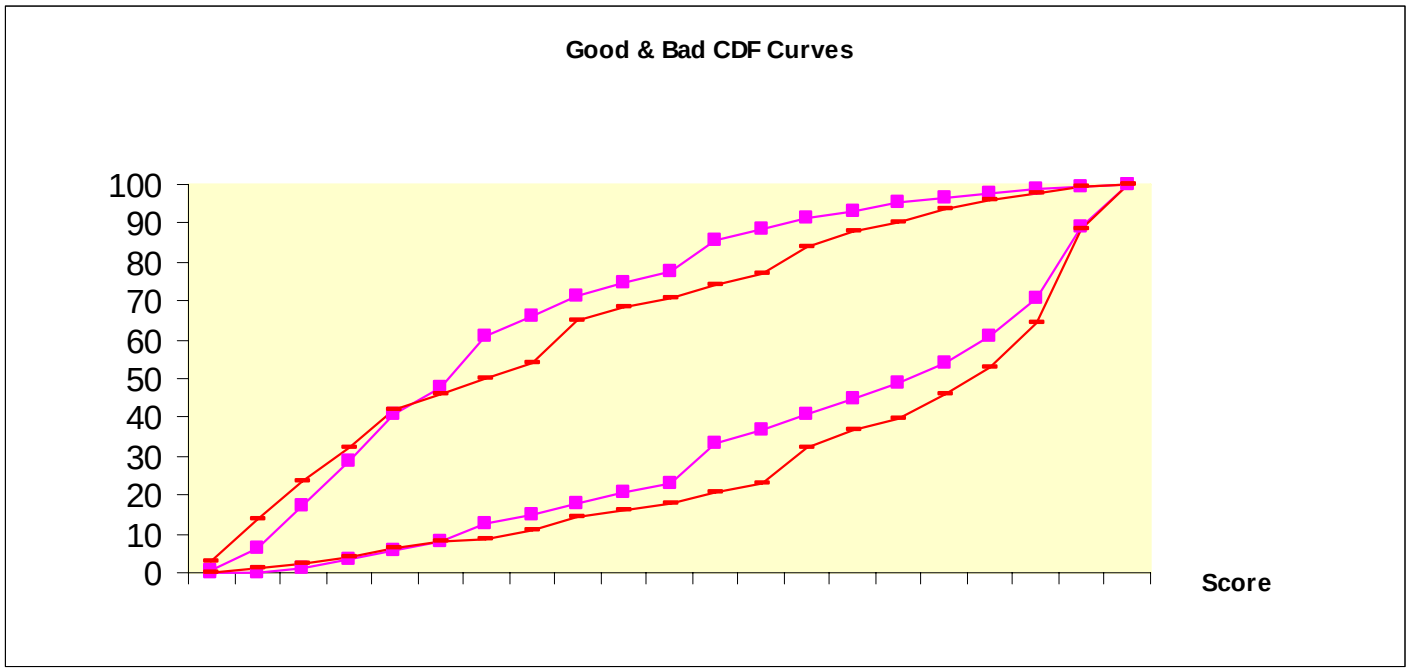
Motivation & Objectives

Scoring Model Output – good & bad accounts cumulative distribution curves



Motivation & Objectives

Two or more scorecards that are pareto-efficient, when used together (e.g., matrix structure), can give better business solutions



Issue: How to generate “pareto-efficient” cards efficiently ?

Stochastic Dominance – definition

FSD - CDF curve for the bad accounts lies above the good CDF curve for all scores: $B(s) \geq G(s) \quad \forall s$

SSD - for any given score, area to the left under the bad CDF curve is greater than or equal to the area under the good curve:

$$\int_{-\infty}^{s} B(t)dt \geq \int_{-\infty}^{s} G(t)dt \qquad \forall \text{s}$$

Other definitions (e.g, TSD) exist. See Hadar & Russell (1969), Whitmore (1970), Hanoch & Levy (1969), Bawa (1982) for review & applications.

Stochastic Dominance – duality results

$S^G(\alpha)$ = α-quantile score for the good accounts (i.e., inverse CDF for goods)

$S^B(\alpha)$ = α-quantile score for the bad accounts

FSD is equivalent to $S^G(\alpha) \geq S^B(\alpha)$ for all $0 \leq \alpha \leq 1$

SSD is equivalent to $\int_0^\alpha S^G(\alpha)d\alpha \geq \int_0^\alpha S^B(\alpha)d\alpha$ for all $0 \leq \alpha \leq 1$

See Ogryczak and Ruszczyski (2002) for references & details.

