Loss Given Default Modelling, within Regulatory Constraints

Laurence Winiarski and Richard Knight

August 2019

Contents Page

Contents

Nationwide Building Society – who are we?

Nationwide Building Society

- UK's largest **building society**, owned by and run for the benefit of our members
- Members hold a mortgage, current account, or savings with society.
- Company focuses on members rather than shareholders
- Last financial year, gave half a billion pounds to members through better value products
- Give at least 1% of our pre-tax profits to local causes
- Second largest mortgage provider in the UK
- C.18,000 employees and over 700 Branches

Disclaimer: The content of the presentation are the views of the modelling team and are not formal recommendations by Nationwide Building Society.

Overview – Reminder of where LGD is used

Expected Loss = PD x LGD x EAD

(PPGD x LGP)

“Two-stage LGD model”
(Leow Mues, 2012)

PPGD

PPGD (Probability of Possession Given Default) – the probability that the property is possessed if the facility is considered to be in default.

LGP

LGP (Loss Given Possession) – the loss that occurs when the property has been possessed and sold, which can also be referred to as Shortfall.

Two key terminology you’ll need today:

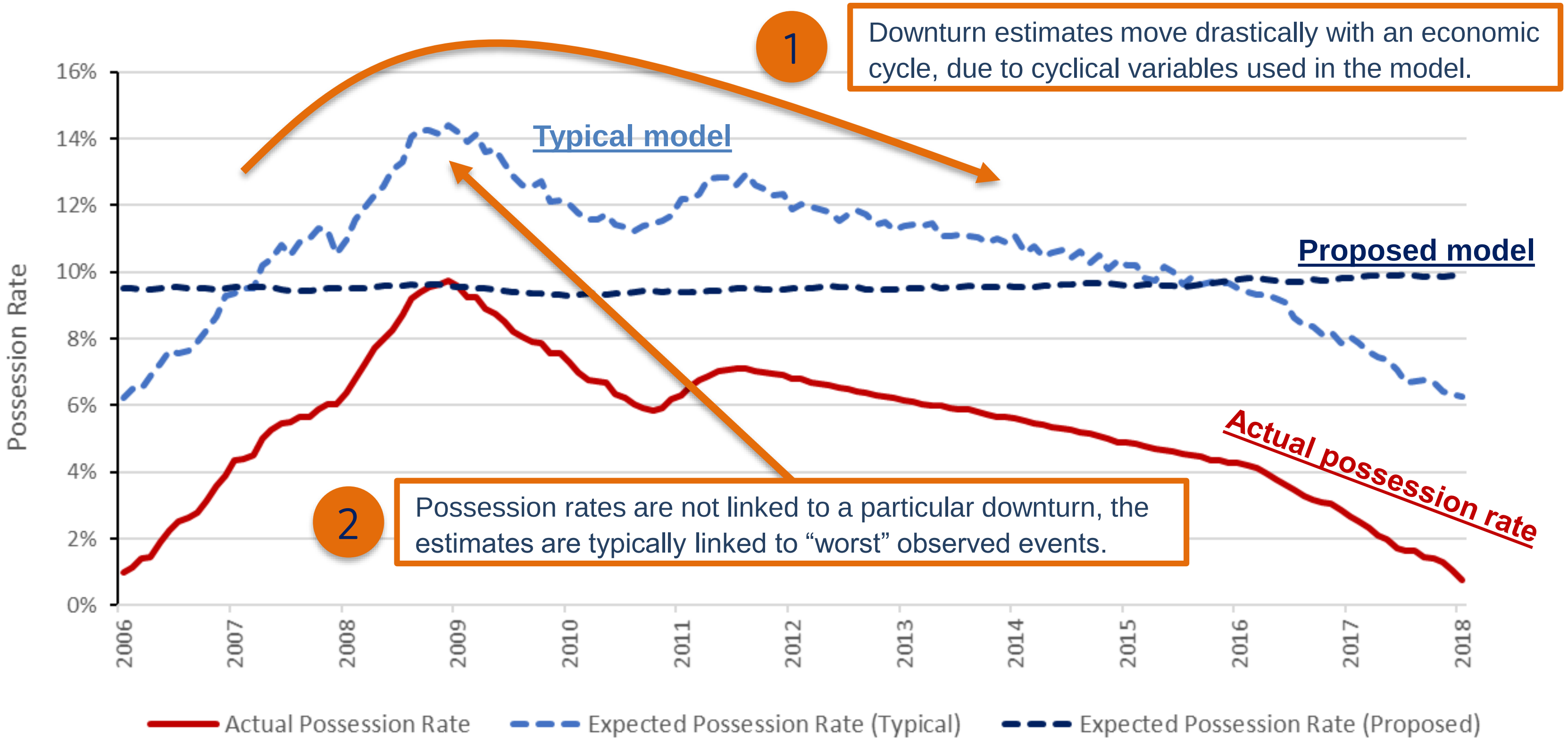
- **Haircut –** the reduction in price of the property following possession.
- **Haircut Coefficient** – the sale price as a proportion of the valuation

