UNIVERSITY OF EDINBURGH
Business School

CRC | Credit Research Centre

AN APPLICATION OF PROFIT SCORING FOR DIFFERENT BEHAVIOURAL TYPES OF CREDIT CARDS HOLDERS WITH USE OF PANEL DATA

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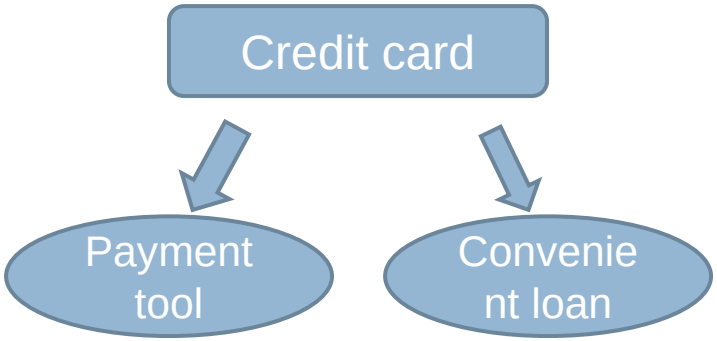
Content

- Objectives and methodology
- General Model
- Interest income prediction - The utilization rate modelling
- Transition income prediction
- Credit Card Holder’s State Transition Probabilities
- Total Income prediction
- Conclusions

Credit card dual nature

Credit Card has dual nature:
Payment tool and Loan



What is the confident definition for revolver?
Positive outstanding balance during 3 months, 6 months, …?
If only once paid off full amount?

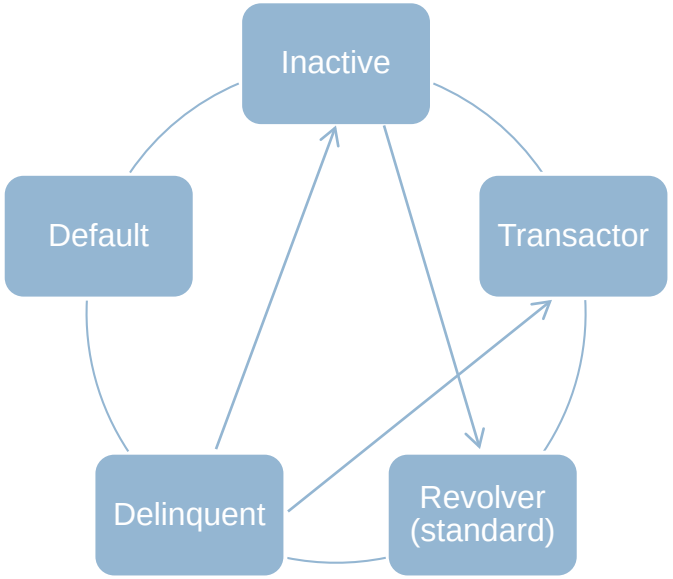
Outstanding balance is stochastic value in the range from 0 to Limit.

Clients is split up two group: revolvers and transactors
Revolver – user, who carry a positive credit card balance and not pay off the balance in full each month – roll over
Transactor – user, who pay in full on or before the due date of the interest-free credit period
Competent user do not incur any interest payments or finance charges

Credit cards dual nature and profitability were investigated by:
Crook, Hamilton, Thomas (1992)
Banasik, Crook, Thomas (2001)
Ma, Crook, Ansell (2010)
So, Thomas (2008)
Cheu, Loke (2010)
Tan, Steven, Yen (2011)

Credit Card Statuses definition

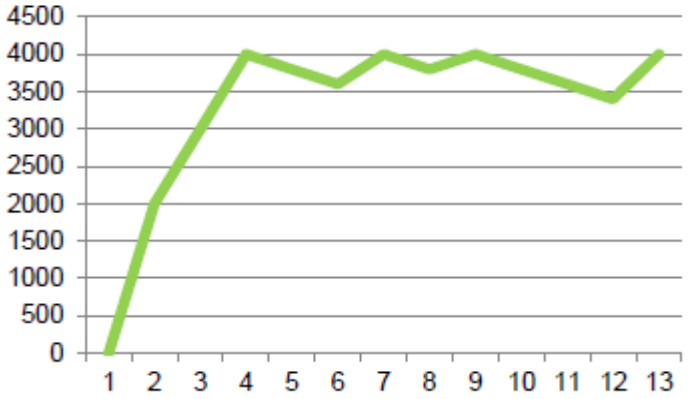
Status	Outstanding balance EOP	Average monthly OB	Turnover Dt (monthy) – expenditure	DPD
Inactive	0	0	0	N/A
Transactor	0	0<	=Turnover Cr (monthly) - payment	N/A
Revolver	0<	0<	0<	0
Cash-user	0<	0<	0	0
Delinquent	0<	0<	N/A	0<
Defaulted	0<	0<	N/A	90<



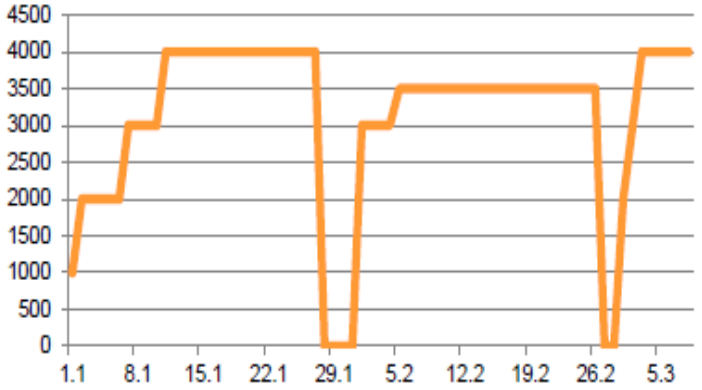
- The main task – to estimate the probability of transition from status t to status t+1 at the account level.
- The problem is that the number of statuses which account can move in is more than two (for example, revolver can be transactor, delinquent, stay revolver, or inactive).

The outstanding balance for different behavioural types of credit card holders

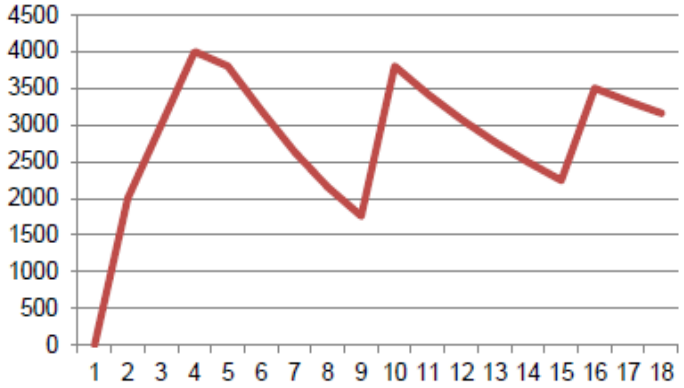
Active revolver outstanding balance by months



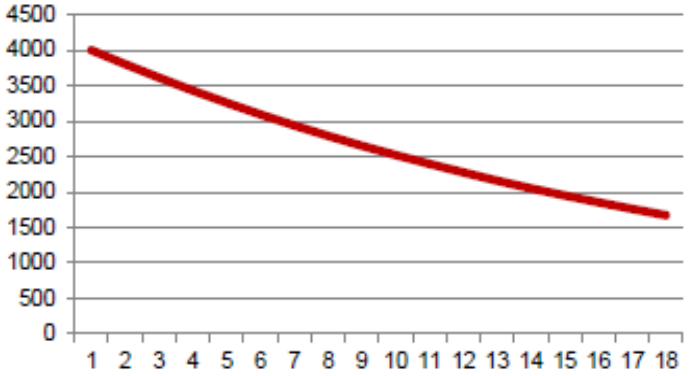
Transactor outstanding balance by days



Non-active revolver outstanding balance by months



Transactor outstanding balance by months



Credit cards income sources

Status	Interest	Fees/Interchange	Penalty
Non active	-	-	-
Transactor	-/+	+	-
Current (revolver)	+	+	-
Delinquent	-	+	+
Defaulted	-	-	-/+

Different income sources can be applied on different credit cycle stages.

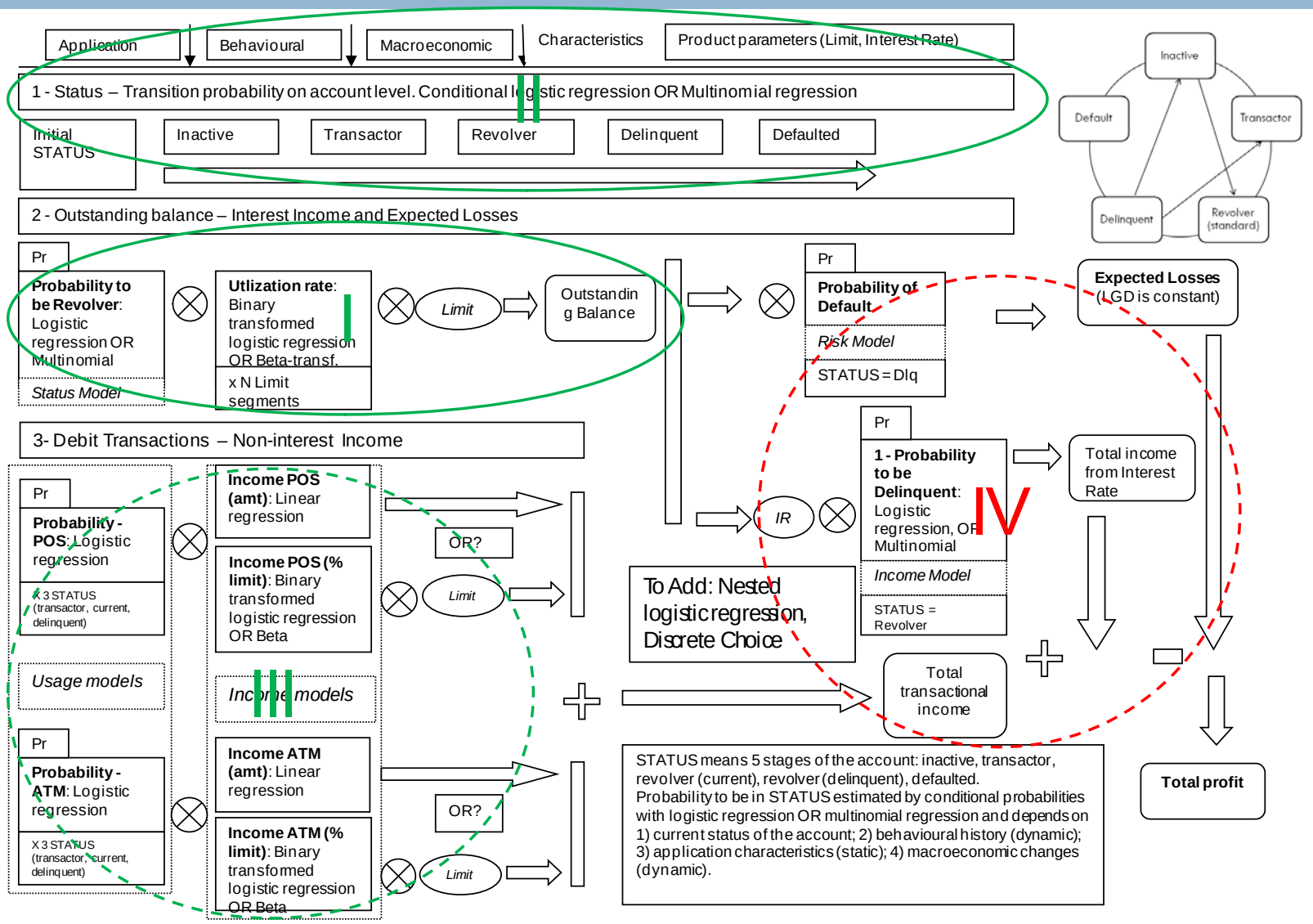
Transactor generates fees/interchange income, the interest income is unexpected, but can be generated due to local policy or accounting rules

For example, delinquent account can generate non-interest income due to interchange fees from merchants and penalty, but doesn't generate interest income because of non-paid debt.

Account state definition and related assessments

Account status	Symbol	Definition	Risk assessment	Revenue assessment
Inactive	NA	Average OB = 0 and Spending Amount = 0	No	No
Transactor	TR	OB end of period (eop) = 0 and Spending Amount > 0	No	Transactional Income Amount
Revolver (current)	RE	OB eop > 0 and DPD = 0	Behavioural (transition) score	Limit x Utilisation Rate x Interest Rate + Transactional Income Amount
Revolver Paid (Repaid)	RP	OB eop (t-1) > 0 and OB eop (t) = 0	No	Limit x Utilization Rate x Interest Rate + Transactional Income Amount
Delinquent 1 (Days Past Due 1-30)	D1	OB eop > 0 and (DPD > 0 and DPD <=30)	Behavioural (transition) score	Transactional Income Amount
Delinquent 2 (Days Past Due 31-60)	D2	OB eop > 0 and (DPD > 31 and DPD <=60)	Behavioural (transition) score	No
Defaulted (DPD 61 and more)	Df	OB eop > 0 and DPD >= 61	LGD	_

General Model Schema



List of behavioural characteristics (fragment)

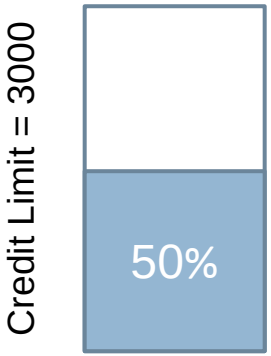
Behavioural Characteristics	
Mob	Month on Balance
Limit	Credit limit at observation point
UT	Utilisation rate at the observation point
b_AvgOB16_to_MaxOB16_ln	Logarithm of average OB to maximum OB for the last six months
b_TRmax_deb16_To_Limit_ln	Logarithm of maximum debit transaction (purchases) for the last six months to the credit limit
b_TRavg_deb16_to_avgOB16_ln	Logarithm of average debit transaction to average OB for the last six months
b_TRsum_deb16tocrd16_ln	Logarithm of sum debit transaction to sum of credit transactions (payments) for the last six months
b_UT1_to_AvgUT16ln	Logarithm of the utilisation rate at the observation point divided by the average utilisation rate for the last six months
b_UT1to2ln	Logarithm of the utilisation rate at observation point to the previous months
b_UT1to6ln	Logarithm of the utilisation rate at observation point to the utilisation rate six months before
b_NumDeb13to46ln	Logarithm of the number of debit transactions for the last three months to 4-6 months before the observation point
b_inactive13	Binary indicator whether the account was inactive last three months
b_avgNumDeb16	Average number of debit transactions for the last six months
b_OB_avg_to_eop1ln	Logarithm of the average outstanding balance for the last month to the outstanding balance at the end of period
b_DelBucket16	Maximum bucket of delinquency for the last six months
b_pos_flag_13	Binary indicator of Point-of-sales (POS) transaction for the last six month
b_atm_flag_13	Binary indicator of ATM cash withdrawals for the last six month

The interest income prediction – The utilization rate modelling

Regression methods for rates prediction

Utilization Rate modelling

- Utilization rate = Outstanding Balance / Credit Limit



X IR = IR Income

IR_Income = Utilization Rate x Limit x IR

$$UT_{it} = \phi_1 UT_{i(t-1)} + \ldots + \phi_T UT_{i(t-T)} + \sum_k^K \beta_b \cdot B_{bi,t-1} + \sum_l^L \alpha_a \cdot A_{ai} + \sum_m^M \gamma_1 M_{m,t-1}$$

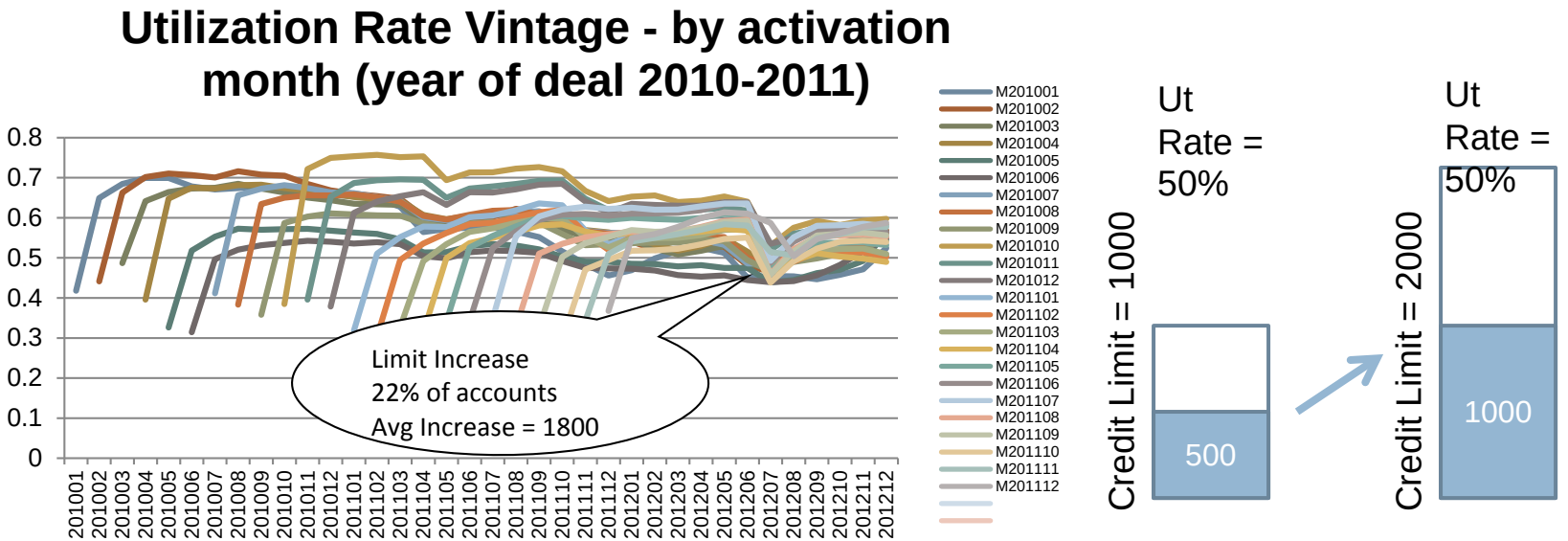
φ, α, β, γ – regression coefficients (slopes)

B *it* – vector of behavioural factors *b* (for example, average balance to maximum balance, maximum debit turnover to limit etc for period *t* observation *i*)

A *i* – vector of application factors - client's demographic, financial and product characteristics like age, education, income etc. for observation *i*

M *t* – vector of macroeconomic factors (GDP, FX, Unemployment rate changes, etc.)