Stochastic dominance enforcement via Cornish-Fisher Approximation

Cornish-Fisher quantile approximation provides expressions for $S^G(\alpha)$ and $S^B(\alpha)$ as functions of respective mean (μ), standard deviation (σ), standardized central skewness (γ_1), and standardized central excess kurtosis (γ_2):

$$S(\alpha) = \mu + \sigma . x_\alpha$$

$$x_\alpha = z_\alpha + 1/6(z_\alpha^2 - 1)\gamma_1 + 1/24(z_\alpha^3 - 3z_\alpha)\gamma_2 - 1/36(2z_\alpha^3 - 5z_\alpha)\gamma_1^2$$

with $z_\alpha =$ std normal α-quantile value.

So, SD requirements can be written in terms of higher moments of good & bad score

Stochastic Dominance constraints –basis for generating alternative scorecards

- Maximize log-likelihood function subject to only a subset of SD constraints
- Mechanism to control shape/distance between the good and bad CDF curves
- Log-likelihood function, SD constraints are functions of score variable weights (w_j = jth variable weight)

$V_j^i =$ value of jth variable for customer i , and $Q_i = \sum_j w_j V_j^i$ be the predictor variate. So, customer score $e^Q/(1+e^Q)$ is function of w_j.

Accordingly, higher moments of good / bad population scores and quantile functions $S^G(\alpha)$ and $S^B(\alpha)$ are functions of w_j.

Nonlinear Programming Formulation P1

$$\underset{w_j}{\text{Maximize}} \sum_j \sum_{i=good} w_j V_j^i - \sum_j \sum_i \log(1+\exp(w_j V_j^i))$$

Subject to $S^G(\alpha) - S^B(\alpha) \geq \kappa$ (α and κ given)

(Again, $S^G(\alpha)$ and $S^B(\alpha)$ are both functions of w's.)

Nonlinear Programming Formulations P2 & P3

Let $V^G(\alpha)$ and $V^B(\alpha)$ be α-quantile expressions for the good & bad predictor variates respectively.

Problem P2:

$$\underset{w_j}{\text{Maximize}} \sum_j \sum_{i=good} w_j V_j^i - \sum_j \sum_i \log(1+\exp(w_j V_j^i))$$

$$\text{Subject to} \quad \frac{e^{V^G(\alpha)}}{1+ e^{V^G(\alpha)}} - \frac{e^{V^B(\alpha)}}{1 + e^{V^B(\alpha)}} \geq \kappa \quad (\alpha, \kappa \text{ given})$$

Problem P3: $\underset{w}{\text{Maximize}}\ S^G(\alpha) - S^B(\alpha) \quad (\alpha \text{ given})$

Solution to NLP

- Proc NLP within SAS 9.1 PC
- Data = option allows straightforward implementation of the P1-P3 objective functions & constraints
- Total solution times for real problems with 9000+ observations under 5 minutes

Examples

Example 1:
-UK Consumer credit bureau data
-Development sample 4879 good, 4323 bads (bad = 90+DPD ever within 2 yrs, reject inference)
-33 indicator variables from 14 distinct bureau variables
-Validation sample 2073 good, 1825 bad

Example 2:
-North American Commercial credit bureau data
-Development sample 2528 good, 1831 bads (bad = 90+DPD ever within 2 yrs, reject inference)
-39 indicator variables from 11 distinct bureau variables
-Validation sample 5983 good, 4279 bad