Probability of Possession Given Default

Probability of Possession – Typical vs Proposed

The Probability of Possession given Default, is the likelihood that a property will be possessed given that they have defaulted on their mortgage. This section focuses on deriving a PPGD estimate for an economic downturn – used to derive the final downturn LGD.

Limitations identified with typical downturn possession rate estimation



① Downturn estimates move drastically with an economic cycle, due to cyclical variables used in the model.

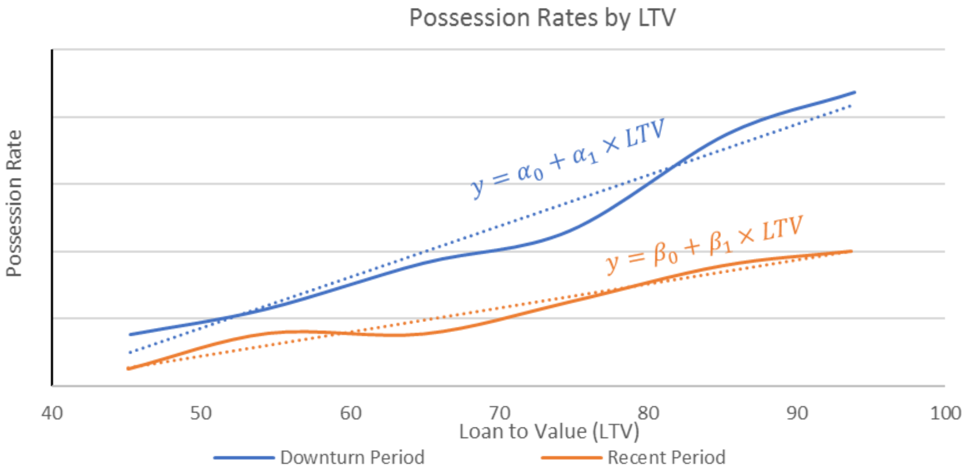
② Possession rates are not linked to a particular downturn, the estimates are typically linked to “worst” observed events.

[1] These values have been randomly generated and are for illustrative purposes only

Deriving the estimates – the use of quantile regression

Step 1 – Identify the predictor

The first step is to identify a non-cyclical variable that rank orders possession rates during an economic downturn and during a more recent period.