Why the utilization rate, not the outstanding balance

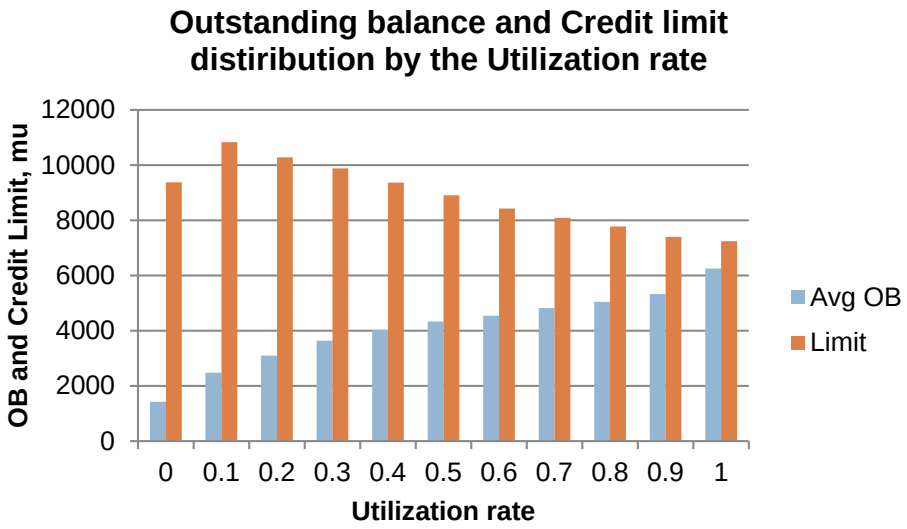


Limit increase process in June has shown the utilization rate drop, but it has stabilized **almost at the same** level in two months.

Assumption: Customer behaviour characteristics and customer profile rather have impact on the utilization rate than on the outstanding balance.

Credit limit depends on credit policy rules and sets up particularly according to the customer risk profile. The same behavioural customer segments have various outstanding balances correlated particularly with the credit limit. Thus customer segment does not has typical outstanding balance, but typical utilization rate as proportion of the credit limit.

Model segments empirical explanation



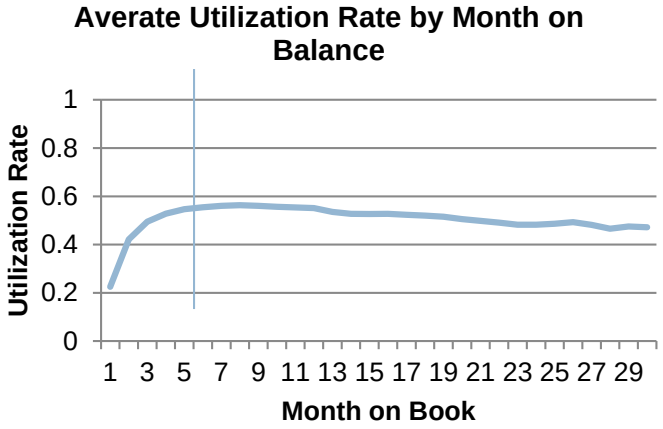
The utilization rate decrease can be caused by the credit limit increase process. However, the customer behaviour can be changed because of the limit changes.

This is the reason why it is reasonable to split the model up two segment:

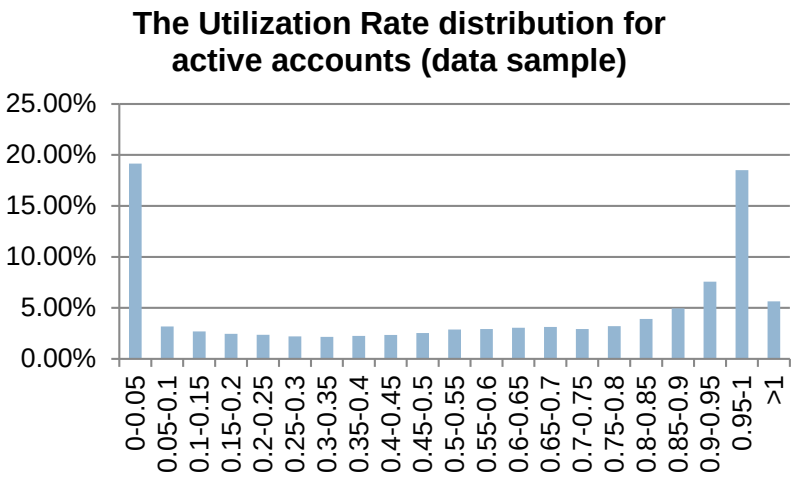
i) credits with unchanged limits and

ii) credits with the limits which have been changed.

Because of the inconsistencies in the behavioural characteristics calculation (lack of history) and the differences in the utilization rate dynamic at the early and late credit history stages it is rational to allocate the separate model for the low MOB period. In our case two periods have been chosen: MOB from 1 to 5 and MOB more than 6.

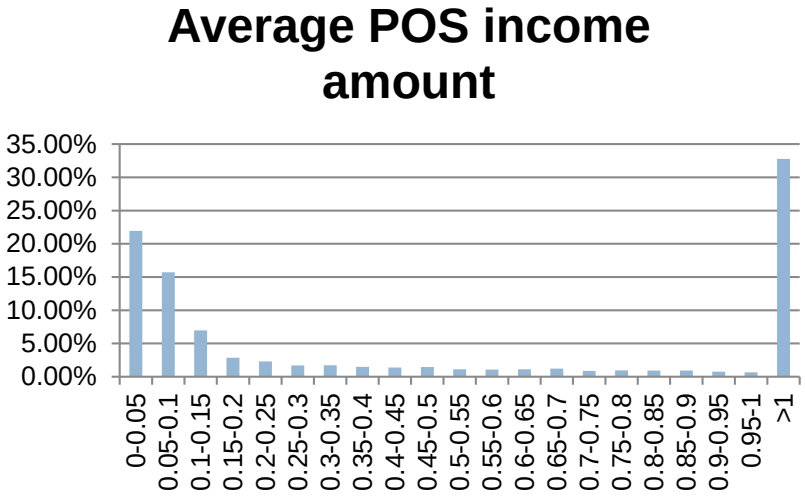


Utilization rate and Income distributions

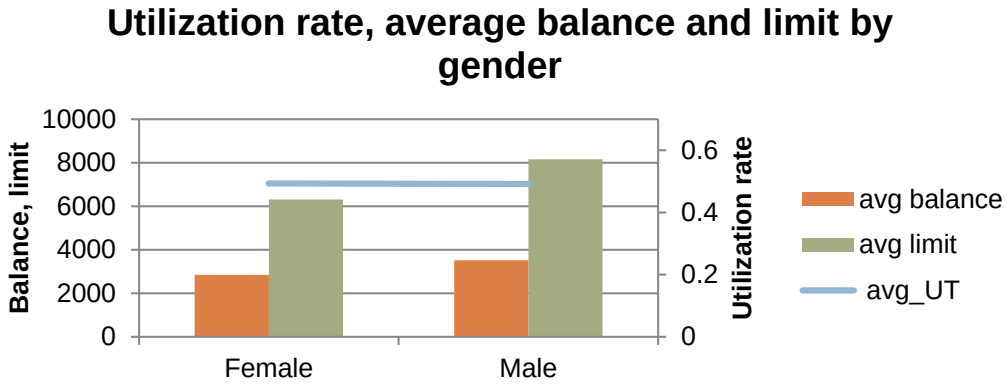


Utilization rate density may have an U-shape distribution,
can be approximated, as option, with beta-distribution.
Values >1 caused by overlimit, are temporary and replaced by 1.

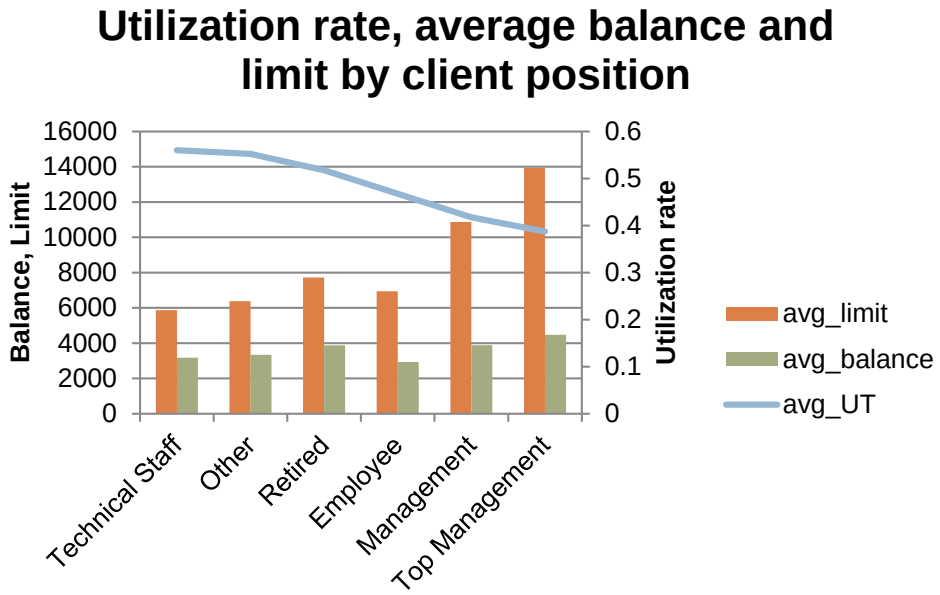
POS income (interchange fees) may have exponential distribution
It's necessary to filter a lot of insufficient amounts and enormous outliers



Utilization rate, balance and limit for gender and age distributions



The utilization rate for gender is not differed significantly, but the average balance for male is higher than for female most likely because of higher limits.



Top managers have the highest limits, but the lowest utilization rate in compare with other positions. This means that the outstanding balance is not so different as the credit limit.

Regression methods (1/2)

Linear regression - OLS

$$y_i = \beta_0 + \boldsymbol{\beta}^T \mathbf{x}_i + \varepsilon_i$$
$$\varepsilon_i \sim N(0, \sigma^2)$$

Beta regression

$$f_X(x) = \frac{1}{\mathrm{B}(\alpha, \beta)} x^{\alpha-1}(1-x)^{\beta-1}$$

Because the outcome is in the range between 0 and 1 the logistic transformation is used to find the dependences between predictors x(a) and regressor

$$Beta(y, \alpha, \beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} y^{\alpha-1}(1-y)^{\beta-1} \qquad \mu(a) = L(\mathbf{x}(a)'\beta) = \frac{e^{\mathbf{x}(a)'\beta}}{1+e^{\mathbf{x}(a)'\beta}}$$

Beta-transformation plus OLS

The algorithm uses the beta distribution to transform the original target.

1 - to find the beta-distribution coefficients (alpha and beta) - fit the sample distribution using the non-linear regression procedures.

2 - replace real target variable by the ideal beta-distributed.

3 -find appropriate normal distributed value.

4 - run OLS or Generalized Linear Mixed Model to find regression coefficients.

Regression methods (2/2)

Fractional logit transformation (quasi-likelihood)

The utilization rate has is bounded between 0 and 1. The Bernoulli log-likelihood function is given by

$$l_i(\mathbf{b}) \equiv y_i \log[G(\mathbf{x}_i\mathbf{b})] + (1 - y_i)\log[1 - G(\mathbf{x}_i\mathbf{b})]$$

$$T_{UT} = \log(UT) - \log(1 - UT)$$ - Transformation function

Weighted Logistic regression with binary transformation (Siddiqi, 2012)

Utilization Rate	Binary recovery – target	Weight
1	1	1
0	0	1
R, 0<r<1	1	r
	0	1-r

Each observation is presented as two observations with the same set of predictors according to the good/bad or 0/1 definition used in logistic regression. The outcome with target 1 corresponds to the rate r – weight r, the outcome with target 0 corresponds to the rate 1-r.

$$\operatorname{logit}(\mathbb{E}[Y_i \mid \mathbf{X}_i]) = \operatorname{logit}(p_i) = \ln\left(\frac{p_i}{1-p_i}\right) = \boldsymbol{\beta} \cdot \mathbf{X}_i$$

The utilization rate – one-stage, prediction period 6 month, OLS

UT and transactions behavioral predictors are significant

Limit parameter is insignificant

Variable	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variance Inflation
Intercept	0.19868	0.00823	24.14	<.0001	0
mob	-0.00328	0.0001604	-20.44	<.0001	1.87755
limit_6	1.59E-07	1.33E-07	1.19	0.2345	2.74067
UT0_6	0.53061	0.00312	170.19	<.0001	4.59573
avg_balance_6	0.00000207	2.37E-07	8.73	<.0001	2.98617
b_AvgOB16_to_MaxOB16_ln	0.04088	0.00134	30.59	<.0001	3.08869
b_TRmax_deb16_To_Limit_ln	0.00699	0.00049122	14.22	<.0001	5.34865
b_TRavg_deb16_to_avgOB16_ln	-0.01841	0.00068774	-26.76	<.0001	3.62388
b_TRsum_deb16_to_TRsum_crd16	0.01087	0.00061894	17.55	<.0001	1.98505
b_UT1_to_AvgUT16ln	-0.00282	0.00040175	-7.01	<.0001	4.58941
b_UT1to2ln	0.00178	0.00030936	5.76	<.0001	1.91965
b_UT1to6ln	-0.0048	0.00025297	-18.99	<.0001	3.08629
b_NumDeb13to46ln	0.00545	0.00033788	16.14	<.0001	2.45803
b_inactive13	0.0824	0.00464	17.77	<.0001	5.54026
b_avgNumDeb16	0.00010706	0.00004173	2.57	0.0103	1.03309
b_OB_avg_to_eop1ln	-0.00132	0.00033009	-4	<.0001	1.15713
b_DelBucket16	0.02571	0.00235	10.96	<.0001	4.35028
b_pos_flag_0	0.01847	0.00174	10.63	<.0001	2.16061
b_pos_flag_13	0.03935	0.00213	18.51	<.0001	3.66705
b_atm_flag_0	0.053	0.00152	34.91	<.0001	1.80498
b_atm_flag_13	0.05824	0.00246	23.7	<.0001	4.5704
b_pos_flag_used46vs13	0.02783	0.0019	14.64	<.0001	1.32548
b_pos_flag_use13vs46	-0.02589	0.00203	-12.78	<.0001	1.27146
b_atm_flag_used46vs13	0.01634	0.0022	7.43	<.0001	1.88301
b_atm_flag_use13vs46	-0.02562	0.00214	-11.96	<.0001	1.34666
b_pos_use_only_flag_13	0.01185	0.00275	4.31	<.0001	2.44702
no_dpd	-0.00524	0.00413	1.27	0.2039	3.49176
max_dpd_60	0.02693	0.00698	3.86	0.0001	1.15943
AgeGRP1	0.01494	0.00189	7.9	<.0001	2.43527
AgeGRP3	-0.00166	0.00172	-0.97	0.3323	2.3734
customer_income_ln	-0.03324	0.00165	-20.1	<.0001	2.86712
Edu_High	-0.02094	0.00175	-11.94	<.0001	2.49428
Edu_Special	-0.00198	0.00169	-1.17	0.2415	1.99299
Edu_TwoDegree	-0.01803	0.00389	-4.63	<.0001	1.24201

Variable	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variance Inflation
position_Man	0.01174	0.00183	6.43	<.0001	1.19365
position_Oth	0.00891	0.00181	4.91	<.0001	1.20526
position_Tech	0.00673	0.0017	3.96	<.0001	1.59924
position_Top	0.00989	0.00336	2.94	0.0033	1.10859
sec_Agricult	0.00428	0.00327	1.31	0.1905	1.09727
sec_Constr	-0.00428	0.00437	-0.98	0.3279	1.05802
sec_Energy	-0.00213	0.00281	-0.76	0.4483	1.17576
sec_Fin	-0.02624	0.00211	-12.45	<.0001	1.28055
sec_Industry	0.00344	0.00552	0.62	0.5328	1.0439
sec_Manufact	0.00805	0.0043	1.87	0.0613	1.07391
sec_Mining	0.00362	0.00299	1.21	0.2259	1.32979
sec_Service	-0.00867	0.00158	-5.49	<.0001	1.37382
sec_Trade	-0.00604	0.00212	-2.84	0.0045	1.3575
sec_Trans	-0.00676	0.00414	-1.63	0.1026	1.06506
car_Own	-0.01099	0.0015	-7.34	<.0001	1.13605
car_coOwn	0.00212	0.00228	0.93	0.3517	1.08877
real_Own	0.00051128	0.00145	0.35	0.7248	1.69724
real_coOwn	-0.00257	0.00154	-1.68	0.0939	1.5081
reg_ctr_Y	-0.00803	0.00223	-3.6	0.0003	3.21889
reg_ctr_N	-0.00653	0.00224	-2.92	0.0035	3.78825
child_1	0.01064	0.0019	5.61	<.0001	2.74174
child_2	0.0065	0.00108	6.02	<.0001	3.09289
child_3	0.03175	0.00362	8.78	<.0001	1.36959
Unempl_lnyoy_6	0.26643	0.02401	11.1	<.0001	2.16435
UAH_EURRate_lnmom_6	-0.07515	0.02953	-2.54	0.0109	1.46217
UAH_EURRate_lnyoy_6	0.16824	0.0185	9.09	<.0001	8.65389
CPI_lnqoq_6	0.62416	0.04887	12.77	<.0001	1.52323
SalaryYear_lnyoy_6	-0.26247	0.04234	-6.2	<.0001	8.33528