Example 1

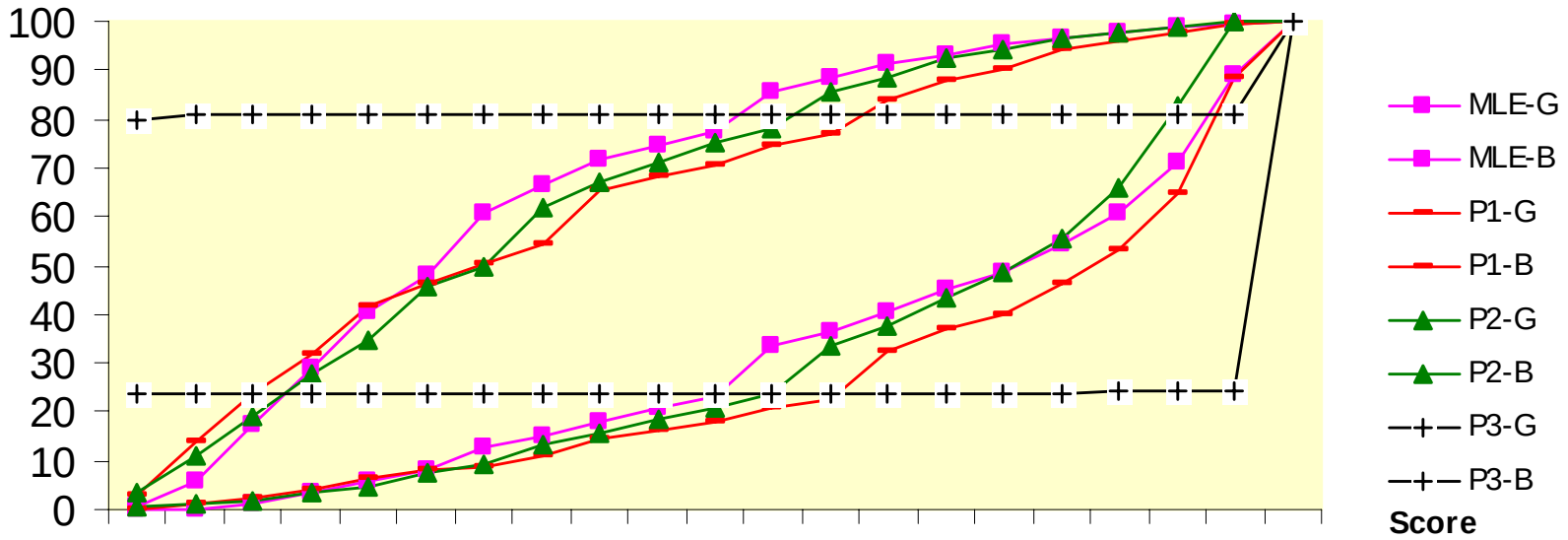
Variable	Logistic regression variable weights (MLE)	MLE with CF: α =0.1, $S^{\alpha}(G) - S^{\alpha}(B) \geq$ 0.4 (score)	MLE with CF: α =0.1, $S^{\alpha}(G) - S^{\alpha}(B) \geq$ 0.4 (variate)	Maximize $S^{\alpha}(G) - S^{\alpha}(B)$ with α =0.5 (score)
W1_1	-0.5772	-0.6792	-0.5836	-95.5685
W1_2	-0.4576	-0.5750	-0.5470	-45.7371
W1_3	0.4056	0.4595	0.3364	-5.2817
W1_4	0.8303	0.9927	0.6631	43.3949
W1_5	1.2155	1.3494	0.9394	111.5168
W2_1	-0.5020	-0.4879	-7.3866	-56.7648
W2_2	-1.1510	-1.4590	-1.3914	-160.1810
W3_1	-0.3324	-0.4807	-0.3060	-7.6186
W3_2	-0.4226	-0.5386	-0.3593	-15.3418
W4_1	-0.3885	-0.5207	-0.3893	-55.8356
W4_2	-0.1440	-0.2535	-0.1026	-36.3506
W5_1	-0.4340	-0.5517	-0.4477	-18.4846
W5_2	-0.9836	-1.4923	-1.0598	-94.6232
W5_3	-1.5920	-2.7095	-1.9979	-132.0503
W5_4	0.5474	0.4704	0.5421	70.5092
W5_5	0.7234	0.5502	0.6742	88.0424
W5_6	0.6296	0.4262	0.6404	39.0659
W5_7	1.6390	1.1107	0.9503	100.3690
Constant	0.5687	1.2716	0.8033	42.8025
KS Value (development sample)	0.544	0.5420	0.5424	0.5744
Validation sample KS (2073 good, 1825 bad)	0.545	0.5470	0.5454	0.5465

Example 1 Continued

Variable	Logistic regression variable weights (MLE)	MLE with CF: α =0.1, $S^{\alpha}(G) - S^{\alpha}(B) \geq 0.4$ (score)	MLE with CF: α =0.1, $S^{\alpha}(G) - S^{\alpha}(B) \geq 0.4$ (variate)	Maximize $S^{\alpha}(G) - S^{\alpha}(B)$ with α =0.5 (score)
W6_1	0.4388	0.5913	0.4779	20.7196
W6_2	0.4901	0.2864	0.3337	48.5796
W6_3	1.0340	0.6168	0.4286	47.4676
W7_1	-0.2799	-0.3277	-0.1910	-8.7908
W7_2	-0.4595	-0.8084	-0.5437	-51.2261
W8	-0.0903	-0.0998	-0.0894	-8.4631
W9_1	-0.3621	-0.3949	-0.3093	-20.5599
W9_2	-0.6188	-0.6081	-0.4327	-54.8516
W10_1	0.4339	0.4456	0.3306	43.9651
W10_2	0.7225	0.7077	0.5424	87.9059
W11_1	-0.2899	-0.2983	-0.2641	-15.1413
W11_2	-0.9690	-1.0914	-0.9061	-92.1342
W12	0.3409	0.3292	0.2060	44.9784
W13	-0.4245	-0.6237	-0.5320	-56.9185
W14	-0.3964	-0.7861	-0.8143	-46.8196
Constant	0.5687	1.2716	0.8033	42.8025
KS Value (development sample)	0.544	0.5420	0.5424	0.5744
Validation sample KS (2073 good, 1825 bad)	0.545	0.5470	0.5454	0.5465