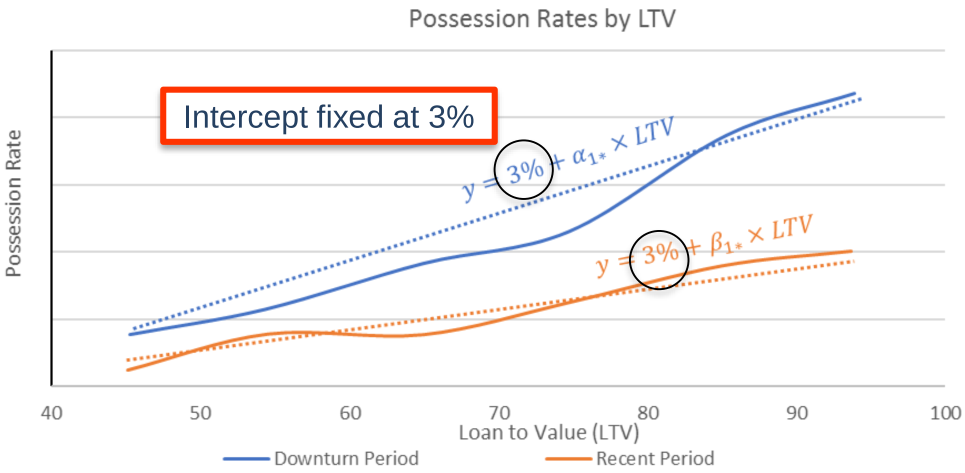


Quantile regression can be used to incorporate addition Margin of Conservatism (Details in Appendix)

Step 2 – Fix the model coefficient(s)

For each period, fix one or none of model coefficients. The figure displayed illustrates an updated linear regression when the intercept is fixed at 3%.

Therefore, the slope coefficient is the only model variable that changes for each period.

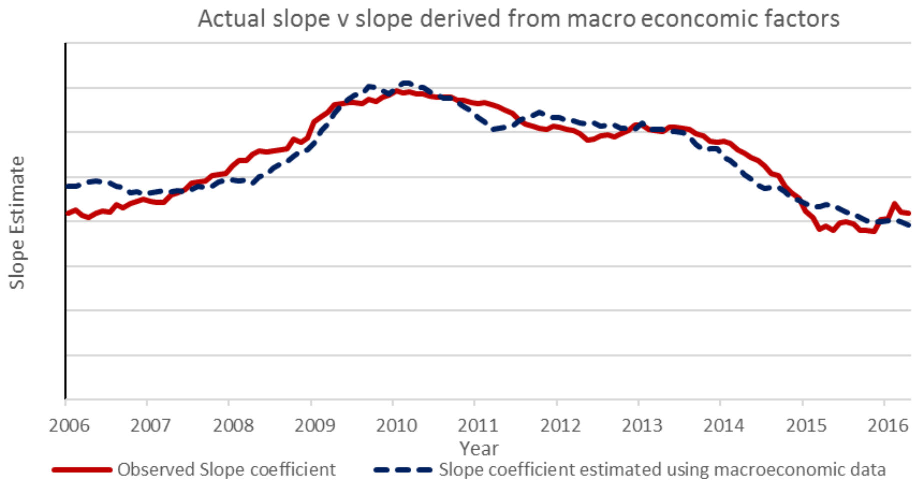

Intercept fixed at 3%

Downturn Possession Rates – incorporating macroeconomics

Step 3 – Model the slope coefficient

Each month will have a difference slope coefficient, therefore, macro economic variables can be used to model the slope.

$$Slope = \rho_0 + \rho_1 \times GDP + \rho_2 \times Unemployment$$



Step 4 – Incorporate downturn conditions

By incorporating the macro economic conditions expected in a downturn, the slope coefficient can be derived. This model then can be applied on to the current book to determine the expected performance during a downturn.

Downturn Scenario	
GDP	-3.7%
Unemployment	8.6%

$$Slope_{DT} = \rho_0 + \rho_1 \times GDP + \rho_2 \times Unemployment$$

$$Expected\ Possession\ Rate = 3\% + Slope_{DT} \times \mathrm{LTV}$$

Regulation - EBA/GL/2019/03 - Guidelines for the estimation of LGD appropriate for an economic downturn