How customer use: POS and ATM transactions are significant

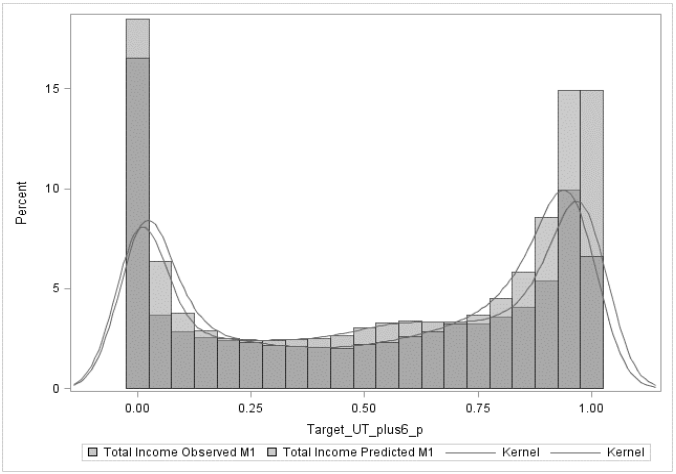
Monthly and annually changes in LCY/EUR are correlated but one is significant

The utilization rate – one stage model – five methods comparison

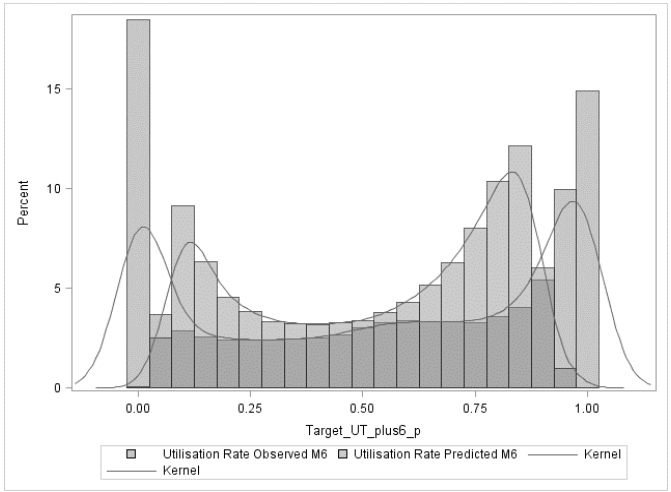
Statistic	OLS	Fractional	Beta regression	Beta+OLS	Weighted Logistic Regrssion
Mean	0.53520	0.53893	0.50482	0.53220	0.53516
Std Deviation	0.28105	0.28297	0.25019	0.37851	0.28243
Skewness	-0.36165	-0.34751	-0.26532	-0.23848	-0.35561
Uncorrected SS	76937	78008	66833	89795	77090
Coeff Variation	52.5123	52.5070	49.5595	71.1215	52.7744
Sum Observations	112680	113465	106284	112048	112671
Variance	0.07899	0.08007	0.06259	0.14327	0.07976
Kurtosis	-1.19212	-1.37168	-1.35845	-1.59755	-1.35032
Corrected SS	16630	16859	13178	30163	16793
Std Error Mean	0.00061	0.00062	0.00055	0.00082	0.00062

Quantile	OLS	Fractional	Beta regression	Beta+OLS	Weighted Logistic Regrssion
100% Max	1.1187	0.9821	0.9495	1.0000	0.9829
99%	0.9440	0.9251	0.8713	0.9962	0.9188
95%	0.8960	0.8845	0.8287	0.9789	0.8804
90%	0.8647	0.8626	0.8053	0.9663	0.8581
75% Q3	0.7843	0.7987	0.7346	0.9079	0.7940
50% Median	0.5985	0.6153	0.5543	0.6285	0.6103
25% Q1	0.2753	0.2551	0.2608	0.0997	0.2568
10%	0.1177	0.1152	0.1333	0.0057	0.1103
5%	0.0770	0.0904	0.1143	0.0025	0.0850
1%	-0.0045	0.0562	0.0872	0.0007	0.0446
0% Min	-0.3780	0.0055	0.0232	0.0000	0.0018

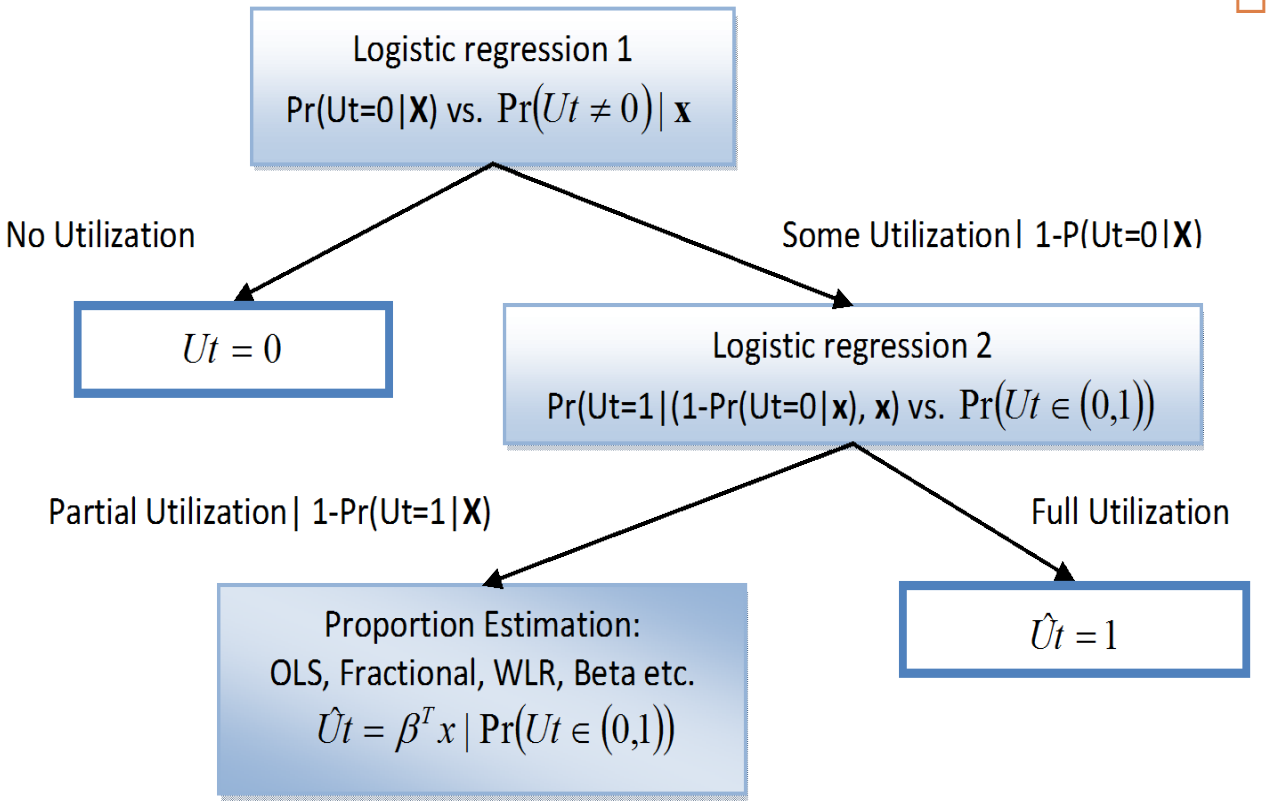
Beta-trans. Plus OLS



Fractional regression



Two-stage model



- Two-stage model consist of two parts:
  - the probability of zero utilization and full utilization with use of logistic regression
  - the proportion estimation with use of the set of the same methods as for one-stage model.

Two-stage model summary

At the first stage the probability that an account has zero utilization (Pr (Ut=0) and then that an account has full utilization (Pr (Ut=1)) in the performance period is calculated with binary logistic regression.

At the second stage the proportion between 0 and 1 excluding 0 and 1 values is calculated according to the set of the approaches used for one-stage direct estimation

$$Ut = \left(1 - \Pr(Ut = 0)\right)\left(\Pr(Ut = 1) + \left(1 - \Pr(Ut = 1)\right) \cdot E(Ut \mid Ut \neq 0, Ut \neq 1)\right)$$

Month on Book	Limit Changes	Stage	Method	Development Sample				Validation Out-of-sample			
Month on Book 6 and more	Limit no change	*Stage 1*	*Probability*	*KS*	*Gini*	*ROC*		*KS*	*Gini*	*ROC*	
		Pr(UT=0)	Logistic Regression	0.6262	0.7479	0.8739		0.6331	0.7547	0.8774	
		Pr(UT=1)	Logistic Regression	0.5931	0.7243	0.8622		0.6036	0.7355	0.8678	
		Stage 2	*Proportion Estimation*	*R2*	*MAE*	*RMSE*	*MAPE*	*R2*	*MAE*	*RMSE*	*MAPE*
		0<UT<1	OLS	0.4310	0.1948	0.2462	4.9151	0.4235	0.1950	0.2462	4.8260
			Fractional(Quasi-Likelihood)	0.4309	0.1946	0.2463	4.9683	0.4235	0.1950	0.2462	4.8260
			Beta regression (nlmixed)	0.4183	0.2102	0.2506	5.0499	0.4108	0.2104	0.2507	4.9075
			Beta transformation + OLS	0.3680	0.1802	0.2673	2.7377	0.3618	0.1809	0.2673	2.6513
			Weighted Logistic Regression	**0.4325**	0.1945	0.2457	4.8937	0.4253	0.1948	0.2456	4.7564
		Two-stage	*Aggregate*	*R2*	*MAE*	*RMSE*	*MAPE*	*R2*	*MAE*	*RMSE*	*MAPE*
		0<= UT <=1	OLS	0.5534	0.1913	0.2535	3.1366	0.5536	0.1910	0.2526	3.0784
			Fractional(Quasi-Likelihood)	0.5527	0.1915	0.2536	3.1590	0.5529	0.1912	0.2528	3.0979
			Beta regression (nlmixed)	0.5366	0.2068	0.2581	3.2109	0.5364	0.2063	0.2574	3.1502
			Beta transformation + OLS	0.4720	0.1773	0.2754	1.7407	0.4724	0.1774	0.2745	1.7019
			Weighted Logistic Regression	**0.5548**	0.1914	0.2531	3.1116	0.5553	0.1910	0.2521	3.0532

Transactional income prediction

Two-stage conditional model:

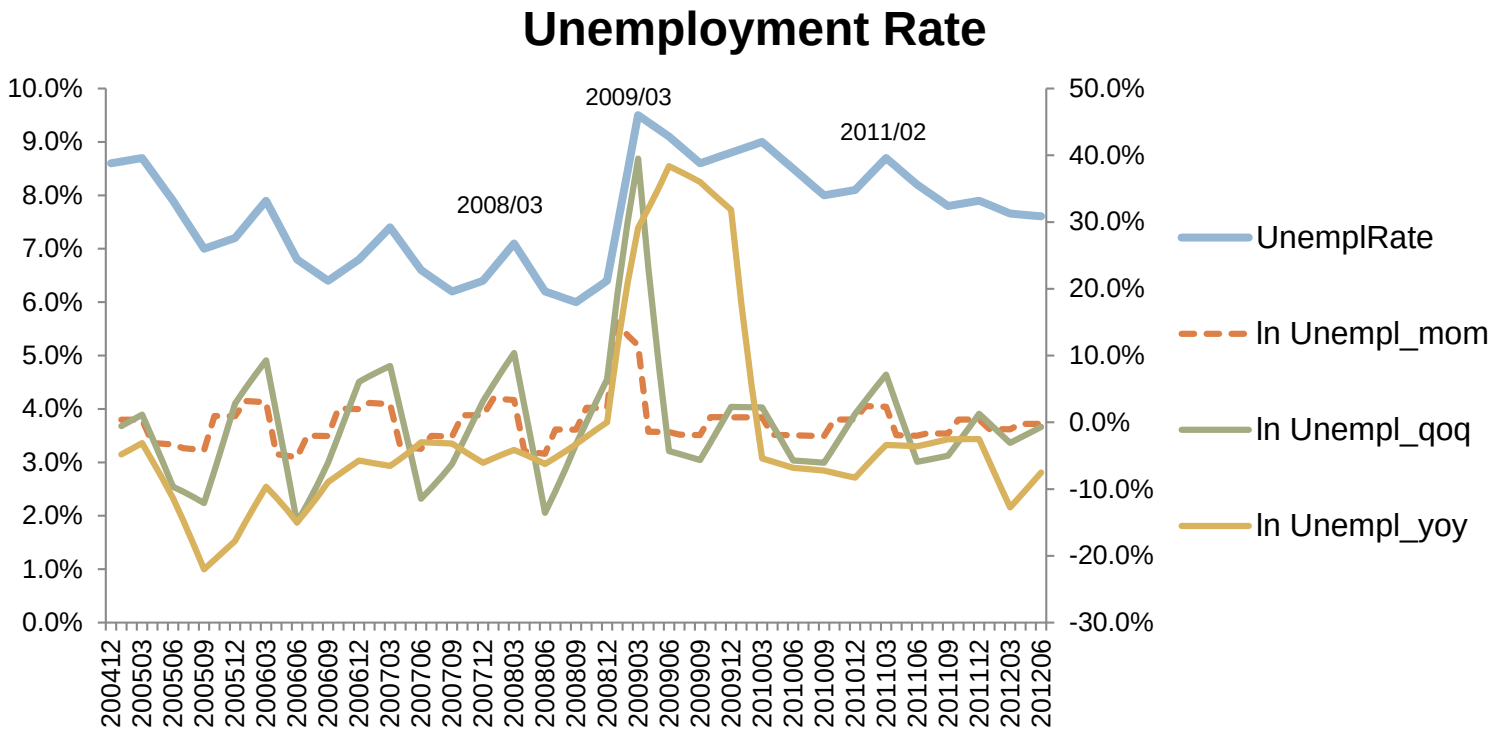
i) Logistic regression as conditional probability

ii) Comparative analysis of panel regression

- pooled and
- random effect

models estimations

Macroeconomics and cycles



Macroeconomic indicators contain cycles and fluctuations.
They have an impact on client behaviour as systematic factor

Panel linear regression

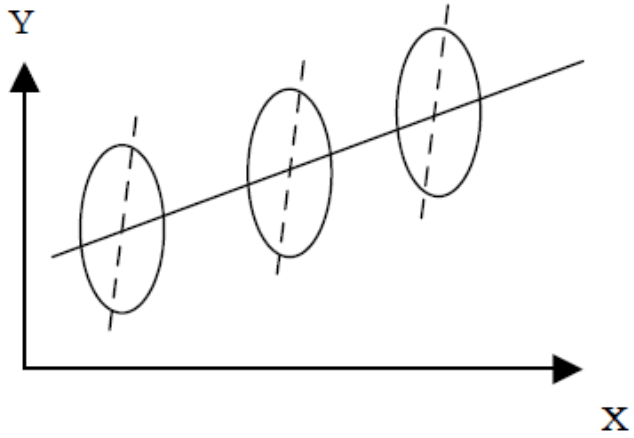
Fixed effect model –

$y_{it} = (\alpha + u_i) + X'_{it}\beta + v_{it}$

α_i - individual intercept (specific effect)

Intercept is varying across groups and/or times

Error variance is constant



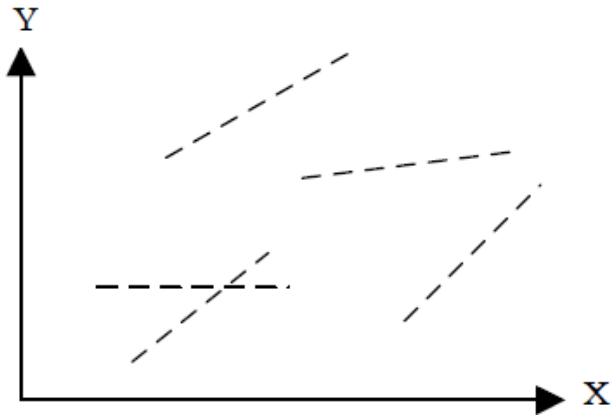
Random effect model –

$y_{it} = \alpha + X'_{it}\beta + (u_i + v_{it})$

$(u_i + v_{it})$ - Random effect

Intercept is constant

Error variance is varying across groups and/or times



Four variance component methods of regression analysis for random-effect models:

Wallace and Hussain, Wansbeek and Kapteyn, Fuller and Battese, Nerlove's

Non-interest income modelling

1st stage – estimation of the probability that the client will use credit cards for POS/ATM transaction during the forecast period

$$\ln\left(\frac{P_i}{1-P_i}\right) = \phi\frac{1}{T}\sum_{l=1}^{T} UT_{i(t-l)} + \sum_{k=1}^{K}\beta_k \cdot B_{ki,t-1} + \sum_{l=1}^{L}\alpha_a \cdot A_{ai} + \sum_{m=1}^{M}\gamma_m M_{m,t-1}$$

2nd stage – income amount for the period

$$POS_{it} = \phi\frac{1}{T}\sum_{n=1}^{T} UT_{i(t-n)} + \sum_{k=1}^{K}\beta_k \cdot B_{bi,t-1} + \sum_{l=1}^{L}\alpha_a \cdot A_{ai} + \sum_{m}^{M}\gamma_m M_{m,t-1}$$

φ, α, β, γ – regression coefficients (slopes)

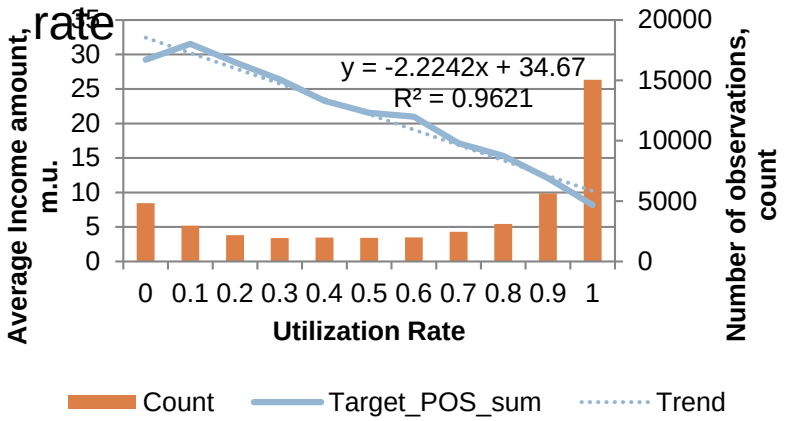
B – vector of behavioural factors (for example, average balance to maximum balance, maximum debit turnover to average outstanding balance or limit, maximum number days in delinquency, etc for some periods of time)

A – vector of application factors - client's demographic, financial and product characteristics like age, education, position, income, property, interest rate, etc.