Example 1- continued

Figure 1 : Good & Bad CDF Curves MLE, P1-P3 Models



Example 1- continued

Bad rate Percentages by MLE & P1 Model Score Deciles

		MLE Score Deciles										
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**Total**
P1 Model Score Decile	**1**	91.73	77.45	-	-	-	-	-	-	-	-	90.64
	2	83.93	82.16	80.90	0.0	-	-	-	-	-	-	81.75
	3	-	71.70	72.19	69.93	-	-	-	-	-	-	71.82
	4	-	-	61.98	73.59	62.34	-	-	-	-	-	71.86
	5	-	-	-	62.16	51.87	42.45	47.37	-	-	-	51.61
	6	-	-	-	-	52.50	40.15	34.33	-	-	-	40.32
	7	-	-	-	-	0.0	38.84	37.02	22.47	-	-	35.20
	8	-	-	-	-	-	-	24.06	17.26	7.94	-	16.89
	9	-	-	-	-	-	-	-	10.67	5.57	4.11	6.16
	10	-	-	-	-	-	-	-	-	6.64	3.6	4.13
Total		91.08	81.16	74.85	72.53	52.68	40.22	35.14	16.85	6.07	3.69	

Example 1 continued

- Objective - Maximize auto decision rate such that (i) bad rate for auto approvals <= 5.15% and (ii) bad rate for auto declines >= 82.15%
- Results (all samples):
  - 50% for MLE score only
  - 49.7% for P1 model scorecard
  - 53.2% using MLE & P1 scorecards
- 6% more auto decision using both scores

Example 2

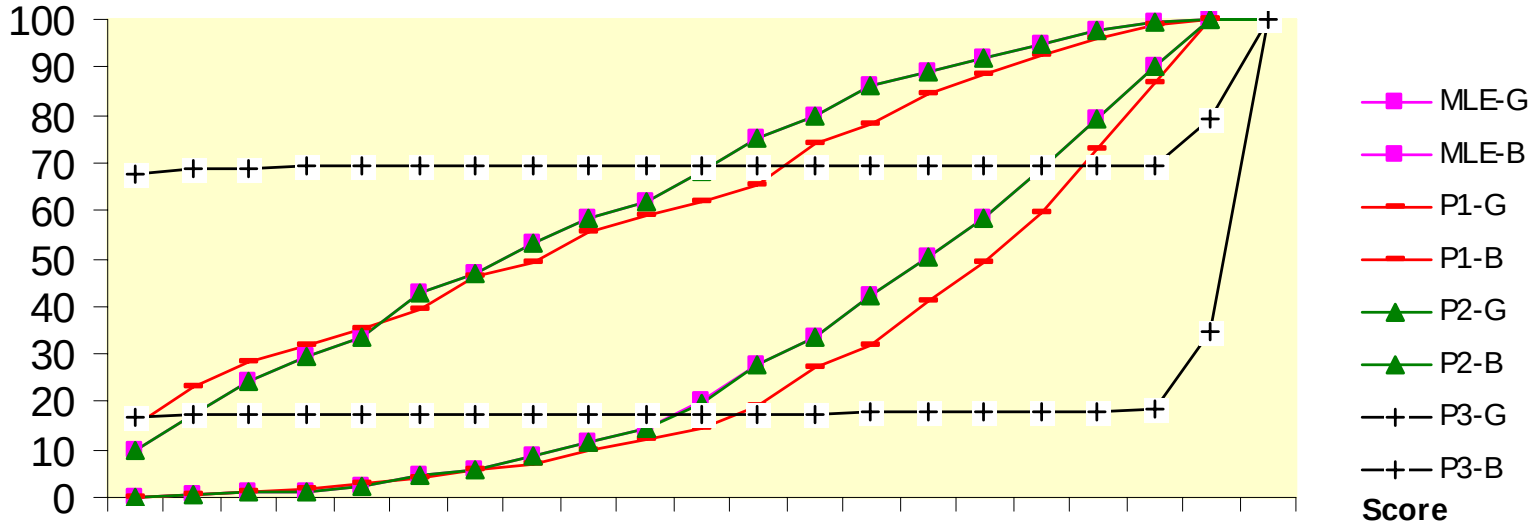
Variable	Logistic regression variable weights (MLE)	MLE with CF: α=0.16, S^{α}(G) - S^{α}(B) $\geq$ 0.5 (score)	MLE with CF: α=0.16, S^{α}(G) - S^{α}(B) $\geq$ 0.5 (variate)	Maximize S^{α}(G) - S^{α}(B) with α=0.5 (score)
W1_1	0.4499	0.4150	0.45	15.1404
W1_2	1.2276	1.3649	1.2294	72.27
W2_1	0.0821	0.0384	0.0819	17.4296
W2_2	0.1369	-0.0201	0.1368	33.955
W2_3	-0.6060	-0.7968	-0.6059	-51.5102
W2_4	-1.0679	-1.5255	-1.0682	-56.8485
W2_5	-1.6864	-2.4085	-1.6873	-75.7523
W3_1	0.1633	0.1622	0.1635	3.2516
W3_2	-0.2334	-0.0453	-0.2340	55.6698
W3_3	-0.1996	-0.2362	-0.1993	3.3786
W3_4	-0.6628	-0.8219	-0.6622	-14.6992
W4_1	0.3711	0.3743	0.3711	25.9623
W4_2	0.9564	0.9372	0.9562	55.9039
W4_3	-0.4742	-0.5466	-0.4745	-26.0535
W5_1	0.388	0.4342	0.3878	25.0167
W5_2	0.8255	0.8832	0.8244	2.9219
W5_3	-0.4354	-0.6314	-0.4377	-45.0829
W6	-0.5408	-0.6606	-0.5400	-27.6243
Constant	0.7011	1.1047	0.7009	18.7720
KS Value (development sample)	0.4802	0.4721	0.4806	0.52
Validation sample KS (5983 good, 4279 bad)	0.48	0.48	0.4802	0.476