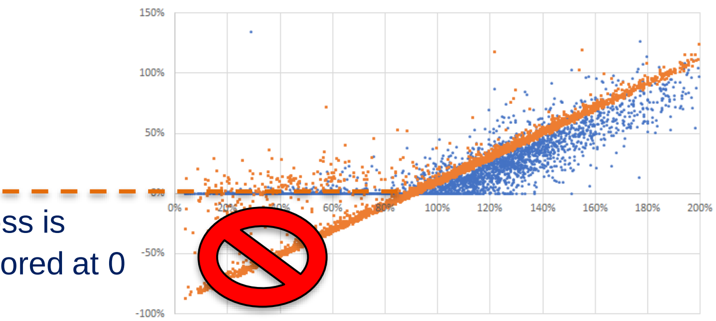
Paragraph 31 - For the purpose of these guidelines, a ‘haircut approach’ refers to an approach for the estimation of the impact of the downturn period on realised LGDs, intermediate parameters or risk drivers in which one or several economic factors are direct or transformed input(s) in the LGD model and where for the purpose of this estimation these input(s) are adjusted to reflect the impact of the downturn period under consideration.

Shortfall / Loss Given Possession (LGP)

Loss Given Possession - Three step approach was taken to find an appropriate solution for estimating loss within regulatory constraints

Reminder: Loss Given Possession (Shortfall) is the balance after using sale proceeds to offset the outstanding balance at the point of sale.

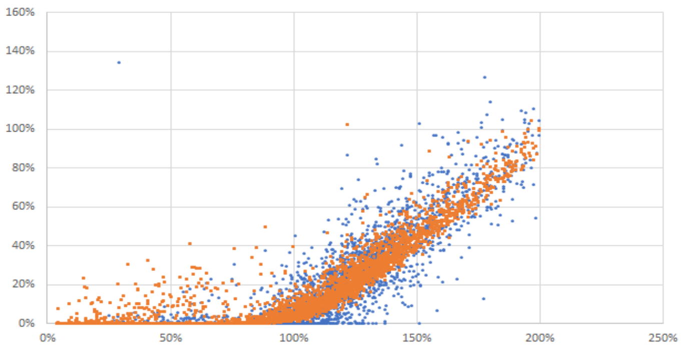
Typical Approach - Empirical



Loss is floored at 0

- **Zero estimates occur**, due to typical approaches ranking by LTV or some other variable.
- **Requires a floor**, limiting business use
- **Blunt approach** and does not have strong differentiation at the account level

Proposed - Statistical



- **Floor is not required** because of the use of a statistical distribution
- **Greater spread of Shortfall estimates** when using a standard deviation model (enabled by property level information)
- **All elements of loss** considered as part of the sale to be within the estimate

The approach to improving the Loss Given Possession was as follows:

Step 1: Assess the distribution of the Haircut

- *The distribution was concluded to be Log-normal*

Step 2: Develop PiT models and confirm the framework is sufficiently robust (not covered in this presentation)

- *Haircut Mean and Standard Deviation models were developed and led to accurate Shortfall estimates*

Step 3: Calibrate a Macroeconomic Variable (MEV) model to create Loss estimates

- *MEV model used alongside a number of assumptions to create the Downturn calibration*

Log-normal Distribution - Quick bit of theory to remind ourselves of the concept of conditional expectation and what log-normal looks like

Assumptions:

- Sale Proceeds and Balance are log-normally distributed.
- Therefore Shortfall can be calculated in the log-normal space.
- Statistical testing was conducted (i.e Anderson-Darling), however mostly leans on business view