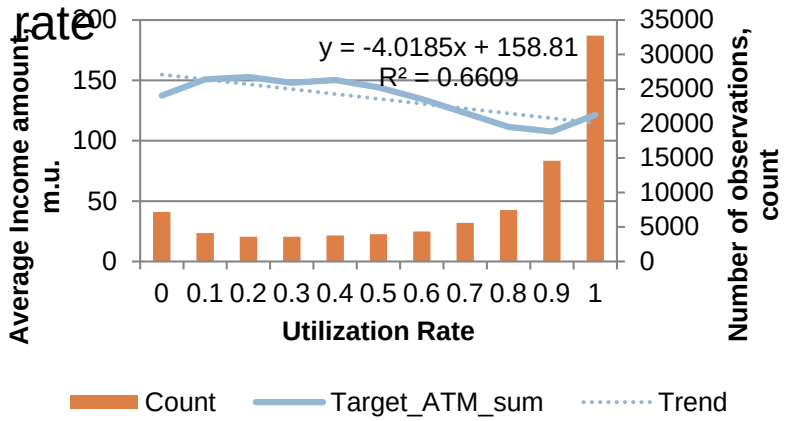
M – vector of macroeconomic factors (GDP, FX, Unemployment rate

Dependencies between POS/ATM Income and some covariates

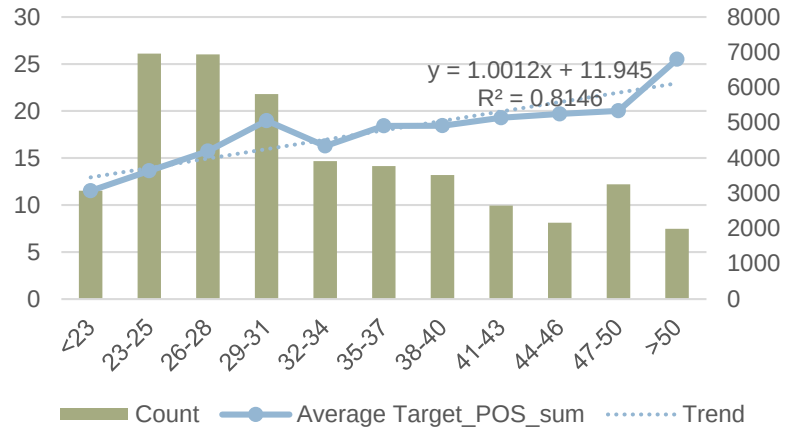
POS income on the utilization rate



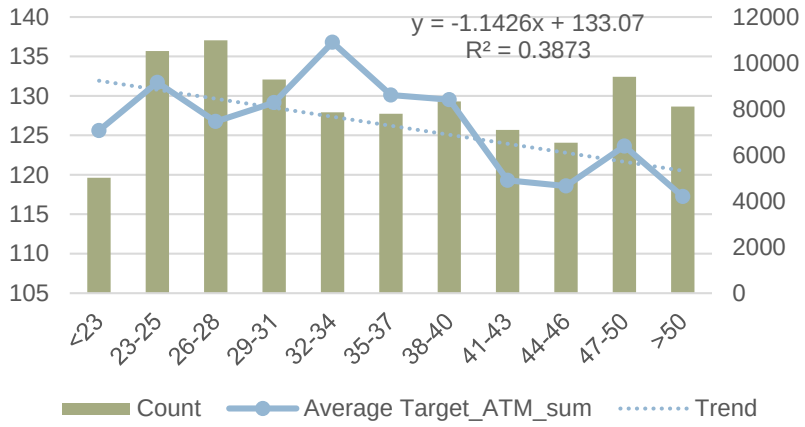
ATM income on the utilization rate



POS income on Age



ATM income on Age



The comparative analysis of pooled and random effect regression coefficient estimations for POS and ATM income (1/2)

Variable	Pooled					Random effect - WK				
	POS		ATM		Signs	POS		ATM		Signs
	Estimate	Pr > |t|	Estimate	Pr > |t|		Estimate	Pr > |t|	Estimate	Pr > |t|	
Intercept	-13.258	-0.0003	-110.263	-0.0906	POS - ATM-	2.065767	-0.5533	104.889	-0.0848	POS + ATM+
mob	0.02774	-0.3116	2.41237	(<.0001)	POS + ATM+	0.237426	(<.0001)	4.645451	(<.0001)	POS + ATM+
limit_6	0.000159	-0.0002	0.000703	-0.0006	POS + ATM+	-0.00004	-0.363	0.000083	-0.7236	POS - ATM+
UT0_6	-6.69484	(<.0001)	-66.2825	(<.0001)	POS - ATM-	-5.97111	(<.0001)	-40.8486	(<.0001)	POS - ATM-
UT0_7	3.669884	-0.0179	45.38276	(<.0001)	POS + ATM+	0.673567	-0.5397	22.1332	(<.0001)	POS + ATM+
UT0_8	2.446091	-0.0673	41.34974	(<.0001)	POS + ATM+	0.581465	-0.5358	16.82168	(<.0001)	POS + ATM+
UT0_9	-0.30908	-0.8108	22.88434	(<.0001)	POS - ATM+	-0.58488	-0.5179	7.568884	-0.059	POS - ATM+
UT0_10	0.103936	-0.933	-1.47977	-0.7635	POS + ATM-	-0.30802	-0.7219	-3.80606	-0.3107	POS - ATM-
UT0_11	-1.5758	-0.0779	17.3266	(<.0001)	POS - ATM+	-0.04502	-0.9452	8.414491	-0.0029	POS - ATM+
b_UT1to2ln	-0.02676	-0.7478	-0.7358	-0.0539	POS - ATM-	-0.0252	-0.6667	-1.07603	-0.0002	POS - ATM-
b_UT1to6ln	0.08954	-0.0718	1.88246	(<.0001)	POS + ATM+	0.090115	-0.0134	2.261019	(<.0001)	POS + ATM+
avg_balance_6	0.000069	-0.7328	0.007904	(<.0001)	POS + ATM+	0.000571	(<.0001)	0.000277	-0.7122	POS + ATM+
avg_balance_7	-0.0008	-0.0006	-0.00416	-0.0001	POS - ATM-	-0.00024	-0.1467	-0.00547	(<.0001)	POS - ATM-
avg_balance_8	-0.00049	-0.0257	-0.00214	-0.0353	POS - ATM-	-0.00027	-0.0844	-0.00259	-0.0008	POS - ATM-
avg_balance_9	0.000113	-0.5999	0.000939	-0.3462	POS + ATM+	-0.00008	-0.5751	0.000054	-0.9433	POS - ATM+
avg_balance_10	0.000052	-0.8061	0.001659	-0.0821	POS + ATM+	-0.00022	-0.1283	0.00067	-0.3584	POS - ATM+
avg_balance_11	0.000576	-0.0009	-0.001	-0.1949	POS + ATM-	-0.00004	-0.7375	0.000771	-0.2032	POS - ATM+
avg_deb_amt_6	0.000127	-0.7854	0.007811	-0.0004	POS + ATM+	-0.00189	(<.0001)	-0.00063	-0.7233	POS - ATM-
avg_deb_amt_7	0.002277	(<.0001)	0.008475	(<.0001)	POS + ATM+	-0.00066	-0.0447	-0.00267	-0.1251	POS - ATM-
avg_deb_amt_8	0.002203	(<.0001)	0.007099	-0.0007	POS + ATM+	-0.00035	-0.2581	-0.00282	-0.0932	POS - ATM-
avg_deb_amt_9	0.001359	-0.0002	0.002934	-0.1071	POS + ATM+	-0.00032	-0.2373	-0.0016	-0.2712	POS - ATM-
avg_deb_amt_10	0.000553	-0.1011	0.000104	-0.9504	POS + ATM+	-0.0005	-0.0418	-0.00361	-0.0066	POS - ATM-
avg_deb_amt_11	0.000602	-0.0364	0.000625	-0.6256	POS + ATM+	-0.00018	-0.3972	-0.00143	-0.1576	POS - ATM-
sum_crd_amt_6	0.001207	(<.0001)	0.013912	(<.0001)	POS + ATM+	0.001036	(<.0001)	0.008854	(<.0001)	POS + ATM+
sum_crd_amt_7	0.000576	-0.0013	0.010262	(<.0001)	POS + ATM+	0.000862	(<.0001)	0.003996	(<.0001)	POS + ATM+
sum_crd_amt_8	0.000044	-0.8237	0.007578	(<.0001)	POS + ATM+	0.000639	(<.0001)	0.000455	-0.5323	POS + ATM+
sum_crd_amt_9	-0.0001	-0.6053	0.0059	(<.0001)	POS - ATM+	0.000329	-0.0146	-0.00221	-0.0013	POS + ATM-
sum_crd_amt_10	-0.00005	-0.7737	0.006921	(<.0001)	POS - ATM+	0.000114	-0.3175	-0.0016	-0.0051	POS + ATM-
sum_crd_amt_11	0.000187	-0.0575	0.005903	(<.0001)	POS + ATM+	-0.00015	-0.0288	-0.00138	-0.0001	POS - ATM-
sum_deb_amt_6	0.002086	(<.0001)	-0.00686	(<.0001)	POS + ATM-	-0.00084	-0.0005	-0.00262	-0.0532	POS - ATM-
sum_deb_amt_7	0.001519	(<.0001)	-0.00519	-0.0051	POS + ATM-	-0.00119	(<.0001)	0.001582	-0.283	POS - ATM+
sum_deb_amt_8	0.001589	(<.0001)	-0.00954	(<.0001)	POS + ATM-	-0.00166	(<.0001)	0.001747	-0.2563	POS - ATM+
sum_deb_amt_9	0.002183	(<.0001)	-0.0091	(<.0001)	POS + ATM-	-0.00153	(<.0001)	0.000831	-0.5743	POS - ATM+
sum_deb_amt_10	0.002049	(<.0001)	-0.00878	(<.0001)	POS + ATM-	-0.0012	(<.0001)	0.002214	-0.0884	POS - ATM+
sum_deb_amt_11	0.002314	(<.0001)	-0.00746	(<.0001)	POS + ATM-	-0.00075	-0.0001	0.00013	-0.9058	POS - ATM+

The comparative analysis of pooled and random effect regression coefficient estimations for POS and ATM income (2/2)

Variable	Pooled					Random effect - WK				
	POS		ATM		Signs	POS		ATM		Signs
	Estimate	Pr > |t|	Estimate	Pr > |t|		Estimate	Pr > |t|	Estimate	Pr > |t|	
b_OB13_to_OB46ln	0.2522	-0.0062	4.11269	(<.0001)	POS + ATM+	-0.00635	-0.9235	0.975792	-0.0055	POS - ATM+
b_OB1_to_OB2_ln	0.195806	-0.2192	3.898535	(<.0001)	POS + ATM+	0.159666	-0.1637	2.16112	-0.0001	POS + ATM+
b_OB2_to_OB3_ln	0.192529	-0.0763	3.405814	(<.0001)	POS + ATM+	0.179691	-0.0209	1.84586	(<.0001)	POS + ATM+
b_OB3_to_OB4_ln	-0.00162	-0.9787	1.603699	(<.0001)	POS - ATM+	0.007994	-0.8547	1.441582	(<.0001)	POS + ATM+
b_OB_avg_to_eop1ln	-0.33916	-0.021	-6.12331	(<.0001)	POS - ATM-	-0.41969	-0.0002	-3.03191	(<.0001)	POS - ATM-
b_pos_flag_use13vs46	-3.22164	(<.0001)	-12.0898	(<.0001)	POS - ATM-	-0.06108	-0.772	2.288242	-0.0198	POS - ATM+
b_pos_use_only_flag_13	7.612892	(<.0001)	14.18524	(<.0001)	POS + ATM+	-0.42913	-0.0503	6.272027	(<.0001)	POS - ATM+
b_TRsum_crd1_to_OB1_ln	0.229749	-0.1523	10.14031	(<.0001)	POS + ATM+	0.146164	-0.2173	3.097325	(<.0001)	POS + ATM+
b_TRsum_crd2_to_OB2_ln	0.050826	-0.7338	6.669577	(<.0001)	POS + ATM+	-0.1271	-0.2469	0.869902	-0.1022	POS - ATM+
b_TRsum_crd3_to_OB3_ln	0.019486	-0.8817	5.264313	(<.0001)	POS + ATM+	-0.018	-0.8531	0.307452	-0.514	POS - ATM+
b_payment_lt_5p_1	-0.1205	-0.6163	-0.97148	-0.2799	POS - ATM-	-0.49178	-0.0062	0.200038	-0.7856	POS - ATM+
b_payment_lt_5p_2	-0.18662	-0.4402	-2.13679	-0.0191	POS - ATM-	-0.55114	-0.0021	-0.99327	-0.1781	POS - ATM-
b_payment_lt_5p_3	-0.01249	-0.9583	-3.22266	-0.0003	POS - ATM-	-0.58614	-0.001	-1.34531	-0.0667	POS - ATM-
b_TRsum_deb1_to_2_ln	0.040732	-0.5334	0.906685	-0.0188	POS + ATM+	0.01449	-0.7526	0.585638	-0.0481	POS + ATM+
b_TRsum_crd1_to_2_ln	0.081813	-0.4876	-1.17756	-0.0313	POS + ATM-	-0.00857	-0.9186	-0.77838	-0.0662	POS - ATM-
l_ch1_ln	2.317187	-0.0776	21.60541	-0.0001	POS + ATM+	0.818812	-0.3884	15.27913	-0.0006	POS + ATM+
l_ch1_flag	-0.62339	-0.3848	-7.60285	-0.007	POS - ATM-	0.466294	-0.3631	-6.75986	-0.0021	POS + ATM-
l_ch6_flag	-1.55193	(<.0001)	-7.46872	(<.0001)	POS - ATM-	-0.56799	-0.011	-7.036	(<.0001)	POS - ATM-
age	0.077702	(<.0001)	-0.34863	(<.0001)	POS + ATM-	0.124086	-0.0047	-0.84843	(<.0001)	POS + ATM-
customer_income_ln	3.570669	(<.0001)	10.80017	(<.0001)	POS + ATM+	8.851626	(<.0001)	62.34363	(<.0001)	POS + ATM+
Edu_High	0.710991	-0.0022	-2.73098	-0.0034	POS + ATM-	1.138817	-0.1833	-7.19017	-0.0414	POS + ATM-
Edu_Special	-0.52136	-0.0314	-0.18141	-0.8368	POS - ATM-	-0.92587	-0.3016	-1.45208	-0.6639	POS - ATM-
Edu_TwoDegree	0.924364	-0.0642	-6.61697	-0.0105	POS + ATM-	2.812073	-0.1278	-5.54633	-0.5704	POS + ATM-
Marital_Civ	-0.19314	-0.5905	5.012159	-0.0006	POS - ATM+	-0.09605	-0.9427	12.17613	-0.0284	POS - ATM+
Marital_Div	-0.09685	-0.7135	-1.15616	-0.257	POS - ATM-	-0.1643	-0.8666	-0.55611	-0.8851	POS - ATM-
Marital_Sin	0.555087	-0.0333	5.213276	(<.0001)	POS + ATM+	0.741467	-0.444	6.356839	-0.1318	POS + ATM+
Marital_Wid	-0.91638	-0.1631	2.619305	-0.1694	POS - ATM+	-1.26513	-0.605	2.562945	-0.7237	POS - ATM+
position_Man	0.596089	-0.0204	2.973025	-0.0059	POS + ATM+	0.755453	-0.4283	3.991961	-0.3284	POS + ATM+
position_Oth	-0.32252	-0.2188	2.226672	-0.0214	POS - ATM+	-0.12061	-0.9006	4.539805	-0.2152	POS - ATM+
position_Tech	-1.5069	(<.0001)	3.960217	(<.0001)	POS - ATM+	-2.13091	-0.0184	5.109176	-0.1335	POS - ATM+
position_Top	-2.14614	(<.0001)	4.257241	-0.0592	POS - ATM+	-3.05558	-0.1245	10.71047	-0.21	POS - ATM+
sec_Agricult	-3.19065	(<.0001)	1.388888	-0.4238	POS - ATM+	-4.81419	-0.0424	3.113879	-0.6353	POS - ATM+
sec_Constr	0.997211	-0.1357	-9.4891	(<.0001)	POS + ATM-	1.017228	-0.6767	-13.7435	-0.1283	POS + ATM-
sec_Energy	-1.97811	(<.0001)	-3.82661	-0.0114	POS - ATM-	-2.15029	-0.143	-4.04122	-0.481	POS - ATM-
sec_Fin	0.744777	-0.0045	-14.6702	(<.0001)	POS + ATM-	2.095365	-0.0285	-15.4275	-0.001	POS + ATM-
sec_Industry	-1.16178	-0.1865	11.92326	-0.0002	POS - ATM+	-0.70728	-0.8274	5.502724	-0.6516	POS - ATM+
sec_Manufact	0.101362	-0.8773	-8.3872	-0.0003	POS + ATM-	-0.99453	-0.6809	-12.2259	-0.1623	POS - ATM-
sec_Service	-0.73702	-0.0008	-2.87355	-0.0007	POS - ATM-	-0.54696	-0.5011	-3.63644	-0.2564	POS - ATM-
sec_Trade	-0.41385	-0.1086	-1.07519	-0.3763	POS - ATM-	-0.0893	-0.9254	5.934037	-0.1952	POS - ATM+

Pooled and random-effect models performance

Assessing the fit of POS income model - second stage: conditional on positive POS transaction (POS Sum 6 month > 0)

	Development			Validation		
Regression Type	R^2	RMSE	MAE	R^2	RMSE	MAE
Pooled	0.2909	15.6793	10.6822	0.2962	15.8619	10.8615
Random effect - One-way						
Wansbeek and Kapteyn	0.1633	17.0360	11.7869	0.1518	17.4598	12.3974
Fuller and Battese	0.1784	16.8774	11.6539	0.1671	17.2891	12.2507
Wallace and Hussain	0.1925	16.7335	11.5346	0.1816	17.1320	12.1148
Nerlove	0.1606	17.0648	11.8113	0.1491	17.4905	12.4237

Assessing the Fit of Two-stage model result – POS Sum X Pr(POS > 0)