Example 2 continued

Variable	Logistic regression variable weights (MLE)	MLE with CF: α=0.16, S^{α}(G) - S^{α}(B) $\geq$ 0.5 (score)	MLE with CF: α=0.16, S^{α}(G) - S^{α}(B) $\geq$ 0.5 (variate)	Maximize S^{α}(G) - S^{α}(B) with α=0.5 (score)
W7_1	-0.0175	0.0196	-0.0170	28.5545
W7_2	0.0603	0.0791	0.0605	4.4645
W8_1	-0.2668	-0.2427	-0.2670	-4.9918
W8_2	-0.333	-0.3168	-0.3339	-17.1341
W8_3	-0.4859	-0.4534	-0.4865	-8.9478
W8_4	-0.2920	-0.1982	-0.2920	-8.1836
W8_5	-0.5996	-0.8201	-0.5998	-59.1433
W8_6	10.2217	6.9583	18.9844	-1.5178
W9_1	-1.1385	-1.2428	-1.1383	-59.191
W9_2	-0.7817	-0.9310	-0.7817	-66.4716
W9_3	-2.252	-2.67	-2.252	-55.7458
W9_4	-3.014	-3.777	-3.014	-119.3731
W10_1	-0.5407	-0.5942	-0.5402	-24.8087
W10_2	-0.6727	-0.7127	-0.6723	-55.3558
W10_3	0.4321	0.4633	0.4316	25.0024
W10_4	0.2742	0.2562	0.2746	21.0402
W10_5	0.7086	0.6582	0.7097	42.0908
W10_6	1.5199	1.4884	1.52	67.5194
W10_7	2.9821	3.2432	2.981	101.2713
W11	-0.2095	-0.3061	-0.2109	-34.4038
Constant	0.7011	1.1047	0.7009	18.7720
KS Value (development sample)	0.4802	0.4721	0.4806	0.52
Validation sample KS (5983 good, 4279 bad)	0.48	0.48	0.4802	0.476

Example 2- continued

Figure 2 : Good & Bad CDF Curves MLE, P1-P3 Models



Conclusions

- Presented methodology to incorporate stochastic dominance constraints efficiently within existing credit scoring model development framework
- Used the framework to formulate nonlinear programming problems whose solutions generate alternate “pareto-efficient” scorecards
- Demonstrated that the nonlinear optimization problems can be readily implemented within SAS making this framework easily accessible to practitioners & researchers
- Solved real-life problems and demonstrated that the use of 2 or more “pareto-efficient” scorecards can lead to better business solutions
- Cornish-Fisher approximation may not work well for extreme quantiles or highly skewed distributions. Quantile approximation of order statistics are being studied (Heidelberger & Lewis (1984), McNeil, Fowler, Mackulak, Nelson (2005))