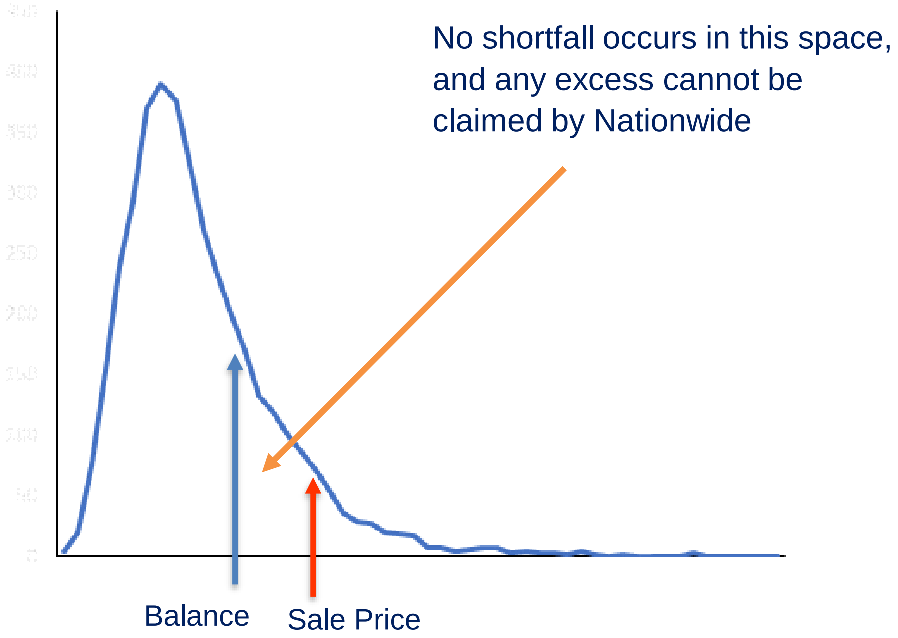
Log-normal Distribution

(where loss occurs)

This is the Shortfall space, which is what we are trying to estimate.

Log-normal Distribution

(where loss does not occur)



No shortfall occurs in this space, and any excess cannot be claimed by Nationwide

- The Haircut Coefficient is modelled (which can be thought of as “Sale Price to Value”)
- This is compared to the LTV (Balance to Value) rather than the balance directly.

The use of the log-normal framework means that **both of the above scenarios** are considered for every account when calculating shortfall, and this enables non-zero estimates for all accounts.

Shortfall Calculation – The estimation of loss is more sophisticated when used within a statistical distribution framework

The calculation of shortfall, within a statistical distribution framework, is as follows:

$$\text{Shortfall \%} \ = \text{Probability of Shortfall} * \text{Severity of Shortfall}$$

Which now means we're now considering a "Three stage approach" (sort of).

$$\text{Shortfall (\%)} \ = \mathit{Haircut\ Coefficient}\ (83.0\%) - \mathit{LTV}\ (85.1\%)$$

Typical approach would put Loss Given Possession at 2.1%

Using a statistical distribution framework, we get the following:

Theoretical Haircut (where Shortfall occurs)

$$\text{Shortfall (\%)} \ = 60.0\% \ * \ \left(1 - \frac{75.5\%}{85.1\%}\right)$$

Probability of Shortfall

LTV (or Loss point)

$$\text{Shortfall (\%)} \ = 60.0\% \ * \ 11.2\%$$

$$:.\ \text{Shortfall (\%)} = \mathbf{6.7\%}$$

Shortfall Calculation – The estimation of loss is more sophisticated when used within a statistical distribution framework

The calculation of shortfall, within a statistical distribution framework, is as follows:

$$\textbf{Shortfall \%} = \textbf{Probability of Shortfall} * \textbf{Severity of Shortfall}$$

Which now means we’re now considering a “Three stage approach” (sort of).

$$\text{Shortfall (\%)} = Haircut\ Coefficient\ (83.0\%) - LTV\ (85.1\%) \longrightarrow$$

Typical approach would put Loss Given Possession at 2.1%

Using a statistical distribution framework, we get the following:

Theoretical Haircut (where Shortfall occurs)

$$\text{Shortfall (\%)} = 60.0\% * \left(1 - \frac{75.5\%}{85.1\%}\right)$$

Probability of Shortfall

LTV (or Loss point)

$$\text{Shortfall (\%)} = 60.0\% * 11.2\%$$

$$:.\ \text{Shortfall (\%)} = \textbf{6.7\%}$$

Nationwide

Downturn Calibration - Macroeconomic model has been used alongside Downturn assumptions to create a downturn view of loss

- **Development of Macroeconomic model** to estimate Haircut through time.
- **Assumptions proposed** and used alongside the model.
- **Creation of non-zero estimates** of Downturn Shortfall for all accounts.