	Development			Validation		
Regression Type	R^2	RMSE	MAE	R^2	RMSE	MAE
Pooled	0.3039	15.1762	7.3214	0.30779	15.8853	7.61005
Random effect - One-way						
Wansbeek and Kapteyn	0.1793	16.4787	8.5385	0.19608	17.1192	8.79218
Fuller and Battese	0.1909	16.3611	8.4428	0.20735	16.9988	8.69906
Wallace and Hussain	0.2018	16.2509	8.3526	0.21776	16.8868	8.61174
Nerlove	0.1772	16.4996	8.5555	0.19405	17.1408	8.80873

Performance quality of the income prediction functions

Model	Regression equation	Target	Results
Probability of POS transaction	Logistic regression $\ln\left(\frac{P_i}{1-P_i}\right)=\sum_{k=1}^{K}\beta_k\cdot B_{ki,t-1}+\sum_{l=1}^{L}\alpha_a\cdot A_{ai}+\sum_{m=1}^{M}\gamma_m M_{m,t-1}$	POS transaction next 6 month	ROC=0.83
Probability of ATM withdrawal	Logistic regression $\ln\left(\frac{P_i}{1-P_i}\right)=\sum_{k=1}^{K}\beta_k\cdot B_{ki,t-1}+\sum_{l=1}^{L}\alpha_a\cdot A_{ai}+\sum_{m=1}^{M}\gamma_m M_{m,t-1}$	ATM withdrawal next 6 month	ROC=0.86
POS income (interchange)	Panel regression: polled $POS_i=\sum_{k=1}^{K}\beta_k\cdot B_{ik}+\sum_{l=1}^{L}\alpha_a\cdot A_{ai}+\sum_{m=1}^{M}\gamma_m M_m$	POS Income next 6 month	R^2~0.29
POS income (interchange)	Panel regression: random-effect $POS_{it}=\sum_{k=1}^{K}\beta_k\cdot B_{bi,t-1}+\sum_{l=1}^{L}\alpha_a\cdot A_{ai}+\sum_{m}^{M}\gamma_1 M_{m,t-1}$	POS Income next 6 month	R^2~ 0.19
ATM withdrawal income	Panel regression: polled $ATM_i=\sum_{k=1}^{K}\beta_k\cdot B_{ik}+\sum_{l=1}^{L}\alpha_a\cdot A_{ai}+\sum_{m=1}^{M}\gamma_m M_m$	ATM withdrawal income next 6 month	R^2 ~ 0.28
ATM withdrawal income	Panel regression: random-effect $ATM_{it}=\sum_{k=1}^{K}\beta_k\cdot B_{bi,t-1}+\sum_{l=1}^{L}\alpha_a\cdot A_{ai}+\sum_{m}^{M}\gamma_1 M_{m,t-1}$	ATM withdrawal income next 6 month	R^`2~ 0.15

Credit Card Holders State Transition Probabilities

Multinomial, ordinal, and conditional logistic regression for non-binary target prediction

Problem definition

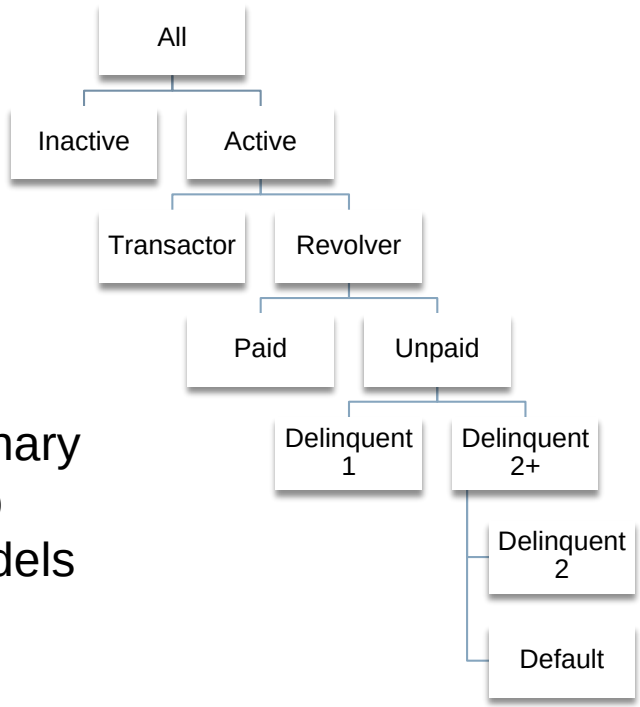
It is required to find all possible transition probabilities for each state to all possible states. For example, for account number 1111, which has 'transactor' state, the task corresponds with multinomial logistic regression problem definition:

Account ID	Transition to State	Probability (predicted)
1111	N	0.01
1111	T	0.17
1111	R	0.67
1111	D	0.15

$$\pi_{ij} = \frac{\exp(x_i^T \beta_j)}{1 + \sum_{k \neq j^*} \exp(x_i^T \beta_k)}$$

Or with ordinal logistic regression if the order of states can be defined

$$\ln \frac{\Pr(Y_i = 1)}{\Pr(Y_i = K)} = \boldsymbol{\beta}_1 \cdot \mathbf{X}_i$$
$$\ln \frac{\Pr(Y_i = 2)}{\Pr(Y_i = K)} = \boldsymbol{\beta}_2 \cdot \mathbf{X}_i$$
$$\cdots\cdots$$
$$\ln \frac{\Pr(Y_i = K-1)}{\Pr(Y_i = K)} = \boldsymbol{\beta}_{K-1} \cdot \mathbf{X}_i$$



However, this problem can be presented as a binary decision tree where number of leaves is equal to number of states S and number of transition models is S-1.

Prediction horizon for this exercise – 1 month

Conditional binary logistic regression

$$\Pr(s_{t+1} = NA) = \boldsymbol{\beta}_{NA}^{\mathrm{T}}\mathbf{x}$$

$$\frac{\Pr(s_{t+1} = TR)}{1 - \Pr(s_{t+1} = NA)} = \boldsymbol{\beta}_{TR}^{\mathrm{T}}\mathbf{x}$$

$$\frac{\Pr(s_{t+1} = RE)}{(1 - \Pr(s_{t+1} = TR))(1 - \Pr(s_{t+1} = NA))} = \boldsymbol{\beta}_{RE}^{\mathrm{T}}\mathbf{x}$$

$$\frac{\Pr(s_{t+1} = RP \mid s_t \in RE)}{(1 - \Pr(s_{t+1} = TR))(1 - \Pr(s_{t+1} = NA))(1 - \Pr(s_{t+1} = RE))} = \boldsymbol{\beta}_{RP}^{\mathrm{T}}\mathbf{x}$$

$$\frac{\Pr(s_{t+1} = D1 \mid s_t \notin (NA, TR, RP))}{(1 - \Pr(s_{t+1} = NA))(1 - \Pr(s_{t+1} = TR))(1 - \Pr(s_{t+1} = RE))} = \boldsymbol{\beta}_{D1}^{\mathrm{T}}\mathbf{x}$$

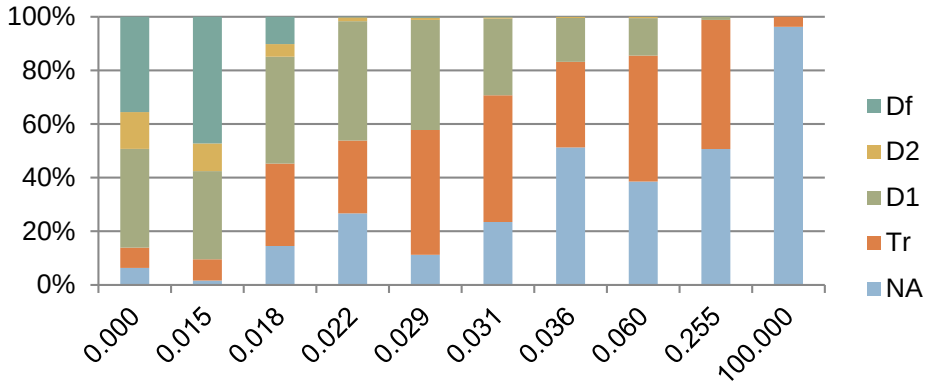
$$\frac{\Pr(s_{t+1} = D2 \mid s_t \notin (NA, TR, RE, RP))}{(1 - \Pr(s_{t+1} = NA))(1 - \Pr(s_{t+1} = TR))(1 - \Pr(s_{t+1} = RE))(1 - \Pr(s_{t+1} = D1))} = \boldsymbol{\beta}_{D2}^{\mathrm{T}}\mathbf{x}$$

$$\frac{\Pr(s_{t+1} = DF \mid s_t \notin (NA, TR, RE, RP, D1))}{(1 - \Pr(s_{t+1} = NA))(1 - \Pr(s_{t+1} = TR))(1 - \Pr(s_{t+1} = RE))(1 - \Pr(s_{t+1} = D1))(1 - \Pr(s_{t+1} = D2))} = \boldsymbol{\beta}_{DF}^{\mathrm{T}}\mathbf{x}$$

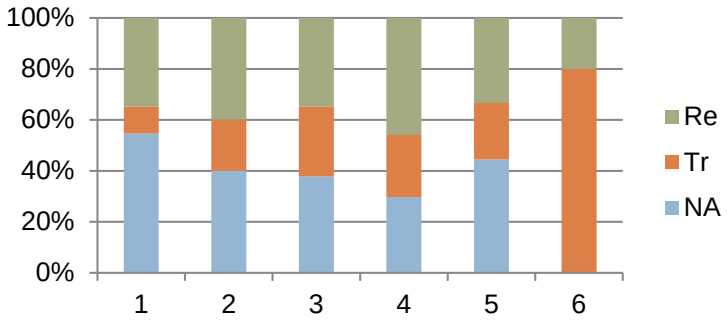
At each stage we predict the probability of transition to one of the states at the next level conditional on the state at the high level. For a full description of all stages we need six equations for non-active, transactor, revolver, delinquent 1, delinquent 2, and defaulted states.

Univariate analysis

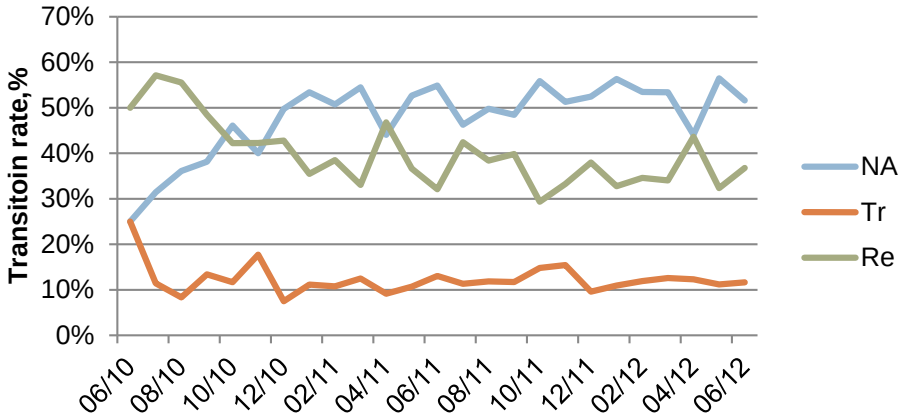
Univariate distiribution from All S t to S t+1: b_TRavg_deb1_to_avgOB1_ln NO-Revolvers



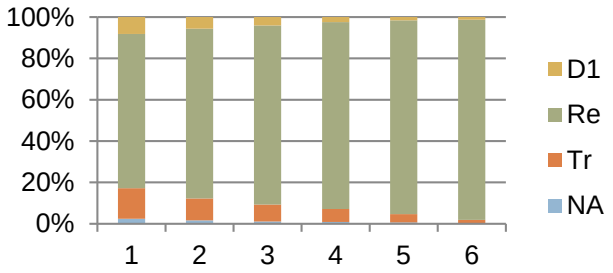
Univariate distiribution from S t=Tr to S t+1: Consecutive Months in Current State



Transitions from Transactor by months



Univariate distiribution from S t=Re to S t+1: Consecutive Months in Current State



Transition Matrix

Number of transitions from state S t to state S t+1

	To						
From	**NA**	**Tr**	**Re**	**D1**	**D2**	**Df**	**Total**
NA	21588	856	3285				25729
Tr	3578	808	2391				6777
Re	753	5079	154023	3201			163056
D1		29	2302	956	743		4030
D2			191	105	65	426	787
Total	25919	6772	162192	4262	808	426	200379

Percent of transitions from state S t to state S t+1

	To						
From	**NA**	**Tr**	**Re**	**D1**	**D2**	**Df**	**Total**
NA	83.91%	3.33%	12.77%	0.00%	0.00%	0.00%	12.84%
Tr	52.80%	11.92%	35.28%	0.00%	0.00%	0.00%	3.38%
Re	0.46%	3.11%	94.46%	1.96%	0.00%	0.00%	81.37%
D1	0.00%	0.72%	57.12%	23.72%	18.44%	0.00%	2.01%
D2	0.00%	0.00%	24.27%	13.34%	8.26%	54.13%	0.39%
Total	**12.93%**	**3.38%**	**80.94%**	**2.13%**	**0.40%**	**0.21%**	**100.00%**

Multinomial Regression Coefficients Estimations - Part 1: Non-active and Transactors

Different signs for different targets

FROM	NA				Tr			
TO	NA		Re		NA		Re	
Characteristic	Estimate	Pr > ChiSq	Estimate	Pr > ChiSq	Estimate	Pr > ChiSq	Estimate	Pr > ChiSq
AgeGRP1	-0.1401	0.2754	0.00529	0.9696	-0.0649	0.6739	0.1653	0.2862
AgeGRP3	-0.0656	0.5703	-0.0331	0.7908	-0.2087	0.124	-0.1713	0.2094
avg_balance_1	-1.0877	0.0745	0.00511	0.9934	-0.00012	0.1388	-0.00014	0.0886
avg_balance_2	-0.00026	0.0096	-0.00028	0.0096	0.000002498	0.9728	0.00008	0.2721
avg_balance_3	0.000096	0.1894	0.000123	0.1078	0.000022	0.7021	-0.00006	0.339
b_atm_flag_1	-1.8317	0	-0.4584	0.021	-0.2975	0.0351	-0.1159	0.4037
b_atm_flag_2	-0.35	0.0686	-0.1669	0.4086	0.0774	0.5802	0.1955	0.1582
b_AvgOB1_to_MaxOB1_l	-0.0369	0.7329	0.0587	0.5736	3.9918	0.2435	-2.9955	0.044
b_AvgOB2_to_MaxOB2_l	0.2188	0.1429	0.2106	0.2747	-0.1817	0.2626	-0.1045	0.5293
b_AvgOB3_to_MaxOB3_l	-0.2635	0.2548	-0.2639	0.268	-0.1918	0.291	0.1185	0.5484
b_fullpaid1	0.2684	0.8777	0.3536	0.8429	0.4419	0.2072	0.519	0.1429
b_inactive2	0.4123	0.5309	-0.3916	0.577	-0.023	0.9758	-0.7323	0.3487
b_inactive3	1.2187	0.0167	0.7085	0.197	0.3896	0.5829	0.831	0.2634
b_maxminOB_avgOB_1_l	0.0733	0.7298	0.1868	0.3898	-1.6626	0.3264	-2.7864	0.0433
b_maxminOB_avgOB_2_l	0.1626	0.1634	0.1613	0.2193	-0.2185	0.0238	-0.1909	0.0542
b_maxminOB_avgOB_3_l	-0.1188	0.281	-0.0973	0.3951	-0.0814	0.3955	0.0859	0.3929
b_maxminOB_limit_1_l	0.00125	0.9738	-0.0488	0.2208	-0.0898	0.0366	-0.0217	0.6253
b_maxminOB_limit_2_l	0.0292	0.3988	0.037	0.3203	0.0443	0.3698	0.0215	0.6688
b_maxminOB_limit_3_l	0.00505	0.8872	-0.0168	0.6566	0.0562	0.2331	-0.00085	0.9858
b_OB_avg_to_eop1ln	-0.0313	0.8497	-0.0764	0.6496	-0.0243	0.8119	-0.0645	0.5364
b_OBbias_1_ln	-0.0948	0.5972	-0.0614	0.7399	-0.2073	0.2417	-0.1974	0.2779
b_OBbias_2_ln	0.0771	0.3805	0.0387	0.7049	0.0306	0.6797	-0.0324	0.6697
b_OBbias_3_ln	-0.0245	0.8093	-0.0541	0.6067	0.0864	0.2591	0.1294	0.1101
b_payment_lt_5p_1	0.5741	0.4554	0.6278	0.4221	-0.7728	0.3219	-0.6128	0.4536
b_pos_flag_1	-1.7162	0	-0.6021	0.0013	-0.5502	0	-0.3985	0.0035
b_pos_flag_2	-0.5217	0.0028	-0.4588	0.013	-0.1066	0.4301	0.2704	0.0461
b_TRavg_deb1_to_avgO	-0.00316	0.9951	1.0052	0.0525	0.237	0.1162	0.0571	0.6797
b_TRavg_deb2_to_avgO	0.4652	0.0693	0.2523	0.3471	0.2386	0.0948	0.0135	0.9217
b_TRavg_deb3_to_avgO	0.3055	0.1234	0.1213	0.559	0.2306	0.106	-0.1216	0.3749
b_TRmax_deb1_To_avgO	1.325	0.5718	-1.5656	0.5052	-0.9042	0.0078	-0.7877	0.0176
b_TRmax_deb1_To_Limi	-4.856	0	0.6618	0.1433	0.117	0.5699	0.2881	0.1095
b_TRmax_deb2_To_avgO	-1.3985	0.0118	-0.589	0.3088	-0.2418	0.4754	0.386	0.2474
b_TRmax_deb2_To_Limi	0.3656	0.4147	0.2313	0.6135	0.0228	0.9045	-0.0344	0.8431
b_TRmax_deb3_To_avgO	-0.1781	0.6809	-0.4546	0.3084	-0.396	0.2322	-0.4472	0.1628
b_TRmax_deb3_To_Limi	-0.1415	0.6309	-0.1188	0.6906	-0.0426	0.8124	-0.204	0.2147
b_TRsum_crd1_to_OB1_	0.225	0	0.2076	0	0.3621	0.002	-0.0736	0.5096
b_TRsum_deb1_to_avgO	-1.5762	0.4881	0.3922	0.8631	0.1076	0.7743	0.7248	0.0484
b_TRsum_deb1_to_TRsu	0.1312	0.0394	0.1363	0.039	0.4341	0.0005	0.0268	0.8265
b_TRsum_deb2_to_avgO	0.8319	0.131	0.2948	0.6079	-0.124	0.7241	-0.4632	0.1831
b_TRsum_deb2_to_TRsu	-0.0202	0.3984	0.0023	0.928	0.041	0.0381	0.0385	0.057
b_TRsum_deb3_to_avgO	-0.2825	0.4943	0.2365	0.5775	0.0316	0.9212	0.4564	0.1422
b_TRsum_deb3_to_TRsu	-0.00828	0.6723	-0.0186	0.3729	0.0249	0.2013	0.03	0.1323

FROM	NA				Tr			
TO	NA		Re		NA		Re	
Characteristic	Estimate	Pr > ChiSq	Estimate	Pr > ChiSq	Estimate	Pr > ChiSq	Estimate	Pr > ChiSq
customer_income_ln	-0.0231	0.7836	-0.0562	0.5395	0.1905	0.051	-0.0253	0.7968
Edu_Special	-0.096	0.4921	-0.117	0.4335	0.0708	0.6133	-0.0726	0.634
Edu_High	-0.2605	0.0594	-0.2517	0.0888	-0.1439	0.3389	-0.1483	0.3243
Edu_TwoDegree	-0.4174	0.0755	-0.375	0.1422	0.4218	0.137	0.1876	0.4972
Intercept	33.4798	0.9896	116	0.964	288.6	0.3261	346.9	0.2352
l_ch1_flag	0.6351	0.3469	0.2946	0.6796	-1.0417	0.0741	-0.5053	0.3709
l_ch1_ln	-2.0827	0.0156	-0.8491	0.3477	0.9629	0.3323	0.6677	0.4923
l_ChangeFlag	0.1413	0.4054	0.1013	0.5762	0.2537	0.1495	0.3392	0.0515
Marital_Civ	0.2618	0.2407	0.3286	0.1608	-0.0186	0.9333	-0.0798	0.7198
Marital_Div	0.1093	0.4282	0.0815	0.5818	0.1325	0.4022	0.2102	0.1841
Marital_Sin	-0.0147	0.9106	-0.1339	0.3448	-0.1124	0.4763	-0.1695	0.2842
Marital_Wid	0.1452	0.5833	-0.0355	0.902	0.1943	0.5764	0.2399	0.4972
mob	0.0333	0.0008	0.00131	0.9038	0.000675	0.9546	0.00483	0.6856
position_Man	0.0331	0.7688	0.0669	0.5829	-0.1598	0.2084	-0.00124	0.9922
position_Oth	-0.0688	0.6015	0.085	0.5468	-0.1552	0.2708	-0.151	0.2915
position_Tech	-0.1826	0.1452	-0.0975	0.4712	0.2278	0.1282	0.0981	0.5204
position_Top	-0.1295	0.453	-0.1334	0.4826	-0.0529	0.7998	0.2592	0.2069
real_coOwn	-0.0234	0.822	-0.0955	0.3965	-0.2493	0.0404	-0.2497	0.0408
real_Own	0.0594	0.5424	0.0394	0.7084	-0.039	0.7375	-0.0403	0.7307
reg_ctr_N	0.1085	0.4468	-0.00543	0.972	0.1909	0.2274	-0.1511	0.3305
reg_ctr_Y	0.0937	0.5039	0.0122	0.9361	0.2802	0.0698	-0.0484	0.7484
SalaryYear_lnyoy_1	3.9776	0.1443	2.1943	0.4565	2.6257	0.4202	5.5404	0.0937
sec_Agricult	0.0818	0.721	0.1372	0.5757	-0.0934	0.727	0.2218	0.4112
sec_Constr	-0.0053	0.986	0.07	0.8297	0.2369	0.5105	-0.0714	0.8492
sec_Energy	0.1082	0.5595	-0.0441	0.8267	0.067	0.756	0.0946	0.6713
sec_Fin	-0.1609	0.1444	-0.1617	0.1775	0.1335	0.3299	0.3251	0.0174
sec_Industry	-0.4463	0.2844	0.059	0.8935	-0.1073	0.8024	0.2143	0.6037
sec_Manufact	0.1441	0.6814	0.1061	0.7787	0.7302	0.1761	1.0287	0.0572
sec_Mining	0.1796	0.4567	0.1052	0.6855	-0.0493	0.8541	0.2288	0.4031
sec_Service	0.0942	0.3796	0.0886	0.4424	-0.0387	0.7418	0.0933	0.4317
sec_Trade	-0.2466	0.0864	-0.2059	0.1882	-0.00678	0.9681	0.1981	0.2411
sec_Trans	-0.6096	0.008	-0.6825	0.0095	0.1127	0.7409	0.4829	0.1596
sum_deb_num_1	0.00598	0.9232	-0.0445	0.4003	-0.0167	0.5556	0.0199	0.3616
sum_deb_num_2	0.1377	0.0386	0.0963	0.1569	0.0127	0.5791	-0.011	0.5806
sum_deb_num_3	0.0561	0.1837	0.0623	0.1431	0.0134	0.5301	-0.0188	0.3154
UAH_EURRate_lnmom_1	4.3869	0.0273	0.9544	0.6598	3.753	0.1062	1.3449	0.5666
UAH_EURRate_lnyoy_1	-2.8508	0.0152	-2.7301	0.0323	-1.0697	0.4529	-2.5095	0.0833
Unempl_lnyoy_1	-0.7767	0.6223	-2.4789	0.1472	0.6729	0.7083	-0.3963	0.8259
UT0_1	0	0	0	0	0.6939	0.3457	0.8848	0.2235
UT0_2	1.4138	0.1216	1.6478	0.0819	-0.1118	0.8558	-0.7462	0.2216
UT0_3	-0.2679	0.6634	-0.177	0.7827	0.0272	0.9578	0.8574	0.0931

Multinomial Regression Coefficients Estimations – Revolvers and Delinquent 1M

FROM	Re				D1					
	Re		D1		Re		D1		D2	
Названия строк	Estimate	Pr > ChiSq	Estimate	Pr > ChiSq	Estimate	Pr > ChiSq	Estimate	Pr > ChiSq	Estimate	Pr > ChiSq
AgeGRP1	0.0431	0.4345	-0.035	0.6943	0.7902	0.2127	0.7207	0.2596	0.9913	0.1235
AgeGRP3	0.038	0.4357	-0.3453	0	0.7203	0.3225	0.7785	0.2889	0.5864	0.4275
avg_balance_1	0.000043	0.0521	-0.00006	0.1856	-0.0016	0.0102	-0.00174	0.0063	-0.00154	0.0179
avg_balance_2	0.000005822	0.8413	0.000037	0.5876	0.00202	0.0177	0.00218	0.0119	0.00194	0.0265
avg_balance_3	0.000002078	0.9279	0.00004	0.4117	-0.00009	0.8145	-0.00007	0.855	-0.00002	0.9533
b_atm_flag_1	0.3541	0	0.5159	0	-0.2066	0.8346	-0.2384	0.8109	-0.1783	0.8598
b_atm_flag_2	0.3215	0	0.2437	0.0148	0.6307	0.4548	1.0319	0.2278	0.7721	0.3705
b_AvgOB1_to_MaxOB1_l	-0.3035	0	1.2137	0.0041	-8.7827	0.6553	-9.8872	0.6211	-9.7357	0.6323
b_AvgOB2_to_MaxOB2_l	0.1264	0.0423	-0.3386	0.1443	6.9322	0.1394	8.4551	0.087	6.5431	0.1928
b_AvgOB3_to_MaxOB3_l	0.1353	0.0242	-0.0414	0.8338	1.2267	0.6859	0.1239	0.9678	2.8972	0.5009
b_fullpaid1	0	0	0	0	0	0	0	0	0	0
b_inactive2	-1.0136	0.0025	-2.4675	0.0853	0	0	0	0	0	0
b_inactive3	0.9021	0.002	3.9043	0.0219	40.9156	0.1195	38.8469	0.524	18.9994	0.8906
b_maxminOB_avgOB_1_l	-0.722	0	-1.2043	0	-24.0473	0.0024	-25.5607	0.0017	-27.564	0.0009
b_maxminOB_avgOB_2_l	-0.1563	0	-0.5273	0	26.4352	0.0008	25.9764	0.0014	28.5432	0.0005
b_maxminOB_avgOB_3_l	-0.012	0.7248	0.0312	0.7923	3.626	0.2628	2.3432	0.4741	0.6288	0.8503
b_maxminOB_limit_1_l	0.3009	0	1.2029	0	24.4757	0.0017	25.8325	0.0013	27.5588	0.0008
b_maxminOB_limit_2_l	-0.0126	0.5623	0.3303	0.0021	-25.0616	0.0012	-24.7011	0.0018	-27.3962	0.0007
b_maxminOB_limit_3_l	-0.1276	0	0.0559	0.5719	-4.0789	0.1757	-2.8052	0.3572	-1.2699	0.6824
b_OB_avg_to_eop1ln	-0.2993	0	-1.686	0	5.9593	0.7343	4.0961	0.8178	12.347	0.4912
b_OBbias_1_ln	0.081	0.0002	-0.0534	0.1229	0.0865	0.8235	-0.0426	0.9135	-0.1821	0.6484
b_OBbias_2_ln	0.0243	0.2394	-0.1469	0	-0.2945	0.3476	-0.23	0.467	-0.3385	0.2874
b_OBbias_3_ln	0.0195	0.3417	-0.0498	0.1296	-0.3549	0.258	-0.3818	0.2267	-0.3979	0.2094
b_payment_lt_5p_1	-0.1712	0.006	-0.1589	0.0855	-0.3399	0.6966	0.7997	0.3624	0.845	0.3437
b_pos_flag_1	-0.0236	0.6458	0.4922	0	-22.8388	0	1.0274	0	-6.136	0.9959
b_pos_flag_2	0.014	0.7841	0.3768	0	0.3285	0.5959	0.1084	0.8626	0.1132	0.8575
b_TRavg_deb1_to_avgO	-0.2567	0	-0.3326	0.0051	-0.3531	0.8625	-0.488	0.8125	-0.4463	0.8304
b_TRavg_deb2_to_avgO	0.0317	0.5278	-0.1889	0.12	-0.0954	0.9318	-0.4377	0.6978	0.0134	0.9906
b_TRavg_deb3_to_avgO	-0.1923	0.0002	-0.3432	0.0015	0.3594	0.7336	0.3064	0.7732	0.0903	0.9332
b_TRmax_deb1_To_avgO	0.5049	0	0.7291	0	0.3271	0.8745	0.0178	0.9932	0.9905	0.6444
b_TRmax_deb1_To_Limi	-0.07	0.0948	0.1696	0.0025	-11.5617	0.2437	-15.212	0.1429	-12.0183	0.2743
b_TRmax_deb2_To_avgO	0.3251	0.005	0.6423	0.0006	-0.9616	0.5152	-0.1157	0.9383	-1.1146	0.4582
b_TRmax_deb2_To_Limi	-0.0984	0.0112	-0.095	0.6321	2.6737	0.2929	0.8104	0.7621	3.0062	0.2404
b_TRmax_deb3_To_avgO	0.1548	0.1497	0.3934	0.0173	0.269	0.8281	0.2481	0.8428	0.181	0.8861
b_TRmax_deb3_To_Limi	-0.0608	0.1177	-0.4589	0.0906	5.0005	0.1416	4.7508	0.1676	2.468	0.4844
b_TRsum_crd1_to_OB1_	0.00358	0.8992	0.1589	0.0083	0.00291	0.9967	0.0811	0.9098	-0.0782	0.916
b_TRsum_deb1_to_avgO	-0.2786	0.024	-1.02	0	0.0809	0.972	0.5941	0.7981	-0.1532	0.9483
b_TRsum_deb1_to_TRsu	0.0842	0.0158	0.706	0	-0.0192	0.9758	0.1878	0.772	0.2249	0.737
b_TRsum_deb2_to_avgO	-0.3092	0.0102	-0.571	0.0046	-1.0602	0.5078	-1.6416	0.3116	-1.1316	0.4883
b_TRsum_deb2_to_TRsu	0.00758	0.3655	0.2388	0	-0.0297	0.6602	-0.053	0.4367	0.1218	0.0757
b_TRsum_deb3_to_avgO	-0.00571	0.9545	-0.2914	0.0523	-1.0863	0.3531	-1.0915	0.3545	-0.5742	0.63
b_TRsum_deb3_to_TRsu	0.000124	0.9882	0.0348	0.0032	-0.0933	0.1735	-0.1053	0.1269	-0.0644	0.3535

Different signs and p-value for different periods

Non-significant

Transition probability backtesting methodology – real response vs. target predicted probability

Multinomial

Response	Target_S1	0-0.1	0.1-0.2	0.2-0.3	0.3-0.4	0.4-0.5	0.5-0.6	0.6-0.7	0.7-0.8	0.8-0.9	0.9-1
Re	***Re***	***7.33%***	***12.57%***	***9.95%***	***6.81%***	***9.42%***	***7.85%***	***2.62%***	***3.14%***	***7.85%***	***32.46%***
	D1	45.71%	29.52%	12.38%	2.86%	3.81%	0.95%	4.76%	0.00%	0.00%	0.00%
	D2	44.62%	26.15%	13.85%	3.08%	7.69%	3.08%	1.54%	0.00%	0.00%	0.00%
	Df	53.05%	23.71%	11.97%	6.34%	2.58%	1.41%	0.70%	0.00%	0.00%	0.23%
Re Total		**40.28%**	**21.98%**	**11.69%**	**5.72%**	**4.83%**	**3.05%**	**1.78%**	**0.76%**	**1.91%**	**8.01%**
D1	Re	70.16%	16.23%	8.90%	2.62%	1.05%	1.05%	0.00%	0.00%	0.00%	0.00%
	D1	***16.19%***	***19.05%***	***19.05%***	***17.14%***	***8.57%***	***10.48%***	***5.71%***	***0.95%***	***1.90%***	***0.95%***
	D2	41.54%	33.85%	12.31%	7.69%	3.08%	1.54%	0.00%	0.00%	0.00%	0.00%
	Df	60.80%	20.89%	9.86%	4.46%	1.88%	1.17%	0.47%	0.47%	0.00%	0.00%
D1 Total		**55.53%**	**20.58%**	**11.05%**	**5.97%**	**2.67%**	**2.41%**	**1.02%**	**0.38%**	**0.25%**	**0.13%**
D2	Re	80.63%	12.57%	3.14%	2.62%	0.52%	0.52%	0.00%	0.00%	0.00%	0.00%
	D1	66.67%	15.24%	14.29%	0.95%	0.95%	1.90%	0.00%	0.00%	0.00%	0.00%
	D2	***21.54%***	***26.15%***	***12.31%***	***9.23%***	***12.31%***	***7.69%***	***4.62%***	***3.08%***	***1.54%***	***1.54%***
	Df	77.70%	12.21%	6.81%	1.64%	1.17%	0.23%	0.00%	0.23%	0.00%	0.00%
D2 Total		**72.30%**	**13.85%**	**7.37%**	**2.41%**	**1.91%**	**1.14%**	**0.38%**	**0.38%**	**0.13%**	**0.13%**
Df	Re	40.31%	6.81%	6.28%	9.95%	8.38%	12.04%	7.33%	3.66%	4.71%	0.52%
	D1	4.76%	8.57%	12.38%	16.19%	19.05%	11.43%	10.48%	9.52%	6.67%	0.95%
	D2	7.69%	15.38%	9.23%	13.85%	18.46%	15.38%	3.08%	9.23%	4.62%	3.08%
	Df Mult	***0.47%***	***1.41%***	***3.52%***	***3.52%***	***7.04%***	***10.80%***	***16.90%***	***17.14%***	***25.35%***	***13.85%***
Df Total		**11.31%**	**4.83%**	**5.84%**	**7.62%**	**9.91%**	**11.56%**	**12.58%**	**12.20%**	**16.14%**	**8.01%**

We predict the probability of transition to ALL possible states
Good model shows concentration in high probabilities for the same response and concentration in low probabilities for other responses.

Multinomial, Ordinal, and Binary Conditional logistic regression validation

		Ordinal		Multinomial		Binary conditional	
		Gini	KS	Gini	KS	GINI	KS
NA	Na	32.33%	25.72%	33.11%	26.05%	35.00%	26.00%
	Tr	32.76%	27.37%	46.20%	35.50%		
	Re	29.68%	21.84%	29.99%	22.14%	30.00%	22.00%
Tr	Na	46.39%	34.79%	46.70%	34.95%	47.00%	35.00%
	Tr	20.00%	15.36%	45.82%	35.04%		
	Re	36.54%	27.04%	38.16%	28.47%	69.00%	29.00%
Re	Na	46.48%	46.28%	53.30%	52.94%	78.00%	68.00%
	Tr	63.47%	60.06%	66.02%	62.16%		
	Re	61.49%	56.88%	64.36%	59.12%	75.00%	64.00%
	D1	77.56%	69.24%	68.41%	67.31%	52.00%	40.00%
D1	Tr	38.15%	34.75%	11.67%	13.12%		
	Re	56.68%	43.24%	58.31%	44.17%	57.00%	44.00%
	D1	32.48%	27.65%	51.47%	38.51%	64.00%	50.00%
	D2	60.04%	44.29%	63.92%	49.01%		
D2	Re	65.55%	50.27%	75.11%	59.67%	80.00%	66.00%
	D1	37.86%	29.05%	62.93%	49.21%	82.00%	70.00%
	D2	26.34%	22.57%	68.96%	55.33%		
	Df	60.28%	48.83%	68.96%	55.33%	63.00%	50.00%

Binary logistic regression has shown better predictive power for the majority of transitions.