Calibration Assumptions

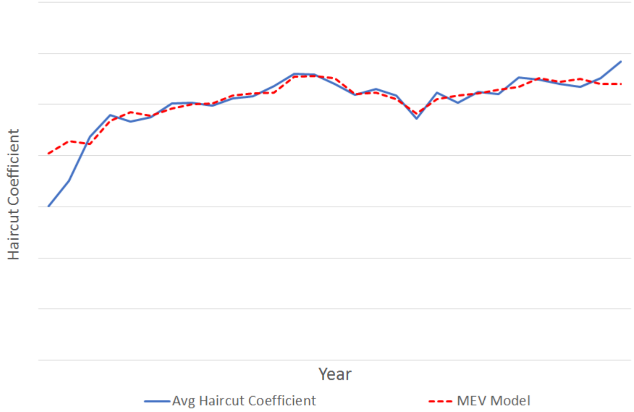
The following assumptions have been used in addition to the macroeconomic model to create a Downturn estimate:

- **HPI shock** - Historic movements were used to create a shock for use within the macroeconomic model
- **Realignment applied** – PiT realignment was used within the Downturn calibration.
- **Downturn view of Balance** – Downturn estimates of components of loss were used (i.e Downturn EAD used within Shortfall calculation).

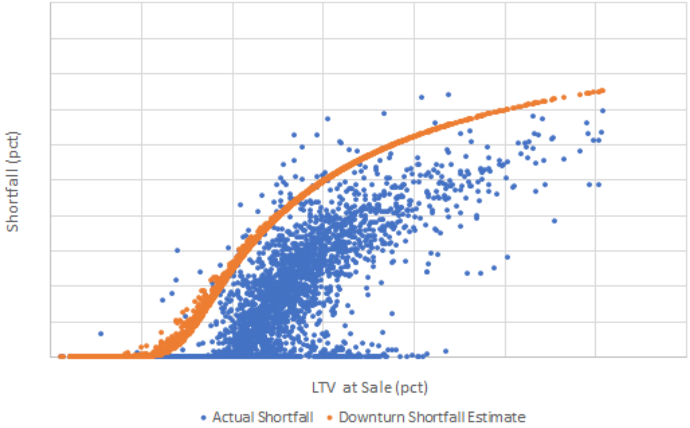
Regulation – SS11/13 - Internal Ratings Based (IRB) approaches

Article 13.8 - The PRA believes that an average reduction in property sales prices of 40% from their peak price, prior to the market downturn, forms an appropriate reference point when assessing downturn LGD for UK mortgage portfolios and expects a firm's rating systems to assume a reduction consistent with this. This reduction captures both a fall in the value of the property due to market value decline as well as a distressed forced sale discount. The PRA expects the assumption for the fall in the value of the property due to house price deflation not to be lower than 25%.

Macroeconomic Model



Shortfall Estimates



LGD – The Conclusion

Conclusion and Recap – Within regulatory constraints, the two major components of LGD have been modelled using alternative techniques

So, how have we innovatively modelled LGD?

- The calculation of PPGD was modelled using **quantile regression**, and **incorporated macroeconomic variables directly** when creating the Downturn estimate.
- Loss Given Possession (Shortfall) was modelled through use of a **statistical distribution** and **incorporated a macroeconomic model** to create a Downturn calibration.

Together, these two models combined:

- Create credible LGD estimates; but also
- Provide a sufficiently conservative view of Downturn performance; which
- Support compliance with regulatory requirements.

The methodology has been tested on Residential and Buy-to-Let secured portfolios, and has shown to be effective for both types of Secured lending.

Thank you for your time.