The probability of transition from Revolver to Non-active has the best results for binary regression

The probability of transition from Revolver to Delinquent has the worst results for binary regression

Total Income aggregation

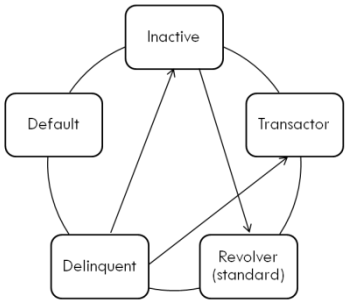
Sum of Income vs. Sum of Weighted Income vs. Direct Estimation

Schema of the modelling

Application | Behavioural | Macroeconomic | Characteristics | Product parameters (Limit, Interest Rate)

1 - Status – Transition probability on account level. Conditional logistic regression OR Multinomial regression

Initial STATUS | Inactive | Transactor | Revolver | Delinquent | Defaulted



2 - Outstanding balance – Interest Income and Expected Losses

Pr

Probability to be Revolver: Logistic regression OR Multinomial

Status Model

Utlization rate: Binary transformed logistic regression OR Beta-transf.

x N Limit segments

Limit

Outstanding Balance

Pr

Probability of Default

Risk Model

STATUS = Dlq

Expected Losses (LGD is constant)

3- Debit Transactions – Non-interest Income

Pr

Probability - POS: Logistic regression

X 3 STATUS (transactor, current, delinquent)

Usage models

Pr

Probability - ATM: Logistic regression

X 3 STATUS (transactor, current, delinquent)

Income POS (amt): Linear regression

OR?

Income POS (% limit): Binary transformed logistic regression OR Beta

Limit

Income models

Income ATM (amt): Linear regression

OR?

Income ATM (% limit): Binary transformed logistic regression OR Beta

Limit

IR

Pr

1 - Probability to be Delinquent: Logistic regression, OR Multinomial

Income Model

STATUS = Revolver

IV

Total income from Interest Rate

To Add: Nested logistic regression, Discrete Choice

Total transactional income

Total profit

STATUS means 5 stages of the account: inactive, transactor, revolver (current), revolver (delinquent), defaulted.
Probability to be in STATUS estimated by conditional probabilities with logistic regression OR multinomial regression and depends on 1) current status of the account; 2) behavioural history (dynamic); 3) application characteristics (static); 4) macroeconomic changes (dynamic).

Matrix of models for the set of account states

Status (s)	Inactive (NA)	Transactor (TR)	Revolver (RE)/ Repaid(RP)	Delinquent (D1)	Defaulted (Df)
S at t+1	Pr(I|s<>D)	Pr(T|s<>D)	Pr(R|s<>D)	Pr(Dlq|s=R)	Pr(D|s=Dlq)
Balance	N/A	N/A	UR x Limit	UR x Limit	UR x CF x Limit
Interest Income	N/A	N/A	UR x Limit x IR	0	0
Debit Turnover	N/A	Pr(POS|s=TR) Pr(ATM|s=TR)	Pr(POS|s=RE/RP) Pr(ATM|s=RE/RP)	Pr(POS|s=D1) Pr(ATM|s=D1)	N/A
Transact. Income	N/A	Inc(TR| Pr(POS)=1) Inc(TR| Pr(ATM)=1)	Inc(RE/RP| Pr(POS)=1) Inc(RE/RP| Pr(ATM)=1)	Inc(D1|Pr(POS)=1) Inc(D1|Pr(ATM)=1)	N/A
Risk	N/A	0	Pr(Df|s=RE)	Pr(Df|s=D2)	LGD x UT x CCF x Limit

Income equations

The final model in general format sum of the products of three estimations such as the probability to be in status S, the probability to do action POS/ATM and income estimation for each status:

Transactor income

$$\mathrm{I}\left(i,t+n \mid s_{i,t+n}=TR\right)=\Pr\left(s_{i,t+n}=TR \mid s_{i,t}\neq Df\right)\times$$
$$\times\left(\begin{array}{l}\Pr\left(a_{i,t+n}=POS \mid s_{i,t+n}=TR\right)\cdot \mathrm{R}\left(\mathbf{x}_{\mathbf{it}} \mid a_{i,t+n}=POS\right)+\\ +\Pr\left(a_{i,t+n}=ATM \mid s_{i,t+n}=TR\right)\cdot \mathrm{R}\left(\mathbf{x}_{\mathbf{it}} \mid a_{i,t+n}=ATM\right)\end{array}\right)$$

Revolver income

$$\mathrm{I}\left(i,t+n \mid s_{i,t+n}=RE\right)=\Pr\left(s_{i,t+n}=RE \mid s_{i,t}\neq DF\right)\times$$
$$\times\left(\begin{array}{l}\mathrm{Ut}\left(\mathbf{x}_{\mathbf{it}} \mid s_{i,t+n}=RE\right)\times IR\times Limit_{it}+\\ +\Pr\left(a_{i,t+n}=POS \mid s_{i,t+n}=RE\right)\cdot \mathrm{R}\left(\mathbf{x}_{\mathbf{it}} \mid a_{i,t+n}=POS, s_{i,t+n}=RE\right)+\\ +\Pr\left(a_{i,t+n}=ATM \mid s_{i,t+n}=RE\right)\cdot \mathrm{R}\left(\mathbf{x}_{\mathbf{it}} \mid a_{i,t+n}=ATM, s_{i,t+n}=RE\right)\end{array}\right)$$

Delinquent income

$$\mathrm{I}\left(i,t+n \mid s_{i,t+n}=D1\right)=\Pr\left(s_{i,t+n}=D1 \mid s_{i,t}\neq Df\right)\times$$
$$\times\left(\begin{array}{l}\Pr\left(a_{i,t+n}=POS \mid s_{i,t+n}=D1\right)\cdot \mathrm{R}\left(\mathbf{x}_{\mathbf{it}} \mid a_{i,t+n}=POS, s_{i,t+n}=D1\right)+\\ +\Pr\left(a_{i,t+n}=ATM \mid s_{i,t+n}=D1\right)\cdot \mathrm{R}\left(\mathbf{x}_{\mathbf{it}} \mid a_{i,t+n}=ATM, s_{i,t+n}=D1\right)\\ +Penalty\end{array}\right)$$

Total Income calculation options

A simple sum of interest and transactional income (TI)

$$TI(i,t+n) = \mathrm{Ut}(\mathbf{x}_{it} \mid s_{i,t+n} \notin (NA, TR, D1, D2, Df)) \times IR \times Limit_{it} +$$
$$+ \Pr(a_{i,t+n} = POS \mid s_{i,t+n} \notin (NA, D2, Df), \mathbf{x}_i) \cdot \mathrm{R}(\mathbf{x}_{it} \mid a_{i,t+n} = POS, s_{i,t+n} \notin (NA, D2, Df)) +$$
$$+ \Pr(a_{i,t+n} = ATM \mid s_{i,t+n} \notin (NA, D2, Df), \mathbf{x}_i) \cdot \mathrm{R}(\mathbf{x}_{it} \mid a_{i,t+n} = ATM, s_{i,t+n} \notin (NA, D2, Df))$$

$\mathrm{Ut}(\mathbf{x}_{it} \mid s_{i,t+n} \notin (NA, TR, D2, Df))$ means the utilization rate function, which depends on the vector of predictors **x** of the account *i* at time *t;*

$\Pr(a_{i,t+n} = POS \mid s_{i,t+n} \notin (NA, D2, Df), \mathbf{x}_i)$ is the probability of POS transaction of the account *i* at time *t+n,* assuming that the account will not be in a state of default at time *t+n*;

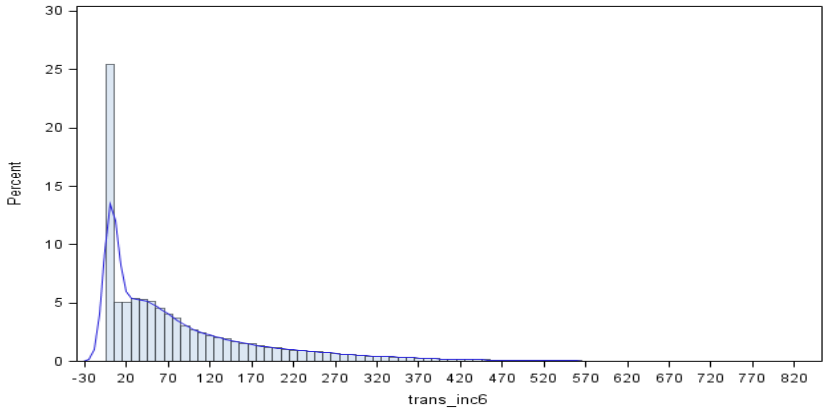
The full formula for the total monthly income *weighted* by the transition probabilities (TIW1)

$$TI(i,t+n) = \mathrm{Ut}(\mathbf{x}_{it} \mid s_{i,t+n} = RE) \times IR \times Limit \times \Pr(s_{i,t+n} = RE \mid \mathbf{x}_{it}) +$$
$$+ \mathrm{Ut}(\mathbf{x}_{it} \mid s_{i,t+n} = RP) \times IR \times Limit \times \Pr(s_{i,t+n} = RP \mid \mathbf{x}_{it}) +$$
$$+ \mathrm{Ut}(\mathbf{x}_{it} \mid s_{i,t+n} = D1) \times IR \times Limit \times \Pr(s_{i,t+n} = D1 \mid \mathbf{x}_{it}) +$$
$$+ \Pr(a_{i,t+n} = POS \mid s_{i,t+n} = TR, \mathbf{x}_{it}) \cdot \mathrm{R}(\mathbf{x}_{it} \mid a_{i,t+n} = POS, s_{i,t+n} = TR) \cdot \Pr(s_{i,t+n} = TR \mid \mathbf{x}_{it}) +$$
$$+ \Pr(a_{i,t+n} = ATM \mid s_{i,t+n} = TR, \mathbf{x}_{it}) \cdot \mathrm{R}(\mathbf{x}_{it} \mid a_{i,t+n} = ATM, s_{i,t+n} = TR) \cdot \Pr(s_{i,t+n} = TR \mid \mathbf{x}_{it}) +$$
$$+ \Pr(a_{i,t+n} = POS \mid s_{i,t+n} = RE, \mathbf{x}_{it}) \cdot \mathrm{R}(\mathbf{x}_{it} \mid a_{i,t+n} = POS, s_{i,t+n} = RE) \cdot \Pr(s_{i,t+n} = RE \mid \mathbf{x}_{it}) +$$
$$+ \Pr(a_{i,t+n} = ATM \mid s_{i,t+n} = RE, \mathbf{x}_{it}) \cdot \mathrm{R}(\mathbf{x}_{it} \mid a_{i,t+n} = ATM, s_{i,t+n} = RE) \cdot \Pr(s_{i,t+n} = RE \mid \mathbf{x}_{it}) +$$
$$+ \Pr(a_{i,t+n} = POS \mid s_{i,t+n} = RP, \mathbf{x}_{it}) \cdot \mathrm{R}(\mathbf{x}_{it} \mid a_{i,t+n} = POS, s_{i,t+n} = RP) \cdot \Pr(s_{i,t+n} = RP \mid \mathbf{x}_{it}) +$$
$$+ \Pr(a_{i,t+n} = ATM \mid s_{i,t+n} = RP, \mathbf{x}_{it}) \cdot \mathrm{R}(\mathbf{x}_{it} \mid a_{i,t+n} = ATM, s_{i,t+n} = RP) \cdot \Pr(s_{i,t+n} = RP \mid \mathbf{x}_{it}) +$$
$$+ \Pr(a_{i,t+n} = POS \mid s_{i,t+n} = D1, \mathbf{x}_{it}) \cdot \mathrm{R}(\mathbf{x}_{it} \mid a_{i,t+n} = POS, s_{i,t+n} = D1) \cdot \Pr(s_{i,t+n} = D1 \mid \mathbf{x}_{it}) +$$
$$+ \Pr(a_{i,t+n} = ATM \mid s_{i,t+n} = D1, \mathbf{x}_{it}) \cdot \mathrm{R}(\mathbf{x}_{it} \mid a_{i,t+n} = ATM, s_{i,t+n} = D1) \cdot \Pr(s_{i,t+n} = D1 \mid \mathbf{x}_{it})$$

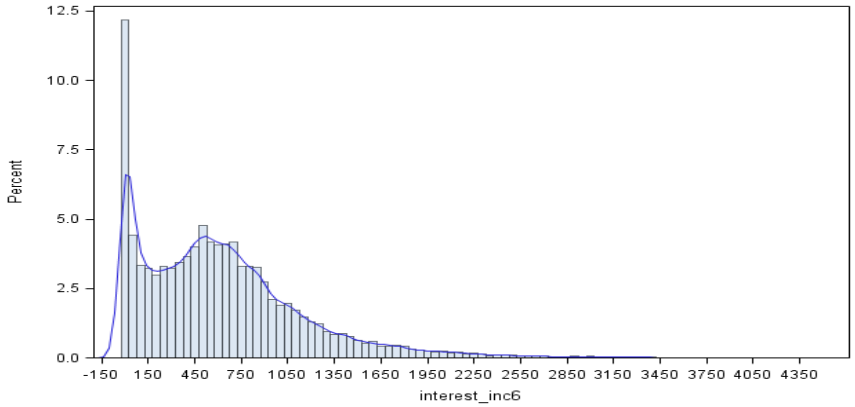
The probability of transition $Pr(s_{i,t+n}=S_i|x_{it})$ is calculated at the account level *i* conditional on the current state of the account and previous behavior unlike the usage of Markov Chains pool level transition probability $Pr(s_{t+n}= S)$

Transactional and Interest Income, observed and predicted distributions

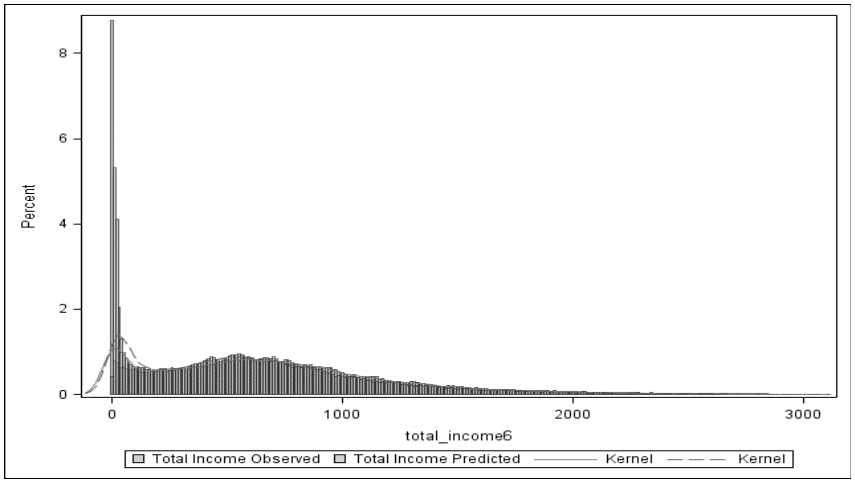
Distribution of *interest income* over 6 months



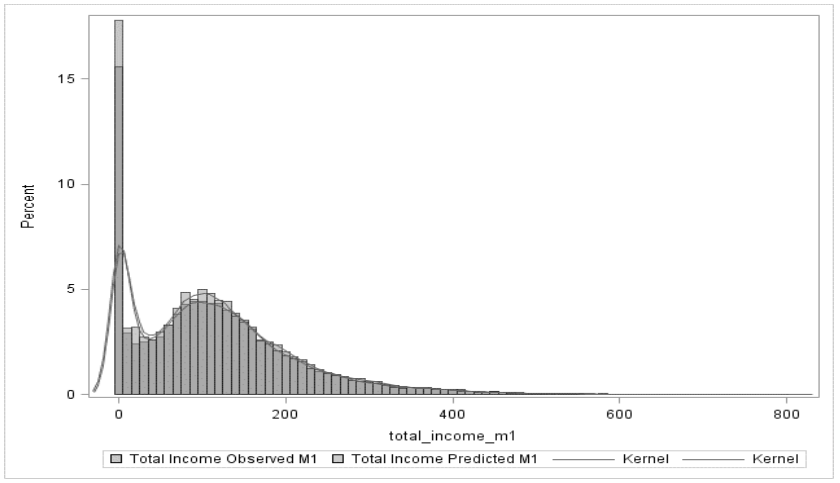
Distribution of *transactional* income over 6 months



Total income observed and total income prediction for *+1 month* period density histogram



Total income observed and total income prediction for *six-month period* density histogram



Validation results of aggregated and direct for the total income prediction

Model name	Development Sample			Validation - Out-of-sample			Validation - Out-of-time		
	R^2	MAE	RMSE	R^2	MAE	RMSE	R^2	MAE	RMSE
Simple Sum of Interest and Transactional Income (TI)									
month 1	0.7673	24.50	47.40	0.7663	24.71	46.97	0.7420	30.18	57.38
month 2	0.7077	29.99	53.26	0.7040	30.27	53.00	0.6538	38.65	67.19
month 3	0.6636	33.88	57.39	0.6553	34.16	57.47	0.5890	45.05	73.69
month 4	0.6239	37.02	60.59	0.6194	37.20	60.36	0.5339	50.18	79.00
month 5	0.6006	39.11	62.23	0.5919	39.38	62.29	0.5030	53.46	81.91
month 6	0.5517	42.50	66.37	0.5422	42.92	66.54	0.4566	56.17	84.72
6 months	**0.8336**	**145.01**	**227.76**	**0.8227**	**148.44**	**232.19**	**0.8181**	**184.41**	**273.37**
Sum of State Incomes Weighted by the States TMM (TIW2)									
month 1	0.7866	23.75	45.39	0.7817	24.08	45.40	0.7693	28.85	54.26
month 2	0.7237	29.72	51.78	0.7182	30.00	51.71	0.6790	37.39	64.70
month 3	0.6746	34.27	56.44	0.6671	34.40	56.47	0.6143	43.56	71.39
month 4	0.6339	37.81	59.78	0.6302	37.85	59.50	0.5613	48.51	76.64
month 5	0.6052	39.98	61.87	0.5980	40.12	61.83	0.5210	52.18	80.41
month 6	0.5572	43.86	65.96	0.5509	44.16	65.91	0.4773	54.80	83.09
6 months	0.7924	164.12	254.40	0.7843	166.31	256.11	0.7588	204.43	314.77
Sum of State Incomes Weighted by the Individual States Transition Probabilities (TIW1)									
month 1	**0.8053**	**23.18**	**43.36**	**0.8011**	**23.52**	**43.33**	**0.7943**	**27.94**	**51.24**
month 2	**0.7323**	**29.12**	**50.97**	**0.7244**	**29.48**	**51.14**	**0.6947**	**36.72**	**63.10**
month 3	**0.6850**	**33.25**	**55.53**	**0.6752**	**33.52**	**55.78**	**0.6300**	**42.78**	**69.92**
month 4	**0.6443**	**36.61**	**58.92**	**0.6374**	**36.79**	**58.92**	**0.5735**	**47.84**	**75.57**
month 5	**0.6136**	**39.16**	**61.21**	**0.6054**	**39.36**	**61.26**	**0.5296**	**51.75**	**79.69**
month 6	**0.5700**	**42.39**	**65.00**	**0.5614**	**42.79**	**65.14**	**0.4877**	**54.14**	**82.26**
6 months	0.7928	161.45	254.17	0.7821	163.96	257.41	0.7464	207.94	322.80
Direct Total Income Estimation (TID)									
month 1	0.7673	24.49	47.397	0.7662	24.71	46.97	0.7420	30.17	57.37
month 2	0.7076	29.98	53.257	0.7040	30.27	52.99	0.6538	38.64	67.18
month 3	0.6635	33.88	57.386	0.6552	34.16	57.46	0.5890	45.05	73.68
month 4	0.6238	37.02	60.589	0.6193	37.19	60.36	0.5338	50.18	78.99
month 5	0.6005	39.11	62.234	0.5918	39.38	62.29	0.5030	53.45	81.91
month 6	0.5516	42.49	66.369	0.5421	42.92	66.54	0.4565	56.17	84.71
6 months	**0.8336**	**145.01**	**227.76**	**0.8227**	**148.44**	**232.19**	**0.8181**	**184.41**	**273.37**

Sum of Interest and Transactional Income (TI) - The highest fitting accuracy for six-month period

Sum of State Incomes Weighted by the Individual States Transition Probabilities (TIW1) - The highest fitting accuracy for monthly prediction

Confusion matrix of number of observed vs. predicted values for total income: t+1 month and 6-months period

TI_{t+1}

Observed \ Predicted	0-49	50-99	100-149	150-199	200-249	250-299	300-349	350-399	400-449	450-499	500-549	550-599	600-649	650-699	700-749	750-800	800-849
0-49	80.75%	12.90%	4.21%	1.18%	0.50%	0.23%	0.11%	0.06%	0.03%	0.01%	0.01%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%
50-99	15.18%	63.72%	18.35%	2.24%	0.34%	0.10%	0.06%	0.01%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
100-149	4.21%	15.66%	66.69%	11.89%	1.24%	0.23%	0.04%	0.03%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
150-199	3.11%	4.29%	21.58%	59.07%	10.40%	1.30%	0.19%	0.04%	0.01%	0.01%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
200-249	2.61%	2.65%	5.87%	22.08%	53.18%	11.36%	1.87%	0.25%	0.07%	0.04%	0.02%	0.01%	0.01%	0.00%	0.00%	0.00%	0.00%
250-299	2.79%	2.01%	3.98%	6.95%	22.26%	47.72%	11.77%	2.00%	0.40%	0.10%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%
300-349	2.15%	1.27%	2.54%	4.64%	7.53%	21.99%	46.44%	10.45%	2.34%	0.47%	0.13%	0.02%	0.04%	0.00%	0.00%	0.00%	0.00%
350-399	1.75%	1.29%	2.11%	2.90%	4.81%	7.28%	19.93%	44.58%	12.29%	2.24%	0.56%	0.20%	0.07%	0.00%	0.00%	0.00%	0.00%
400-449	1.14%	0.69%	1.26%	1.77%	3.03%	4.75%	7.67%	23.87%	39.27%	12.76%	2.46%	1.14%	0.06%	0.06%	0.06%	0.00%	0.00%
450-499	0.89%	0.53%	0.98%	1.87%	1.87%	4.01%	5.08%	7.13%	20.23%	42.60%	11.59%	2.32%	0.62%	0.27%	0.00%	0.00%	0.00%
500-549	0.15%	0.76%	1.07%	0.92%	1.22%	1.68%	3.36%	4.27%	6.26%	19.24%	42.14%	15.88%	2.75%	0.15%	0.15%	0.00%	0.00%
550-599	0.22%	0.22%	0.66%	0.22%	0.88%	0.88%	1.76%	3.96%	7.05%	7.27%	21.81%	35.02%	17.84%	1.54%	0.66%	0.00%	0.00%
600-649	0.00%	0.00%	0.00%	0.00%	0.85%	0.85%	0.85%	2.56%	2.99%	5.56%	10.26%	25.21%	33.76%	14.10%	2.56%	0.43%	0.00%
650-699	0.88%	0.00%	1.77%	0.00%	0.00%	0.88%	1.77%	0.88%	1.77%	3.54%	6.19%	7.96%	16.81%	38.94%	17.70%	0.88%	0.00%
700-749	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	6.10%	8.54%	8.54%	26.83%	30.49%	19.51%	0.00%
750-799	0.00%	0.00%	0.00%	0.00%	2.70%	2.70%	0.00%	0.00%	0.00%	2.70%	0.00%	2.70%	5.41%	21.62%	18.92%	43.24%	0.00%
800-849	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	14.29%	0.00%	0.00%	0.00%	0.00%	14.29%	28.57%	42.86%	0.00%

$\sum_{k=1}^{6} TI_{t+k}$

Observed \ Predicted	<100	100-299	300-499	500-699	700-899	900-1099	1100-1299	1300-1499	1500-1699	1700-1899	1900-2099	2100-2299	2300-2499	2500-2699	2700-2899	2900-3099	3100-3299	3300-3500	>3500
<100	77.79%	16.09%	3.87%	1.42%	0.50%	0.21%	0.06%	0.03%	0.02%	0.01%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
100-299	27.90%	40.38%	20.78%	7.11%	2.28%	0.86%	0.33%	0.14%	0.10%	0.05%	0.02%	0.02%	0.02%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%
300-499	10.23%	19.59%	45.22%	17.85%	4.58%	1.54%	0.46%	0.28%	0.13%	0.04%	0.04%	0.01%	0.01%	0.02%	0.00%	0.00%	0.00%	0.00%	0.00%
500-699	3.98%	7.81%	23.77%	47.63%	12.23%	2.93%	0.97%	0.36%	0.15%	0.08%	0.03%	0.03%	0.01%	0.01%	0.00%	0.01%	0.00%	0.00%	0.00%
700-899	2.22%	4.41%	8.84%	31.00%	40.87%	9.33%	2.08%	0.74%	0.25%	0.12%	0.06%	0.03%	0.03%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
900-1099	1.61%	2.52%	5.29%	11.07%	35.72%	32.85%	7.68%	2.12%	0.59%	0.24%	0.12%	0.08%	0.05%	0.04%	0.01%	0.01%	0.00%	0.00%	0.00%
1100-1299	1.11%	1.93%	3.06%	5.72%	12.32%	35.39%	29.37%	7.77%	2.13%	0.69%	0.23%	0.11%	0.07%	0.01%	0.03%	0.03%	0.03%	0.01%	0.01%
1300-1499	0.73%	1.35%	2.18%	3.46%	5.78%	13.30%	35.60%	26.89%	6.92%	2.26%	0.77%	0.31%	0.21%	0.09%	0.05%	0.02%	0.03%	0.03%	0.00%
1500-1699	0.66%	1.25%	1.46%	2.70%	4.14%	6.84%	14.90%	32.73%	25.11%	6.47%	2.14%	0.76%	0.37%	0.18%	0.19%	0.05%	0.03%	0.00%	0.02%
1700-1899	0.54%	0.92%	1.16%	1.77%	2.46%	3.95%	6.64%	15.03%	30.74%	26.54%	6.10%	2.22%	0.80%	0.52%	0.28%	0.14%	0.02%	0.12%	0.05%
1900-2099	0.31%	0.59%	0.83%	1.56%	1.66%	2.74%	4.47%	7.77%	15.95%	35.58%	18.65%	5.58%	2.84%	0.59%	0.42%	0.31%	0.07%	0.07%	0.00%
2100-2299	0.10%	0.46%	0.76%	1.22%	1.73%	1.98%	3.20%	4.77%	7.97%	16.69%	31.56%	20.50%	5.63%	1.73%	1.01%	0.36%	0.05%	0.20%	0.10%
2300-2499	0.24%	0.39%	0.63%	0.55%	0.71%	1.97%	1.57%	2.44%	4.87%	7.86%	15.09%	37.81%	16.75%	5.42%	1.65%	1.02%	0.31%	0.39%	0.31%
2500-2699	0.11%	0.77%	0.66%	0.44%	1.21%	0.99%	0.88%	1.54%	3.63%	3.74%	7.71%	18.94%	31.50%	19.49%	4.41%	2.42%	0.99%	0.44%	0.11%
2700-2899	0.65%	0.78%	0.39%	1.16%	0.39%	1.55%	1.55%	2.20%	2.58%	3.10%	4.39%	6.72%	15.50%	36.82%	15.25%	3.49%	1.55%	1.42%	0.52%
2900-3099	0.23%	0.23%	0.45%	0.23%	1.13%	0.00%	0.23%	1.80%	1.13%	2.03%	4.28%	6.08%	9.23%	22.07%	21.62%	16.89%	5.86%	4.28%	2.25%
3100-3299	0.00%	0.27%	0.27%	0.54%	0.27%	0.00%	0.27%	1.35%	1.62%	2.97%	3.51%	3.78%	2.70%	10.27%	18.92%	32.43%	14.05%	4.86%	1.89%
3300-3500	0.00%	0.30%	0.00%	0.30%	0.30%	0.61%	0.00%	0.61%	0.61%	1.52%	2.12%	2.73%	3.03%	3.64%	15.15%	17.88%	27.58%	19.09%	4.55%
>3500	0.00%	0.00%	0.00%	0.24%	0.00%	0.95%	0.00%	0.00%	0.47%	0.24%	0.71%	1.65%	0.71%	1.18%	2.60%	2.84%	11.35%	26.71%	50.35%

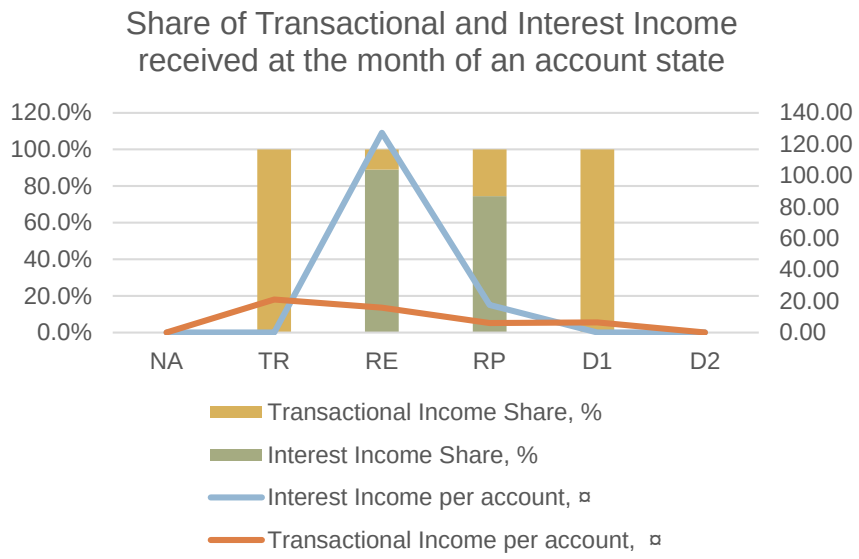
Transactional and Interest Income by Account States

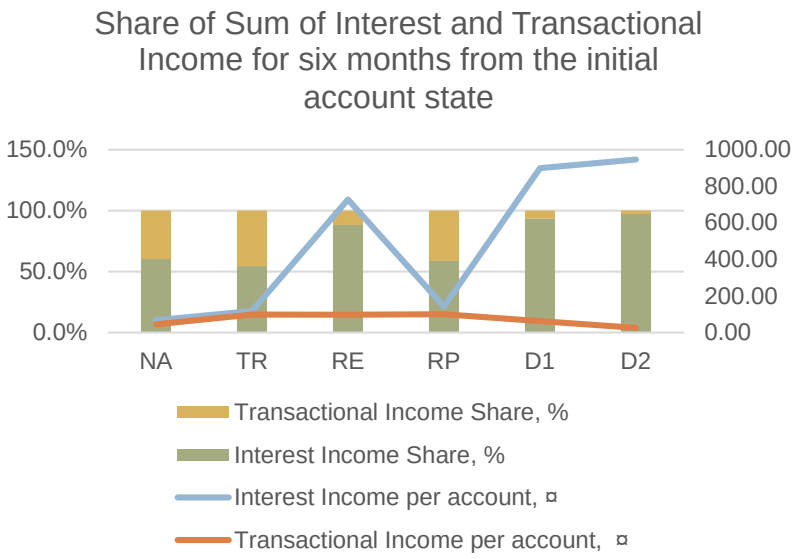
Share of Transactional and Interest Income received at the month of the current account state

Current State	Interest Income per account, ¤	Transactional Income per account, ¤	Total Income per account, ¤	Share of Interest Income, %
NA	0.00	0.00	0.00	-
TR	0.00	20.98	20.98	0.0
RE	127.26	15.81	143.08	88.9
RP	17.56	6.01	23.57	74.5
D1	0.00	6.46	6.46	0.0
D2	0.00	0.00	0.00	-

Share of Sums of Transactional and Interest Income received for six month from the initial account state

State at the first month	Interest Income per account, ¤	Transactional Income per account, ¤	Total Income per account, ¤	Share of Interest Income, %
NA	67.87	44.66	112.53	60.3
TR	117.27	97.87	215.14	54.5
RE	727.43	96.95	824.39	88.2
RP	141.72	99.89	241.62	58.7
D1	898.78	62.64	961.42	93.5
D2	946.18	26.00	972.18	97.3





Example of the total income calculation

	Case 1	2	3	4
Product parameters				
Credit Limit	10000	10000	10000	10000
Interest Rate	36%	36%	36%	36%
Transition Probabilities				
Inactive	0.1	0.1	0.1	0.1
Transactor	0.2	0.2	0.6	0.6
Revolver	0.6	0.6	0.25	0.25
Delinquent	0.1	0.1	0.05	0.05
Interest Income				
Utilization Rate	0.5	0.1	0.5	0.1
Interest Rate Income	*150*	*30*	*150*	*30*
Transactional income				
For Revolver				
Probability of transaction				
Probability of POS transaction	0.8	0.8	0.8	0.8
Probability of ATM transaction	0.5	0.5	0.5	0.5
Transactional income amount				
POS	100	100	100	100
ATM	50	50	50	50
Transactional income POS	80	80	80	80
Transactional income ATM	25	25	25	25
Total transactional income	*105*	*105*	*105*	*105*

	Case 1	2	3	4
For Transactor				
Probability of transaction				
Probability of POS transaction	0.9	0.9	0.9	0.9
Probability of ATM transaction	0.2	0.2	0.2	0.2
Transactional income amount				
POS	200	200	200	200
ATM	20	20	20	20
Transactional income POS	180	180	180	180
Transactional income ATM	4	4	4	4
Total transactional income	*184*	*184*	*184*	*184*
Weighted Income				
Interest Income Weighted	90	18	37.5	7.5
Total transactional income Weighted Revolver	21	21	63	63
Total transactional income Weighted Transactor	36.8	36.8	110.4	110.4
Total income	***147.8***	***75.8***	***210.9***	***180.9***

For the same income estimation different transition probabilities give different estimations for total income

Contribution

Contribution

	Transactor	Revolver	Competent Revolver
Interest Income	Low/ No	High	No
Non-Interest Rate	High	Low	Low
Risk Level	Low	Moderate	Moderate/ High

Competent Revolver – the worst client from the profitability, but not sales, point of view

Expected Loss calculation: use the utilization rate for Exposure at Default for credit card

$$EaD = L \cdot UR + \left(L\left(1 - UR\right)\right) \cdot CF$$

The 'risk – revenue' strategies in Card business help to maximize the profitability.
Areas of application:
✓Limit management – segmentation by revolver/transactor – risk limitation and usage motivation
✓Pricing – not only risk-based, but use motivation
✓Marketing – differentiate target groups

Use of panel data helps to avoid the impact of time heterogeneity on model results.

An empirical testing of different approaches to the profit modelling

Business Contribution 2 - Credit Limit Management

Segment	Utilization	Profitability	PD									
			0.0%	1.0%	2.0%	3.0%	4.0%	5.0%	6.0%	7.0%	8.0%	9.0%	10.0%
1	10.2%	3%	5%	5%	0%	0%	0%	0%	0%	-5%	-5%	-5%	-5%
2	16.3%	5%	10%	5%	5%	5%	5%	0%	0%	0%	0%	-5%	-5%
3	18.0%	7%	10%	10%	10%	10%	5%	5%	5%	5%	0%	0%	0%
4	19.5%	9%	15%	15%	10%	10%	10%	10%	5%	5%	5%	5%	0%
5	39.5%	11%	20%	20%	15%	15%	15%	10%	10%	10%	10%	5%	5%
6	36.3%	12%	20%	20%	20%	20%	15%	15%	15%	10%	10%	10%	10%
7	85.0%	13%	25%	20%	20%	20%	20%	15%	15%	15%	10%	10%	10%
8	78.4%	14%	25%	25%	20%	20%	20%	20%	15%	15%	15%	10%	10%
9	60.3%	16%	30%	30%	30%	25%	25%	25%	20%	20%	20%	15%	15%
10	35.3%	18%	35%	35%	35%	30%	30%	25%	25%	25%	20%	20%	20%
11	78.7%	19%	40%	40%	35%	35%	30%	30%	30%	25%	25%	25%	20%
12	77.6%	21%	45%	45%	40%	40%	35%	35%	35%	30%	30%	25%	25%
13	91.9%	24%	55%	50%	50%	45%	45%	45%	40%	40%	35%	35%	30%
14	93.3%	25%	60%	55%	55%	50%	50%	45%	45%	45%	40%	40%	35%

Credit Limit changes depends on the profitability and probability of default segment. The utilization rate illustrates the fact that credit line profitability does not depend on the utilization rate pro rata.

Authors' papers (SAS Global Forum)

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- Osipenko, D (2016). Credit Card Interest and Transactional Income Prediction with an Account-Level Transition Panel Data Model Using SAS/STAT® and SAS/ETS® Software. https://sasglobalforum2016.lanyonevents.com/connect/sessionDetail.ww?SESSION_ID=11845

Thank you for your attention!

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