Any questions?

Appendix

Appendix I.a – Quantile Regression

Quantile regression is a linear modelling technique that models a specified percentile rather then the conditional mean (which is used for OLS modelling).

This method can naturally incorporate conservatism into the model component by selecting an quantile that tends to generate estimates that are larger than the mean.

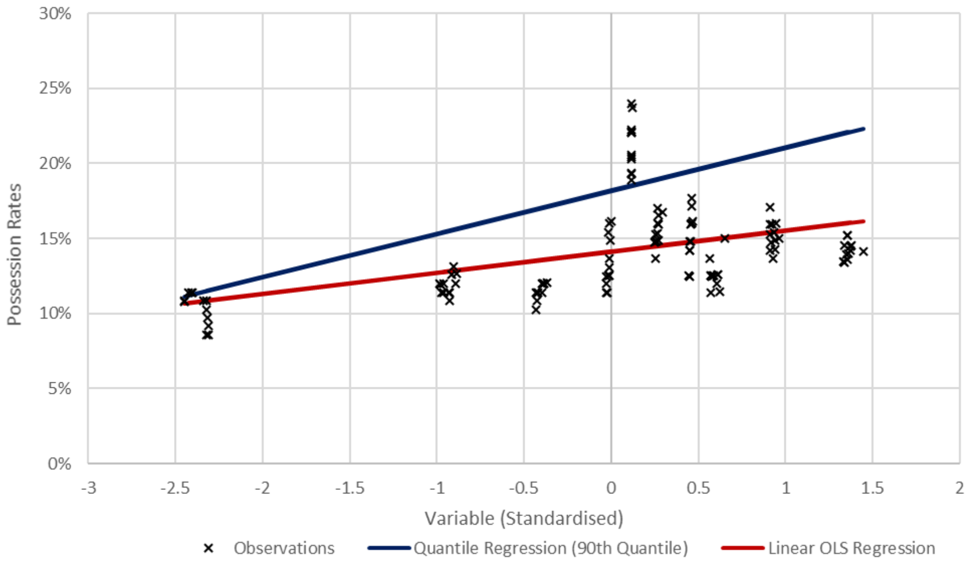
The optimisation function is:

$$Min\left[\sum_{y_i \geq \hat{y}_i} Q|y_i - \hat{y}_i| + \sum_{y_i < \hat{y}_i} (1-Q)|y_i - \hat{y}_i|\right]$$

where Q is the percentile modelled.

Illustrated on the right is a Quantile regression that is modelling the 90th Percentile. It can be observed that 90% of the observations are below the regression line

Difference in using Quantile and OLS Regression



Appendix II.a - Additional Shortfall Calculation Detail

Shortfall is reported as a £ amount but can be expressed as a % of the balance:

I. $$\frac{Shortfall}{Balance} = \frac{Balance}{Balance} - \frac{Sale\ Proceeds}{Balance}$$

The calculation of Shortfall as a % can therefore be defined as follows:

II. $$\text{Shortfall \%} = \mathbf{1} - \frac{Sale\ Proceeds}{Balance}$$

Where the sale proceeds are lower than the balance, a shortfall will occur

The Haircut Coefficient (HC) is multiplied by the Valuation to derive expected Sale Proceeds:

III. $$\text{Haircut Coefficient \%} = \frac{Sale\ Proceeds}{Valuation}$$

Where the sale proceeds are lower than the Valuation, a loss of value (or Forced Sale Discount) will occur

By dividing the right-hand terms in Equation II by Valuation, we can substitute in Equation III to make the calculation of Shortfall a function of Haircut and LTV.

IV. $$\text{Shortfall \%} = \mathbf{1} - \frac{Sale\ Proceeds}{Balance} = 1 - \frac{\frac{Sale\ Proceeds}{Valuation}}{\frac{Balance}{Valuation}}$$

Equation III defined this as Haircut Coefficient

Standard definition of LTV

Which simplified, demonstrates that Shortfall can be expressed as follows:

V. $$\text{Shortfall \%} = 1 - \frac{Haircut\ Coefficient\ \%}{LTV}$$

Note: Haircut Coefficient % can also be defined as:

$$\text{Haircut Coefficient \%} = 1 - Forced\ Sale\ Discount\ (\%)$$

Appendix II.b - Additional Shortfall Calculation Detail

Within a statistical distribution framework, Equation V can be split into possibility of loss and non-loss. However, only the first part is relevant for calculating Shortfall.

VI. Shortfall % $= \mathrm{P}(HC < LTV) * \left(1 - \frac{(Haircut\ Coefficient\ \% \mid HC<LTV)}{LTV}\right) + \boxed{\mathrm{P}(HC \geq LTV) * \left(1 - \frac{(Haircut\ Coefficient\ \% \mid HC\geq LTV)}{LTV}\right)}$

Shortfall would be negative for this outcome, and is therefore taken as 0.

The next step is to substitute the relevant components so that Shortfall can be calculated in the log-normal space.

VII. Probability that Haircut is less than LTV

$$\mathrm{P}(HC < LTV) = \Phi\left(\frac{\ln(k) - \mu}{\sigma}\right)$$

VIII. Conditional expectation of Haircut (or, Haircut when lower than LTV)

$$(Haircut\ Coefficient\ \% \mid HC < LTV) = e^{\mu+\frac{\sigma^2}{2}} \cdot \frac{\Phi\left(\frac{\ln(k) - \mu - \sigma^2}{\sigma}\right)}{\Phi\left(\frac{\ln(k) - \mu}{\sigma}\right)}$$

Substituting into Equation VI yields the calculation for Shortfall within a log-normal space:

IX. Shortfall % $=\Phi\left(\frac{\ln(k)-\mu}{\sigma}\right) * \left(1 - \frac{e^{\mu+\frac{\sigma^2}{2}} \cdot \frac{\Phi\left(\frac{\ln(k)-\mu-\sigma^2}{\sigma}\right)}{\Phi\left(\frac{\ln(k)-\mu}{\sigma}\right)}}{LTV}\right)$

Haircut Components – Considering normal and log-normal

Using the two models we previously developed, we need to use them as part of the shortfall calculation.

$E[X] = m$	**The "expected mean" (or arithmetic mean), given X**
$\sqrt{var[X]} = s$	**The expected (or arithmetic) standard deviation, given X**

These two components then feed into two equations, which yield the parameters for using the log-normal distribution.

I. $$\mu = \ln\left(\frac{\mathrm{E}[X]^2}{\sqrt{\mathrm{E}[X^2]}}\right) = \ln\left(\frac{\mathrm{E}[X]^2}{\sqrt{\mathrm{Var}[X] + \mathrm{E}[X]^2}}\right) = \ln\left(\frac{\boldsymbol{m^2}}{\sqrt{\boldsymbol{s^2+m^2}}}\right)$$

II. $$\sigma^2 = \ln\left(\frac{\mathrm{E}[X^2]}{\mathrm{E}[X]^2}\right) = \ln\left(1 + \frac{\mathrm{Var}[X]}{\mathrm{E}[X]^2}\right) = \ln\left(\boldsymbol{1} + \frac{\boldsymbol{s^2}}{\boldsymbol{m^2}}\right)$$

These will then be used as part of two different functions, the **conditional expectation** and the **log-normal probability**.

Conditional expectation

E[X]

III. $$E[X \mid X < k] = e^{\mu + \frac{\sigma^2}{2}} \cdot \frac{\Phi\left[\frac{\ln(k)-\mu-\sigma^2}{\sigma}\right]}{\Phi\left[\frac{\ln(k)-\mu}{\sigma}\right]}$$

CDF function,
where M=0, S=1

Log-normal probability

IV. $$P(X < k) = \Phi\left[\frac{\ln(k)-\mu}{\sigma}\right]$$